Common Diagonal Stability of Second Order Interval Systems

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Abstract: In this paper for second order interval systems we obtain necessary and sufficient conditions for the existence of a common diagonal solutions to the Lyapunov (Stein) inequality. Hurwitz and Schur cases are considered separately. One necessary and sufficient condition is given for $n \times n$ interval family of Z-matrices. The obtained results also give diagonal solution in the case of existence.

1 INTRODUCTION

Consider $n \times n$ real matrix $A = (a_{ij})$. If all eigenvalues of A lie in the open left half plane (open unit disc) A is said to be Hurwitz (Schur) stable. Necessary and sufficient condition for Hurwitz (Schur) stability is the existence of a symmetric positive definite matrix P (i.e. $P > 0$) such that

$$A^T P + PA < 0 \quad (A^T PA - P < 0) \tag{1}$$

where $B < 0$ means $-B > 0$. If in (1) the matrix P can be chosen to be positive diagonal then A is called Hurwitz (Schur) diagonally stable.

Diagonal stability problem have many applications (see (Arcat and Sontag, 2006; Johnson, 1974; Ziolko, 1990; Kaszkurewicz and Bhaya, 2000)).

The existence of diagonal type solutions are considered in the works (Khalil, 1982; Kaszkurewicz and Bhaya, 2000; Mason and Shorten, 2006; Pastravanu and Voicu, 2006; Dzhafarov and Büyükköroğlu, 2006; Büyükköroğlu, 2012) and references therein.

For a single 2×2 real matrix

$$A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \tag{2}$$

algebraic characterization of Hurwitz and Schur diagonal stability are available.

Fact 1.1 ((Cross, 1978; Kaszkurewicz and Bhaya, 2000)). *The matrix (2) is Hurwitz diagonally stable if and only if $a_1 < 0$, $a_4 < 0$ and $a_1 a_4 - a_2 a_3 > 0$.*

Fact 1.2 ((Mills et al., 1978; Kaszkurewicz and Bhaya, 2000)). *The matrix (2) is Schur diagonally stable if and only if $|a_1 a_4 - a_2 a_3| < 1$, $|a_1 + a_4| < 1 + a_1 a_4 - a_2 a_3$ and $|a_1 - a_4| < 1 - (a_1 a_4 - a_2 a_3)$.*

Consider an 2×2 interval family

$$\mathcal{A} = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} : a_i \in [a_i^-, a_i^+], i = 1,2,3,4 \right\}. \tag{3}$$

There are many important results for stability of interval systems (see for example (Kharitonov, 1978; Barmish, 1994; Deng et al., 1999)). Define the following 4-dimensional box

$$Q = [a_1^-, a_1^+] \times \cdots \times [a_4^-, a_4^+].$$

Without loss of generality all 2×2 positive diagonal matrices may be normalized to have the form $D = \operatorname{diag}(\lambda, 1)$ where $\lambda > 0$.

Definition 1.1. The family (3) is robust Hurwitz (Schur) diagonally stable if every matrix in (3) is Hurwitz (Schur) diagonally stable, i.e. for every $a = (a_1, a_2, a_3, a_4) \in Q$ there exist $\lambda > 0$ such that

$$A^T D + DA < 0 \quad (A^T DA - D < 0)$$

where

$$A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}, \ D = \begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix}.$$

Robust Hurwitz (Schur) diagonal stability of the family (3) can be easily tested due to multilinearity of the diagonal stability conditions (see Fact 1.1 and Fact 1.2).

Recall that a function $f : Q \to \mathbb{R}$ is said to be multilinear if it is affine-linear with respect to each component of $a \in Q$. The following theorem expresses the well-known property of a scalar multilinear function defined on a box.

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Theorem 1.1 ((Barmish, 1994, page 245)). *Suppose that Q is a box with extreme points a^i, $f : Q \to \mathbb{R}$ is multilinear. Then both the maximum and the minimum are attained at extreme points of Q. That is*

$$\max_{a\in Q} f(a) = \max_i f(a^i), \quad \min_{a\in Q} f(a) = \min_i f(a^i).$$

2 HURWITZ CASE

In this section for the family (3) we give a necessary and sufficient condition for the existence of a common diagonal solution to the Lyapunov inequality, i.e. the existence of $D = \mathrm{diag}(\lambda_*, 1)$ with $\lambda_* > 0$ such that

$$A^T D + DA < 0$$

for all $a \in Q = [a_1^-, a_1^+] \times \cdots \times [a_4^-, a_4^+]$. Necessary condition for the existence of a common diagonal solution is the robust diagonal stability.

Proposition 2.1. *The family (3) is robust Hurwitz diagonally stable if and only if*

$$a_1^+ < 0,\ a_4^+ < 0,\ \text{and}\ a_1^+ a_4^+ - \max\{a_2 a_3\} > 0 \quad (4)$$

where the maximum is calculated over extreme points a_2^-, a_2^+, a_3^- and a_3^+.

Proof. By Theorem 1.1 for every $a = (a_1, a_2, a_3, a_4) \in Q$

$$a_1 < 0,\ \ a_4 < 0,\ \ a_1 a_4 - a_2 a_3 > 0$$

or

$$\max a_1 < 0,\ \ \max a_4 < 0,\ \ \min\{a_1 a_4 - a_2 a_3\} > 0$$

or

$$a_1^+ < 0,\ \ a_4^+ < 0,\ \ \min\{a_1 a_4\} + \min\{-a_2 a_3\} > 0.$$

Obviously $\min\{a_1 \cdot a_4\} = a_1^+ \cdot a_4^+$, $\min\{-a_2 \cdot a_3\} = -\max\{a_2 \cdot a_3\}$ and (4) follows. The maximum in (4) is calculated over extreme points by Theorem 1.1. $\square$

Now we proceed to the necessary and sufficient condition for the existence of common diagonal solutions. Assume that the family (3) is robust Hurwitz diagonally stable, that is (4) is satisfied and we are looking for condition for the existence of common diagonal solution.

The existence of a common $D = \mathrm{diag}(\lambda_*, 1)$ ($\lambda_* > 0$) means that

$$A^T D + DA = \begin{bmatrix} 2a_1\lambda_* & a_2\lambda_* + a_3 \\ a_2\lambda_* + a_3 & 2a_4 \end{bmatrix} < 0$$

or

$$2a_1\lambda_* < 0, \quad 4a_1 a_4 \lambda_* > (a_2\lambda_* + a_3)^2 \quad (5)$$

for all $a = (a_1, a_2, a_3, a_4) \in Q$. The first condition of (5) is satisfied automatically since by (4), $a_1^+ < 0$. The second condition is equivalent to the following

$$\min_{(a_1,a_4)} (4a_1 a_4)\lambda_* > \max_{(a_2,a_3)} (a_2\lambda_* + a_3)^2$$

or

$$(4a_1^+ a_4^+)\lambda_* > \max\left\{(a_2^-\lambda_* + a_3^-)^2, (a_2^+\lambda_* + a_3^+)^2\right\}$$

or

$$(4a_1^+ a_4^+)\lambda_* > (a_2^-\lambda_* + a_3^-)^2,$$
$$(4a_1^+ a_4^+)\lambda_* > (a_2^+\lambda_* + a_3^+)^2$$

or

$$\begin{aligned} &(a_2^-)^2\lambda_*^2 + (2a_2^- a_3^- - 4a_1^+ a_4^+)\lambda_* + (a_3^-)^2 < 0, \\ &(a_2^+)^2\lambda_*^2 + (2a_2^+ a_3^+ - 4a_1^+ a_4^+)\lambda_* + (a_3^+)^2 < 0. \end{aligned} \quad (6)$$

Consider the function

$$f(x) = (a_2^-)^2 x^2 + (2a_2^- a_3^- - 4a_1^+ a_4^+)x + (a_3^-)^2 \quad (x \geq 0)$$

which corresponds to the first condition in (6). Since $f(0) \geq 0$ and the family (3) is robust Hurwitz diagonally stable there exists a positive solution interval (α_1, α_2) of the inequality $f(x) < 0$. For example, if $a_2^- \neq 0$ then

$$\alpha_1 = \frac{(2a_1^+ a_4^+ - a_2^- a_3^-) - \sqrt{(a_2^- a_3^- - 2a_1^+ a_4^+)^2 - (a_2^- a_3^-)^2}}{(a_2^-)^2}$$
$$\alpha_2 = \frac{(2a_1^+ a_4^+ - a_2^- a_3^-) + \sqrt{(a_2^- a_3^- - 2a_1^+ a_4^+)^2 - (a_2^- a_3^-)^2}}{(a_2^-)^2}$$

(the discriminant $\Delta = (a_2^- a_3^- - 2a_1^+ a_4^+)^2 - (a_2^- a_3^-)^2$ is positive by the robust Hurwitz diagonal stability of (3)).

Analogously the exists an open interval (β_1, β_2) which corresponds to the second condition in (6). If $a_2^+ \neq 0$ then

$$\beta_1 = \frac{(2a_1^+ a_4^+ - a_2^+ a_3^+) - \sqrt{(a_2^+ a_3^+ - 2a_1^+ a_4^+)^2 - (a_2^+ a_3^+)^2}}{(a_2^+)^2}$$
$$\beta_2 = \frac{(2a_1^+ a_4^+ - a_2^+ a_3^+) + \sqrt{(a_2^+ a_3^+ - 2a_1^+ a_4^+)^2 - (a_2^+ a_3^+)^2}}{(a_2^+)^2}$$

(the discriminant is positive).

Now we give the main result of this section.

Theorem 2.2. *Assume that the family (3) is robust Hurwitz diagonally stable. There exists a common diagonal solution to the Lyapunov inequality if and only if the intervals (α_1, α_2) and (β_1, β_2) have nonempty intersection, i.e.*

$$\max\{\alpha_1, \beta_1\} < \min\{\alpha_2, \beta_2\}.$$

In this case for every $\lambda \in (\alpha_1, \alpha_2) \cap (\beta_1, \beta_2)$ the matrix $D = \mathrm{diag}(\lambda, 1)$ is a common solution.

Example 2.1. Consider the family

$$\begin{bmatrix} [-3,-2] & [1,2] \\ [-5,-4] & -1 \end{bmatrix}. \tag{7}$$

Is there a common diagonal solution $D = \mathrm{diag}(\lambda, 1)$? The family (7) is robust Hurwitz diagonally stable by Proposition 2.1, since $a_1^+ = -2 < 0$, $a_4^+ = -1 < 0$, $a_1^+ a_4^+ - \max\{a_2 a_3\} = (-2)\cdot(-1) - (-4) = 6 > 0$. Corresponding to (6) inequalities are

$$x^2 - 18x + 25 < 0, \quad 4x^2 - 24x + 16 < 0$$

and $\alpha_1 = 9 - 2\sqrt{14}$, $\alpha_2 = 9 + 2\sqrt{14}$, $\beta_1 = 3 - \sqrt{5}$ and $\beta_2 = 3 + \sqrt{5}$. $(\alpha_1, \alpha_2) \cap (\beta_1, \beta_2) = (9 - 2\sqrt{14}, 3 + \sqrt{5})$. For every $\lambda \in (9 - 2\sqrt{14}, 3 + \sqrt{5})$ the matrix $D = \mathrm{diag}(\lambda, 1)$ is a common diagonal solution.

Example 2.2. Consider the family

$$\begin{bmatrix} [-2,-1] & [0,1] \\ -1 & [-3,-2] \end{bmatrix}.$$

The family is robust Hurwitz diagonally stable by Proposition 2.1, since $a_1^+ = -1 < 0$, $a_4^+ = -2 < 0$, $a_1^+ a_4^+ - \max\{a_2 a_3\} = (-1)\cdot(-2) - 0 = 2 > 0$. Inequalities, corresponding to (6) are $-8x + 1 < 0$ and $x^2 - 10x + 1 < 0$ with common solution interval $(1/8, 5 + \sqrt{24})$, every λ from this interval gives common diagonal solution.

3 SCHUR CASE

Here we give a necessary and sufficient condition for the existence of common diagonal solution in the Schur case, i.e. the existence of $D = \mathrm{diag}(\lambda_*, 1)$ with $\lambda_* > 0$ such that

$$A^T D A - D < 0$$

for all $a \in Q = [a_1^-, a_1^+] \times \cdots \times [a_4^-, a_4^+]$.

In order to have a common diagonal solution a family must be robust diagonally stable. From Fact 1.2 we obtain

Proposition 3.1. *The family (3) is robust Schur diagonally stable, i.e. every member is Schur diagonally stable if and only if the following six conditions are satisfied*

$$\begin{array}{c} 1 + a_2 a_3 - a_1 a_4 > 0, \\ 1 + a_1 a_4 - a_2 a_3 > 0, \\ 1 + a_1 a_4 - a_1 - a_4 - a_2 a_3 > 0, \\ 1 + a_1 + a_4 + a_1 a_4 - a_2 a_3 > 0, \\ 1 + a_4 + a_2 a_3 - a_1 - a_1 a_4 > 0, \\ 1 + a_1 + a_2 a_3 - a_4 - a_1 a_4 > 0 \end{array} \tag{8}$$

for all $(a_1, a_2, a_3, a_4) \in Q$.

These conditions can be easily checked through the extremal points of Q by using multilinearity of the left-hand sides of (8) and Theorem 1.1.

Assume that the family (3) is robust Schur diagonally stable. Again, the existence of a common $D = \mathrm{diag}(\lambda_*, 1)$ $(\lambda_* > 0)$ means that

$$A^T D A - D = \begin{bmatrix} \lambda_*(a_1^2 - 1) + a_3^2 & \lambda_* a_1 a_2 + a_3 a_4 \\ \lambda_* a_1 a_2 + a_3 a_4 & \lambda_* a_2^2 + a_4^2 - 1 \end{bmatrix} < 0$$

or

$$\begin{array}{r} \lambda_*(a_1^2 - 1) + a_3^2 < 0, \\ \left[\lambda_*(a_1^2 - 1) + a_3^2\right]\left[\lambda_* a_2^2 + a_4^2 - 1\right] - \\ (\lambda_* a_1 a_2 + a_3 a_4)^2 > 0 \end{array} \tag{9}$$

for all $(a_1, a_2, a_3, a_4) \in Q$. From the robust Schur diagonal stability it follows that $|a_1| < 1$ ((Kaszkurewicz and Bhaya, 2000, page 78)). Therefore the first condition of (9) gives $\lambda_* \cdot \min(1 - a_1^2) > \max(a_3^2)$, which in turn gives $\lambda_* > \alpha = (\max a_3^2)/(1 - \max(a_1^2))$. The second condition gives

$$(a_2^2)\lambda_*^2 - \left[a_3^2 a_2^2 + (a_4^2 - 1)(a_1^2 - 1) - 2a_1 a_2 a_3 a_4\right]\lambda_* + a_3^2 < 0.$$

Consider the function

$$g(x) = (a_2^2)x^2 - \left[a_3^2 a_2^2 + (a_4^2 - 1)(a_1^2 - 1) - 2a_1 a_2 a_3 a_4\right]x + a_3^2 \quad (x \geq 0).$$

To avoid a division by zero, without loss of generality assume that $0 \notin [a_2^-, a_2^+]$. From $g(0) \geq 0$ and robust Schur stability of (3) it follows that there exist positive, continuous root functions $r_1(a_1, a_2, a_3, a_4)$ and $r_2(a_1, a_2, a_3, a_4)$ such that the inequality $g(x) < 0$ is satisfied for all $x \in (r_1, r_2)$. The continuous functions r_i $(i = 1, 2)$ can be written explicitly by using the discriminant which is positive by the robust Schur diagonal stability.

Finally, we arrive at the main result of this section.

Theorem 3.2. *Assume that the family (3) is given and* $0 \notin [a_2^-, a_2^+]$. *Let (3) be robust Schur diagonally stable. There exits a common Schur diagonal solution if and only if the following two conditions are satisfied:*

i)

$$\gamma_1 := \max_{(a_1,a_2,a_3,a_4)} r_1(a_1, a_2, a_3, a_4) < \gamma_2 := \min_{(a_1,a_2,a_3,a_4)} r_2(a_1, a_2, a_3, a_4)$$

ii) $(\alpha, \infty) \cap (\gamma_1, \gamma_2) \neq \emptyset$

In this case for every $\lambda \in (\alpha,\infty)\cap(\gamma_1,\gamma_2)$ the matrix $D = \mathrm{diag}(\lambda,1)$ is a common solution to the Stein inequality.

Example 3.1. Consider the following interval family

$$\begin{bmatrix} \left[0,\frac{1}{2}\right] & \left[\frac{1}{3},\frac{1}{2}\right] \\ \left[-\frac{1}{10},\frac{1}{10}\right] & \frac{1}{2} \end{bmatrix}.$$

This family is robust Schur diagonally stable by Proposition 3.1 and Theorem 1.1:

$\min_{a\in Q}(1+a_2a_3-a_1a_4) = 0.7$,
$\min_{a\in Q}(1+a_1a_4-a_2a_3) = 0.95$,
$\min_{a\in Q}(1+a_1a_4-a_1-a_4-a_2a_3) = 0.2$,
$\min_{a\in Q}(1+a_1+a_4+a_1a_4-a_2a_3) = 1.45$,
$\min_{a\in Q}(1+a_4+a_2a_3-a_1-a_1a_4) = 0.7$,
$\min_{a\in Q}(1+a_1+a_2a_3-a_4-a_1a_4) = 0.45$

where $Q = \left[0,\frac{1}{2}\right]\times\left[\frac{1}{3},\frac{1}{2}\right]\times\left[-\frac{1}{10},\frac{1}{10}\right]\times\frac{1}{2}$. The maximization of the left root function $r_1(a_1,a_2,a_3,a_4)$ of $g(x)$ over Q gives $\gamma_1 = 0.019$, and the minimization of the rgiht root function $r_2(a_1,a_2,a_3,a_4)$ over Q gives $\gamma_2 = 2.141$. Since $\alpha = 0.0134$, for every $\lambda \in (0.019, 2.141)$ the matrix $D = \mathrm{diag}(\lambda,1)$ is a common diagonal solution.

4 *Z*-MATRICES

In this section for real $n\times n$ interval Z-matrices we give necessary and sufficient condition for the existence of a common diagonal solution to the Lyapunov inequality.

A real $n\times n$ matrix $A = (a_{ij})$ is said to be Z-matrix if $a_{ij} \geq 0$ for all $i \neq j$. The following properties of Z-matrices are well-known (see (Horn and Johnson, 1991)):

Assume that A is a Z-matrix. Then:

1) If A is Hurwitz stable then it is diagonally stable
2) The spectral abscissa $\rho(A) = \max_i \mathrm{Re}\lambda_i(A)$, i.e. the maximum of the real parts of eigenvalues of A is a real eigenvalue of A. Therefore A is Hurwitz stable if and only if every real eigenvalue is negative.

Assume that the following interval Z-matrix family is given

$$\mathcal{A} = \left\{(a_{ij}): \quad a_{ij}\in[a_{ij}^-, a_{ij}^+]\right\} \tag{10}$$

with $a_{ij}^- \geq 0$ for all $i \neq j$. Denote by U the right-end matrix of the family $\mathcal{A}$: $U = (a_{ij}^+)$.

Theorem 4.1. *Let the family $\mathcal{A}$ (10) be given. There exists a common diagonal solution to the Lyapunov inequality if and only if the matrix U is Hurwitz stable (equivalently Hurwitz diagonally stable).*

Proof. The implication $\Rightarrow$ follows from the inclusion $U \in \mathcal{A}$.
$\Leftarrow)$: Since U is Hurwitz diagonally stable there exists a positive diagonal D_* such that

$$U^T D_* + D_* U < 0 \tag{11}$$

Take an arbitrary $A\in\mathcal{A}$. Then

$$A^T D_* + D_* A \leq U^T D_* + D_* U$$

where the symbol "$\leq$" means componentwise inequality. The matrices $A^T D_* + D_* A$ and $U^T D_* + D_* U$ are Z-matrices. Take sufficiently large $\alpha > 0$ such that componentwisely

$$A^T D_* + D_* A + \alpha I \geq 0, \quad U^T D_* + D_* U + \alpha I \geq 0.$$

Then from

$$A^T D_* + D_* A + \alpha I \leq U^T D_* + D_* U + \alpha I$$

and (Bernstein, 2005, page 160, Fact 4.11.7) it follows that

$$\sigma(A^T D_* + D_* A + \alpha I) \leq \sigma(U^T D_* + D_* U + \alpha I)$$

where $\sigma(\cdot)$ denote the spectral radius. For a nonnegative (componentwise) matrix B, $\sigma(B) = \lambda_{\max}(B)$ where $\lambda_{\max}(B)$ denotes the greatest real eigenvalue of B. Therefore

$$\lambda_{\max}(A^T D_* + D_* A + \alpha I) \leq \lambda_{\max}(U^T D_* + D_* U + \alpha I). \tag{12}$$

On the other hand for a symmetric matrix C

$$\lambda_{\max}(C+\alpha I) = \lambda_{\max}(C) + \alpha,$$

therefore from (11) and (12) we obtain

$$\lambda_{\max}(A^T D_* + D_* A) \leq \lambda_{\max}(U^T D_* + D_* U) < 0.$$

Since $A\in\mathcal{A}$ is arbitrary, the last inequality means that the family $\mathcal{A}$ has a common diagonal solution D_* to the Lyapunov inequality (recall that D_* is a positive diagonal solution of (11)). $\square$

The matrix D_* can be evaluated by different ways: by direct solution of (11), or by the linear matrix inequality (LMI) techniques, or by the algorithm from (Khalil, 1982), or by the Perron-Frobenius theory.

Example 4.1. Consider the following interval Z-matrix family

$$\mathcal{A} = \begin{bmatrix} [-8,-6] & [1,5] & [1,2] \\ [2,3] & [-9,-8] & [1,3] \\ [2,4] & [2,5] & [-9,-8] \end{bmatrix}.$$

Here

$$U = \begin{bmatrix} -6 & 5 & 2 \\ 3 & -8 & 3 \\ 4 & 5 & -8 \end{bmatrix}$$

which is Hurwitz stable. The matrix $D_* = (0.249, 0.308, 0.183)$ evaluated by LMI technique is solution of (11) and a common solution for the family $\mathcal{A}$.

5 CONCLUSIONS

In this paper we consider common diagonal Lyapunov function problem for interval systems. For second order interval systems we obtain necessary and sufficient conditions for the existence of common diagonal solutions to Lyapunov and Stein inequalities. Necessary and sufficient condition is given for $n \times n$ interval Z-matrix family. The obtained results also give diagonal solutions in the case of existence.

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