Modeling and $H_{\infty}$ Composite Control of the Coupled Hysteretic Dynamics in Piezoelectric Micro-displacement Systems

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Abstract: This paper investigates the modeling and $H_{\infty}$ composite control of the coupled hysteretic dynamics in a piezoelectric micro-displacement system (PMS). First, the coupled multi-field hysteretic dynamics with physical meanings is presented for PMS. Next, the composite control analysis of the hysteretic dynamics is proposed. Then, a $H_{\infty}$ synthesis controller is designed by using the simplified hysteretic dynamics. To enhance the $H_{\infty}$ performance, the inversion-based feedforward compensation is augmented. The proposed $H_{\infty}$ feedback control and the inversion-based feedforward can be designed separately. Finally, the experimental studies are provided to demonstrate the proposed $H_{\infty}$ composite control approach.

1 INTRODUCTION

Piezoelectric micro-displacement systems (PMSs) are widely investigated to suppress jitters and micro vibrations produced by reaction flywheels, control momentum gyroscopes, Stirling coolers and step motors of precision spacecrafts, such as inter-satellite laser communication, space telescope and missile warning satellite with staring camera (Kamesha and Ghosalb, 2010; Maillarda and LeLettya, 2009; Nagashima and Agrawal, 2014; Dewella and Blaurockb, 2005; Maillarda and LeLettya, 2009; Laneand and Lacy, 2008; Neat and Goullioud, 1998; McMickell and Hansen, 2007). For instance, piezoelectric fast steering mirrors are employed to suppress jitters of line-of-sight (LOS) in inter-satellite laser communication and space telescopes. To reject jitters and micro vibrations, broadband control of PMS is increasingly appealing, but most of the operating bandwidth of PMS is still insufficient.

To enhance the bandwidth and performance of PMS, various controllers were designed (Devassa and Moheimani, 2007). If tracking signals are at low frequencies, proportional-integral-derivative (PID) and notch filter are adequate (Fleming, 2010). As the reference signal frequency increases, model based controllers are alternatively designed, such as $H_{\infty}$ feedback control (Wu and Zou, 2009), inversion-based feedforward control (Liu and Lee, 2013b; Tan and Ang, 2009; Liu and Lee, 2013a). Accurate modeling over a broad frequency range is necessary to enhance the performance of model-based control.

At broadband frequencies, the hysteretic dynamics of PMS has multi-field effects. It is required to model the coupled hysteresis, creep, electric and vibration dynamics. Hysteresis is a strongly nonlinear element with global memory (Brokate and Sprekels, 1996). Preisach model is typically to describe the static hysteresis (Mayergozy, 2003). Creep is slow dynamics and can be represented by spring-damping model (Devasia and Moheimani, 2007).

In this paper, the multi-field dynamics with physical meanings is developed for PMS. The non-hysteretic creep model is used. Electrical and vibration dynamics of PMS are fast dynamics and can be represented using transfer functions. In PMS, the time constant of electric dynamics is in order of $0.002$ seconds, and the first resonance frequency of the vibration dynamics is generally in the order of $1kHz$. To represent the PMS at broadband frequencies, this paper employs a cascade connection of static and dynamic components. The static hysteresis is represented using classical Preisach model. The non-hysteretic creep, electric and vibration dynamics are represented using transfer functions.

To compensate PMS dynamics at broadband frequencies, various modern controllers were investigated. Clayton reviewed feedforward approaches
which were mainly based on linear dynamical models (Clayton and Devisia, 2009). Wu presented a 2-DOF feedforward-feedback controller (Wu and Zou, 2009). Leaning also proposed a notch filter and an inversion-based feedforward controller to enhance the high-gain feedback (Leaning and Devisia, 2007). Intelligent feedback controllers were also investigated. Liaw used neural network to enhance the motion tracking of piezo-based flexible mechanisms (Liaw and Shirinzadeh, 2009). Shieh and Hsu investigates the adaptive control (Shieh and Hsu, 2008). Additionally, dynamic hysteresis models were investigated to achieve high bandwidth tracking (Jiang and Chen, 2011; Janaiden and Rakheja, 2008). Based on rate-dependent Prandtl-Ishlinskii (P-I) hysteresis, Tan proposed the hysteresis-based inversion to extend the tracking bandwidth (Tan and Ang, 2009), but it is difficult to design modern control techniques using rate-dependent hysteresis. Alternatively, most modern controllers are designed using non-hysteretic models.

In this paper, the $H_{22}$ composite control is designed using the proposed hysteretic dynamics of PMS. The proposed composite controller comprises an separate $H_{oo}$ feedback controller and an inversion-based feedforward controller. More accurate tracking is thus presented at high frequencies.

This paper is organized as follows. First, Section 2 presents the modeling of the coupled hysteretic dynamics with physical meanings. Next, Section 3 provides the analysis of the composite control strategy of the hysteretic dynamics. Then, the $H_{oo}$ composite control strategy is developed in Section 4. To validate the proposed modeling and control approaches, the experimental studies are provided in Section 5. Finally, Section 6 makes a conclusion of this paper.

## 2 COUPLED HYSERETIC DYNAMICS IN PMS

In this section, the multi-field modeling of the hysteretic dynamics in PMS is presented. The hysteretic dynamics model is derived from the material, electrical and mechanical fields. The complete model of the hysteretic dynamics consists of the static Preisach hysteresis effect, creep effect, and electrical and vibration dynamics.

Fig. 1 shows the complete hysteretic model structure of PMS. The hysteretic model is derived as follows. First, the electrical model of the voltage amplifier is presented. Next, the hysteresis effect due to the lead zirconate titanate (PZT) stack is proposed using the classical Preisach model. Additionally, the creep effect is presented using a transfer function. Then, the electrical model of PZT stack is proposed. Moreover, the mechanical vibration dynamics is derived using stiff and damping parameters. Finally, the characteristics of the hysteretic dynamics are proposed.

![Model structure of PMS](image)

**Figure 1:** Model structure of PMS ($\theta_0$ denotes the input voltage of the voltage amplifier, $u$ denotes the output voltage of the voltage amplifier, $u_p$ denotes the voltage of the PZT stack, $f$ denotes the actuating force due to the inverse piezoelectric effect, $x$ denotes the displacement of PMS, and $y$ denotes the displaced displacement of PMS).

### 2.1 Electrical Model of the Voltage Amplifier

The power and bandwidth of voltage amplifiers are limited. As the input frequency increases, the current reduces and the phase delay increases. To describe this dynamic response, the deduced electrical dynamics of the voltage amplifier is presented.

Fig. 2 shows the sketch of deduced RLC electrical dynamics of the voltage amplifier where the amplifying factor is not presented. $R$, $L$, and $C$ represent the resistance, inductance, and capacitance of the voltage amplifier, respectively. $\theta_0$ and $u$ represents the input and output voltage, respectively.

\[
\frac{U(s)}{\theta_0(s)} = \frac{1}{s^2 + \frac{1}{L C} s + \frac{R}{L}}.
\]

Let $\omega_n = 1/\sqrt{LC}$ and $\xi_n = 1/(2 \sqrt{L C})$, equation (1) can be rewritten as

\[
\frac{U(s)}{\theta_0(s)} = \frac{\omega_n^2}{s^2 + 2\xi_n \omega_n s + \omega_n^2}.
\]

### 2.2 Preisach Hysteresis Model

The hysteresis effect of PZT material (stack) are described using Preisach model. Fig. 3 shows the hysteresis effect and RC electrical dynamics in the PZT stack. $\Gamma$ represents the hysteresis effect. $R$ and $C$ represent the resistance and capacitance of the PZT stack, respectively. $T_{em}$ represents the electromechanical transformer ratio of the PZT material. $i$ is the conductor current, $u$ is the input voltage of the PZT stack, $u_p$ is the effective voltage for the PZT stack.

The hysteresis between the input voltage and the effective PZT voltage can be represented as the following Preisach model (Mayergozy, 2003):
Figure 2: Electrical dynamics of the voltage amplifier.

\[ u_p = \Gamma(u) = \int \mu(\alpha, \beta) \gamma_{\alpha \beta}[u(t)] d\alpha d\beta, \]  
(3)

where \( \mu(\alpha, \beta) \) and \( \gamma_{\alpha \beta} \) are the density function and hysteron output of point \( \alpha, \beta \) on the Preisach plane, respectively. The Preisach model is rate-independent, i.e. it is a static model.

Fig. 4 shows the Preisach plane. The shadowing area \( S^+ \) is activated with the \( \gamma_{\alpha \beta} \) of one. The blank area \( S^- \) is unactivated with the \( \gamma_{\alpha \beta} \) of zero.

2.3 RC Electrical Model of the PZT Stack

In the electrical field of the PZT stack, the voltage drop \( u_p \) is represented by

\[ u_p = iR + u_c, \]  
(4)

where \( R \) is the resistance and \( i \) is the current. \( u_c \) is the voltage of the equivalent capacitor \( C \) of the PZT stack. \( u_c \) can be represented by

\[ u_c = QC, \]  
(5)

where \( Q \) is the charge.

Figure 3: Hysteresis effect and electrical dynamics in the PZT stack.

Additionally, the conduction current \( i \) is represented by

\[ i = \frac{dQ}{dt}, \]  
(6)

By combining equations (4), (5) and (6), the electrical dynamics is written as

\[ Q(s) U_p(s) = C \left( 1 + \tau s \right). \]  
(7)

where \( s \) is the Laplace operator \( \tau = RC \).

In summary, the electrical dynamics in PMS consists of the electrical dynamics of the voltage amplifier and the electrical dynamics of the PZT material. By combining equations (2) and (7), the electrical model of PMS is given by

\[ G_e(s) = \frac{C}{1 + \tau s} \left( \frac{\omega_n^2}{s^2 + 2\xi_n\omega_n s + \omega_n^2} \right). \]  
(8)

The force \( F \) due to the inverse piezoelectricity effect of the PZT stack is written as

\[ F = T_{em} Q, \]  
(9)

where \( T_{em} \) the electromechanical transformer ratio due to the inverse piezoelectric effect.

2.4 Mechanical Vibration Dynamics

A typical mechanical strut with motion amplification is considered in this paper, as shown in Fig. 5. The proposed PMS can be used to compensate jitters and micro vibrations of spacecrafts. The stiff and damping of the PZT stack as well as the passive isolator and flexible joints are contained in the mechanical vibration dynamics. Rubber cushion between the working platform and the base can be used to introduce damping for passive isolation.

The mechanical motions of PMS are driven by the
force $F$ due to the inverse piezoelectric effect. Fig. 5 shows the mechanical vibration dynamics. $K_p$ and $C_p$ represent the stiff and damping of the PZT stack, respectively. $K_f$ and $C_f$ represent the stiff and damping of the passive isolation and the flexure guide. $x$ represents the displacement of the arm tip which is generally attached to a working platform. $\theta$ represents the tilt angle due to the piezo displacement. $L$ represents the length of the motion amplifying arm. $N$ represents the amplifying value of the motion amplifying arm.

For PMS, only micro displacement is provided. For instance, the maximum displacement of a typical PMS is 50$\mu$m. Compared with its arm length $L = 10$cm, $\tan \theta = x/L < 0.0005$. Thus, $\theta \approx \tan \theta$. $x = L\theta$. According to Newton’s law, the dynamics of $M$ can be written as

$$J\ddot{x} + K_p xL + C_f xL + K_p xL + C_p xL = FL / N$$

where $J = ML^2$ and $x_p = x/N$.

Equation (10) can be written as

$$M\ddot{x} + (C_f + C_p / N^2)x + (K_f + K_p / N^2)x = F / N.$$  

Then,

$$G_v(s) = k_v \frac{s^2 + 2\omega_n s + \omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}.$$  

where $G_v(s) = X(s) / Q(s)$, $\omega_n = \sqrt{(C_f + C_p / N^2) / M}$, $2\omega_n^2 = (K_f + K_p / N^2) / M$, $K_v = T_{em} / (K_f + K_p / N^2)$

Finally, there exists the creep effect (also named drift) in PZT material. The creep effect can be represented by (Devasia and Moheimani, 2007)

$$G_c(s) = k_c \prod_{i=1}^{m} \frac{s + z_{ci}}{s + p_{ci}}$$

where $k_c$ is the creep gain when $s$ goes to infinity, i.e. $k_c$ represents the creep gain at infinite frequencies. $m$ is the creep order. $p_{ci}$ and $z_{ci}$ are the poles and zeros of the creep dynamics, respectively.

### 2.5 Multi-field Hysteretic Dynamics

The multi-field hysteretic dynamics of PMS can be divided into the static hysteresis and the non-hysteretic dynamics. In this paper, the static hysteresis is represented by classical Preisach model. The non-hysteretic dynamics comprises of the creep, electrical and vibration dynamics.

By combining equations (8) and (12), the electric and mechanical dynamics can be presented as

$$G_{ev}(s) = \frac{k_{ev}}{1 + 2\xi s^2 + 2\omega_{n1}s + \omega_{n1}^2 / s^2 + 2\omega_{n2}s + \omega_{n2}^2}$$

where $k_{ev} = k_c C$.

Fig. 6 shows the model sketch of the multi-field hysteretic dynamics in PMS. The cascade connection is used to represent the relationship among the components of the hysteretic dynamics. The hysteresis effect and creep effect are built in the material field. The electrical dynamics is built in the electrical field. The vibration dynamics is built in the mechanical field. The non-hysteretic dynamics $G$ can be summarized as $G = G_e G_v G_c$.

### 2.6 Characteristics of the Coupled Hysteretic Dynamics

In this section, the responses of typical PMS are proposed. Fig. 7 shows the response of the static Preisach hysteresis under sinusoidal inputs. Compared with phase delay in linear dynamics, the Preisach hysteresis achieve its peak value simultaneously with the input signal, i.e., there is not delay at the peak point. Moreover, the Preisach output is not differential at the peak point.

Fig. 8 shows the response of the creep, electrical and vibration dynamics under square inputs. It can be seen that the electrical and vibration dynamics behaves fast, but the creep dynamics behaves slow. Moreover, the creep, electrical and vibration dynamics are coupled.

Fig. 9 shows the response of the hysteresis, creep, electrical and vibration dynamics under slow sinusoidal inputs with varying amplitudes. As the input continues, the drift due to the creep and low frequency electrical and vibration dynamics are obvious.
Figure 7: Response of the Preisach hysteresis under sinusoidal inputs.

Figure 8: Response of the creep, electrical and vibration dynamics.

Figure 9: Response of creep effect under the sinusoidal input with varying amplitudes.

3 COMPOSITE CONTROL ANALYSIS

The proposed composite controller is analyzed in this section. First, using the reference signal \( r \), the model-based inversion feedforward controller \( K_{FF} \) of PMS can be written as

\[
K_{FF} = G^{-1} \hat{\Gamma}^{-1}(r),
\]

(15)

where the hysteresis estimation \( \hat{\Gamma} \) is strong nonlinearities with global memories (Mayergozy, 2003). \( \hat{\Gamma}(r) \) is computed using Preisach model in equation (3).

\[
G^{-1} \text{ and } \Gamma^{-1} \text{ can be represented as}
\]

\[
\begin{cases}
G^{-1} = G^{-1}(1 + \delta_t) \\
\Gamma^{-1} = \Gamma^{-1}(1 + \delta_h)
\end{cases},
\]

where \( \delta_t \) denotes the inversion error of the non-hysteretic dynamics and \( \delta_h \) denotes the inversion error of the rate-independent hysteresis. \( \delta_t \) and \( \delta_h \) are bounded uncertainties and determined by the identification accuracy of PMS. Then, \( \hat{\Gamma}^{-1}(r) \) and \( \hat{\Gamma}^{-1} \) can be rewritten as

\[
G^{-1} \hat{\Gamma}^{-1}(r) = (1 + \delta_t + \delta_h + \delta_h \delta_t) G^{-1} \Gamma^{-1}(r). 
\]

(16)

Let \( \delta = \delta_t + \delta_h + \delta_h \delta_t \), the model-based inversion feedforward controller of PMS is rewritten as

\[
K_{FF} = (1 + \delta) G^{-1} \Gamma^{-1}(r). 
\]

(17)

With only the feedforward controller \( K_{FF} \) in (17), the relative error in \( e/r \) is given by

\[
\frac{e}{r} |_{K_{FB}=0} = \delta + \frac{d}{r}. 
\]

(18)

Equation (18) indicates that the tracking performance of feedforward relies on the identification accuracy and the output disturbances are not suppressed. Thus, feedback control is necessary to guarantee the stability and robustness under modeling error \( \delta \) and disturbance \( d \).

Fig. 10 shows the proposed composite control strategy where \( G^{-1} \) and \( \Gamma^{-1} \) are represented by \( G^{-1} \) and \( \Gamma^{-1} \) according to (17), respectively. With the proposed composite control, the relationship between the reference \( r \) and PMS displacement output \( y \) is written as

\[
\frac{y}{r} = 1 + \frac{1}{G \Gamma(u) K_{FB} + 1} \delta + \frac{G \Gamma(u) K_{FB} n}{G \Gamma(u) K_{FB} + 1} \frac{n}{r} + \frac{1}{G \Gamma(u) K_{FB} + 1} \frac{d}{r}, 
\]

(19)

where \( K_{FB} \) denotes the feedback controller, \( n \) and \( d \) are the measurement noise and output disturbance, respectively. \( \Gamma(u) \) is computed using Preisach model in equation (3).
4 \( H_\infty \) COMPOSITE CONTROL

The proposed \( H_\infty \) composite control consists of a \( H_\infty \) feedback controller and an inversion-based feedforward controller.

4.1 \( H_\infty \) Controller Design

The loop shaping is employed to design the feedback controller as shown in Fig. 11. The performance and stability requirements are satisfied by specifying \( L_1 \) and \( L_2 \). \( \omega_s \) is the cross frequency of \( GK_{FB} \) and is related and close to feedback bandwidth, \( \omega_p \) is related to disturbance rejection performance, and \( \omega_n \) is related to the robust stability under modeling errors. Weighting functions are suitable for specifying different requirements at different frequencies as shown in Fig. 11. It is convenient to achieve multi objectives using weighting functions (Skogestad and Postlethwaite, 2005). The robust \( H_\infty \) controller is designed based on the non-hysteresis dynamics, while the rate-independent hysteresis \( \Gamma \) can be regarded as an input uncertainty consisting of the nominal gain \( k_h \) and the weighting function \( w_u \).

Fig. 12 shows the sketch of multi-objective robust \( H_\infty \) control. \( w_1 \) is the performance weighting function to specify performance requirements and achieve fine tracking. Significant vibrations are easily induced by high gain at high frequencies. Then, an integral action is added to \( w_1 \) to reduce the feedback bandwidth and enhance the disturbance suppressing at low frequencies. \( w_2 \) and \( w_3 \) denote the noise and reference weighting functions, respectively. \( w_2 \) is the control weighting function to limit the control gain and suppress noise at high frequencies, \( w_u \) denotes the uncertainty due to the hysteresis nonlinearity. \( \Delta_u \) is an unit complex uncertainty with norm \(|\Delta_u| < 1\).

4.2 Inversion-based Feedforward Compensation

The feedforward controller is used to overcome the bandwidth limitation of the feedback controller. In this section, an inversion-based feedforward controller is used to enhance the \( H_\infty \) feedback performance.
The inversion-based feedforward controller encompasses the inverse non-hysteretic dynamics and the inverse hysteresis. First, the reference signals pass through the inverse non-hysteretic dynamics \( \hat{G}^{-1} \), then the inverse hysteresis \( \hat{\Gamma}^{-1} \). The details of the model-based inversion \( \hat{G}^{-1} \) can be found in Refs. (Liu and Lee, 2012). The Preisach-based inversion \( \hat{\Gamma}^{-1} \) is shown in (Liu and Lee, 2012). The inversion of the non-hysteretic dynamics can be represented as

\[
\hat{G}^{-1}(s) = \frac{s + 1}{k_{ev}} \prod_{i=1}^{m} \frac{s + \hat{p}_i}{s + \hat{z}_i} \prod_{i} \frac{s^2 + 2\hat{\omega}_ni s + \hat{\omega}_n^2}{\hat{\omega}_n^2},
\]

where \( \hat{k}_{ev}, \hat{\xi}, \hat{\xi}_{ni} \) and \( \hat{\omega}_ni \) are the identified parameters of the electric and vibration dynamics, respectively, \( \hat{z}_i \) and \( \hat{p}_i \) are the estimated zeros and poles of the creep dynamics, respectively.

5 EXPERIMENTAL STUDIES

5.1 Experimental Setup

The experimental setup consists of a piezoelectric actuator with motion amplification, an voltage amplifier, a linear variable differential transformer (LVDT) and a DSPACE 1104 board. Fig. 13 shows the piezoelectric actuator. The actuator has a travel span of 80 \( \mu m \). The amplifier is E-662 with the output voltage range of \([-20, 120]\) V. The LVDT sensor has white noise, and the RMS value of sensor noise is 0.01 \( \mu m \). MATLAB/Simulink and a DSPACE DS1104 board are used to implement the model-based controller.

The electric and vibration dynamics are identified as

\[
\hat{G}_{ev}(s) = \frac{1}{0.000474s + 1} \left( \frac{8.111 \times 10^6}{s^2 + 3786s + 8.111 \times 10^6} \right) \left( \frac{2.478 \times 10^7}{s^2 + 809.1s + 2.478 \times 10^7} \right).
\]

The creep dynamics is identified as

\[
\hat{G}_{c}(s) = \frac{(s + 0.0146)(s + 0.172)(s + 0.241)}{(s + 0.0142)(s + 0.169)(s + 0.2402)} \left( \frac{(s + 1.07)(s + 18.29)}{(s + 1.053)(s + 17.57)} \right).
\]

Fig. 14 shows the identified density function \( \mu(\alpha, \beta) \) in equation (3).

5.2 Controller Parameters

The performance weighting function \( w_1 \) and the control weighting function \( w_2 \) are set to

\[
w_1 = \frac{350\pi}{s + 0.0001},
\]

\[
w_2 = \frac{0.1}{s + 100000000\pi}.
\]

The reference signal and measurement noise are represented using the weighting functions \( w_r, w_d \) and \( w_n \), respectively

\[
w_r = 0.1, w_d = 0.0001.
\]

To reduce the conservation, the discrete D-K iteration with structured singular value (SSV) is used to solve the controller (Skogestad and Postlethwaite, 2005). After 6 iterations, the SSV is less than 0.98, and the order of the \( H_m \) controller is 9. To easily implement the controller in DSP, the \( H_m \) controller with order of 4 is given by

\[
K_{FB} = 1903075\left( \frac{\frac{s + 31360}{s + 5669}(s + 0.313)}{(s + 19830)(s + 2951)(s + 923.7)} \right).
\]

5.3 Experimental Result

In this paper, square and sinusoidal references are used to demonstrate the effectiveness of the proposed
composite control. Further, the root-mean-square (RMS) error $e_{\text{rms}}$ is used to measure the tracking errors. Fig. 15 shows the tracking performance of the square reference at 20Hz. The RMS tracking error is 0.19 $\mu$m (To grantee the differential of the reference signal, a pre-filer is used for inversion-based feedforward). Fig. 16 shows the control voltage. Further, Fig. 17 shows the tracking performance of the sinusoidal trajectory at 600Hz. The RMS tracking error is 0.78 $\mu$m. Fig. 18 shows the control voltage of the sinusoidal tracking at 600Hz.

The experimental results demonstrate that the proposed composite control provides precision tracking performance at broadband frequencies.

6 CONCLUSIONS

It is increasingly demanded to present broadband accurate tracking of PMS. The modeling and $H_\infty$ composite control of the coupled hysteretic dynamics is thus provided in this paper. The Preisach hysteresis, creep, electrical and vibration dynamics are developed to describe the complex behaviors of PMS. The proposed hysteretic dynamics has physical meanings which is useful for deep developments of PMS. The proposed $H_\infty$ composite control provides high-speed and precision tracking. The experimental studies demonstrate the effectiveness of the proposed modeling and control approaches.

The proposed modeling and control approaches of PMS are beneficial to the suppression of jitters and micro-vibrations in precision spacecrafts, such as inter-satellite laser communication, staring cameras, space-based interferometers and space telescopes.

REFERENCES


