Networks and Imitations in an Agent based Asset Market

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1 STAGE OF THE RESEARCH
Revising and proofreading the draft.

2 OUTLINE OF OBJECTIVES
The basic argument for mimetic behavior is that economic agents can be heavily influenced by the behavior of their neighbors where neighbors are defined as those with whom they have sufficient contact to be informed about the forecasting strategy that they follow and their success. Their anticipations of the evolution of prices are the result of their evaluation of the forecasting strategies’ success rather than on their own personal anticipations. So agents can switch from one forecasting strategy to another as the success of strategies changes. We study how the imitation of the rules of others as well as the structure of their interaction can influence the market share of agents and the prices of the asset traded on this market.

3 RESEARCH PROBLEM
How do the structures of information exchange regarding the success of investment strategies among investors influence the price volatility in financial markets? How do the different ways in which investors change their investment strategies based on the acquired information from their “friends” and “colleagues” affect the answer to this question? These are the questions we try to answer in this paper by conducting computational experiments in an artificial asset market in which investors who exhibit mimetic behavior operate and switch from one forecasting strategy to another.

4 STATE OF THE ART
Understanding the relationship between the structure of interactions among investors who imitate each other and the dynamics of prices in financial markets are of interest to us for, at least the following three reasons (1) There is a growing awareness among economists and policy markers alike that we need to deal better with heterogeneity across agents and the interaction among those (boundedly rational) heterogeneous agents. (2) Among various dimensions of bounded rationality, there has been increasing interests among economists and researchers in understanding the consequences of mimetic behavior not only in financial markets (Föllmer et al., 2005; Kirman, 1993; Lux, 1995; Lux and Marchesi, 1999; Topol, 1991) but also in other fields in management such as Marketing (Choi et al., 2010; Zhou, 2006) and Strategy (Giarratana and Mariani, 2013; Posen et al., 2013). Imitation impact is also finding applications in various contexts such as anthropology (Goodwin and Heritage, 1990), social psychology (Levine et al., 1993), political environment (McKinley, 1901). Furthermore, (3) rapidly cumulating evidence shows the importance of better understanding the ways local interaction structures influence aggregate outcomes (Panchenko et al., 2013; Jackson, 2008; Shiller, 1995; Shiller and Pound, 1989).

5 METHODOLOGY
In this paper, we embed the model of Föllmer et al (2005) into a family of network structures that can be generated simply by a model of Watts and Strogatz (1998) namely regular one dimensional lattice networks, small world networks, and random networks. In the model of Föllmer et al (2005), asset market prices are determined as temporary equilibria and agents’ excess demand is a function of the prices they expect in the next period. For simplicity, agents (who are investors) in our model use, at any point in
time, one of the following two forecasting strategies: “chartists” and “fundamentalists.” Agents switch between these two strategies by mimicking the “successful” strategies employed by their local neighbors. We consider two mimicking rules, the most profitable rule and the average rule, by varying the definition of “successful” strategies.

Under the most profitable rule, an agent copies the forecasting strategy used by the most successful neighbor. Under the average rule, an agent adopt the forecasting strategy that resulted in the higher average profit among their neighbors who were using it. These two appear particularly adapted to a model with boundedly rational agents (Ellison and Fudenberg, 1995; Schlag, 1998; Selten and Ostromann, 2001). In addition, we introduce a small noise in the mimicking rule so that with a small probability agents fail to employ the forecasting strategy that these mimicking rules specify, and use the other rule.

We employ computational experiments because, as noted by the pioneering research in this field (Arthur, 1994; Palmer et al., 1994), traditional analytical approaches have difficulty in taking into account changes individual behaviors and in considering realistic market microstructures except in a very few special cases. Computational models are very often used in analyzing behavioral models with heterogeneous agents (Brock and Hommes, 1998; Hommes, 2006; LeBaron, 2006; Tedeschi et al., 2010).

5.1 Anticipations and Price Formation Process

Our model progresses in the following way. We initialize the model by creating interaction network among agents, and assign investment strategies among agents so that a half of investors are using chartist investment strategy and remaining half are using fundamentalist investment strategy. The agents using each investment strategies are spread randomly throughout the network. In each period, each investor forms price expectation based on the forecasting strategy s/he uses. The price expectation of each investor determines his or her demand schedule in that period, which in turn determines the market price. The realized market price determines the profit of each investor. Agents update their forecasting strategies based on the mimicking rule and the model enters the next period. This process is summarized in Figure 1.

This process is summarized in.

In this section, we first describe the forecasting strategies, and price determination process. The creation of network structure is described in the next section.

We consider a model that involves two different types of forecasting strategies: “chartists” and “fundamentalists.”

Fundamentalists are investors who follow the fundamental rule, which predicts that market prices will (gradually) return to their “fundamental” level. Thus, the expected price for fundamentalist $n$ for the period $t + 1$ is given by

$$s^n_{t+1} = P_t + \alpha (f - P_t)$$

(1)

where $f$ is the fundamental value of the asset, $\alpha$ is a constant that represents the fundamentalist’s estimate of the speed of price adjustments. We suppose that it is the same for everybody (in our simulations we set $\alpha = 0.1$). As is clear from equation (1), if $P_t$ is below what traders believe to be the fundamental value, they anticipate a price increase. On the other hand, if $P_t$ exceeds the fundamental value, they anticipate a price decrease.

Chartist’s strategy relies only on the history of previous prices observed in financial markets and we shall refer to investors following this strategy as chartists. The latter attempt to extrapolate the past movements of the stock price to predict its future
price and form their expectations on the basis of the history of the asset’s prices. To simplify matters we have supposed that agents use only one observation from the past. However they could be sophisticated time series econometricians and this would not change the analysis. Thus, the expected price for chartist $n$ for the period $t+1$ is given by

$$E^n_{t+1} = P_t + \alpha (P_t - P_{t-1})$$

(2)

where $\alpha$ is the investor’s current estimates of the speed of the trend-cycle. Equation (2) shows that if $P_t$ is above $P_{t-1}$, a chartist anticipates a price increase. On the other hand, if $P_t$ is below $P_{t-1}$, she anticipates a price decrease.

Given the price forecast and an investor’s idiosyncratic liquidity demand in each period, the net demand schedule for the investor is determined. Following the specification used by Föllmer et al. (2005), we define net demand schedule for agent $n$, $D^n(t)$ in period $t$ as

$$D^n(t) = (\log E^n_t - \log P_t) + \eta^n_t$$

(3)

where $E^n_t$ is the expected price for agent $n$ in period $t$, $P_t$ is the asset price and $\eta^n_t$ is the exogenous random liquidity demand. We assume that $\eta^n_t \in [-1;1]$.

In other words, the net demand of agent $n$ involves an exogenous liquidity demand and an endogenous amount reflecting the deviation of the price from the expected price. A positive net demand at price $P_t$ of the investor reflects his intention to buy the stock at the price. If her net demand is negative, she sells the stock at the price.

The equilibrium asset price in period $t$, $P_t$ is defined as

$$\sum_{n \in N} D^n_t(P_t) = 0$$

(4)

Applying Equation (3) to Equation (4) gives the asset price $P_t$

$$P_t = e^{\frac{1}{N} \sum_{n \in N} \log E^n_t + \eta^n_t}$$

(5)

where $\eta^n_t = \frac{1}{N} \sum_{n \in N} \eta^n_t$ . All agents obtain profits in period $t$ based on the order they have placed in period $t-1$, $D^n_{t-1}(P_{t-1})$ at the market price as follows

$$\Pi^n_t = (P_t - P_{t-1}) \times D^n_{t-1}(P_{t-1})$$

(6)

Agents then forecast next period’s price at each step and form their excess demand, which in turn determines the next period’s price. In the next period given the realised price, and profits, a new forecasting rule is chosen, forecasts are made and excess demand is determined.

Let us first show the outcomes of the model when agents do not change their forecasting strategies. These outcomes will serve as benchmarks when we introduce mimicking and local interaction structures. shows the time series of prices for three different scenarios: (1) 0% chartists, (2) 50% chartists and (3) 100% chartists. In each case, we assume that there are 100 investors in the market.

We can see from that figure that prices become more volatile when there are more chartists in the market.

We quantify price volatility by computing the coefficient of variation of prices $c_v$: a neutral measure often used to gauge dispersion’s degree.

$$c_v = \frac{\sigma}{\mu}$$

(7)

Figure 2: Difference in price evolution between a chartist regime, a fundamentalist regime and a fifty-fifty regime.

The coefficient of variation of prices is defined as the ratio of the standard deviation of prices $\sigma$ for a given period to the mean of prices $\mu$ for the same period. This coefficient of price variation is between 14.5% and 25.6% when 50% of investors in the market are chartists. This is relatively high because it does not exceed 17.6% in case of complete absence of chartists. When there are 100% chartists, the coefficient of price variation can reach 127%. The presence of chartists makes the price less stable and sometimes drives it far away from the fundamental value.

Our artificial asset market, even when there is no switching between strategies, is thus able to reproduce the stylized facts observed in real financial market and illustrates in particular that excess volatility of stock prices occurs when the market is dominated by chartists (Kirman 2010 Chapter 4, Kirman and Teyssiére 2002, Föllmer et al., 2005, Lux and Marchesi, 1999). However, while these models allowed for switching of agents’ strategies, they did not consider the structure of local interactions.
interactions between agents. This is what this paper introduces as we describe in the next section.

5.2 Network Topologies

As soon as we recognize that local interaction between agents conditions their behavior, we have to specify the network which governs that interaction. The fact that networks play a significant role in agents’ behavior and interaction in financial markets and thus, in replicating the stylized facts of financial time series has been recognized in the literature (See e.g. Cont and Bouchaud 2000, Alfarano and Milaković 2008.)

We want to investigate to what extent network structure matters when, as in our model agents mimic their neighbors. We employ the model of Watts and Strogatz (1998) to generate a family of network structures that spans regular network (one dimensional lattice) and random network. The method they used to generate various is as follows: They consider a random rewiring method. They start from a one-dimensional regular network with k degree. That is each node is connected to k/2 closest nodes on each side by undirected edges. And each link is randomly rewired with probability p.

For p = 0, the graph network remains to be a regular lattice. The degree of disorder increases with increasing p. When p = 1 all edges are rewired randomly. The graph is referred to as a “small-world” network for intermediate values of p (0 < p < 1).

We want to show how the differences in the network structure determined by the various values of p influences the market share of agents and the asset price volatility. To generate the different graphs, we choose the method of Watts and Strogatz and the formal notion of clustering coefficient that they introduced. We study three different network topologies (at both extreme values of p and for intermediate values of p).

The Figure 3 below shows examples of networks generated according to this model for three different values of rewiring probability p. A regular network (when the rewiring probability is equal to 0 (p = 0)) which is highly clustered and has a large characteristic path lengths, a random network (when the rewiring probability equals 1 (p = 1)) which is poorly clustered but has a short characteristic path length and a small world network (p = 0.1) which exhibits two properties, small average shortest path length and large clustering coefficient. Watts and Strogatz (1998) vary the basic parameter and show that it is only in a small range of values that a small world network is observed.

In our model, unlike Watts and Strogatz (1998), we consider directed links to allow for the possibilities that while agent i is observing carefully about forecasting strategies used by agent j and its performance, agent j does not pay attention to what agent i does.

![Network Examples](image)

Figure 3: Network structure with N=100 nodes and k=4 neighbors.

5.3 Network Structure, Mimicking Strategies and Prices Volatility

The basic argument for mimetic behavior is that economic agents can be heavily influenced by the behavior of their neighbors where neighbours are defined as those with whom they have sufficient contact to be informed about the forecasting strategy that they follow and their success. Their anticipations of the evolution of prices are the result of their evaluation of the forecasting strategies’ success rather than on their own personal anticipations. So agents can switch from one forecasting strategy to another as the success of strategies changes. We assume that all agents update their forecasting rule at the same time and in every period of time t. This is not entirely a passive behavior in that the agents can choose whom to imitate. We study how the imitation of the rules of others can influence the prices of the asset traded on this market.

As we have mentioned we examine the consequences of two mimicking rules: the most profitable rule and the average rule. When the most profitable rule is adopted, traders follow the strategy used by the most successful neighbor; each of them compares the profit recorded in the previous period of all his neighbors and copies the strategy of the most profitable one. The average rule is designed to copy the forecasting strategy, which resulted in the highest average profit for those using it; they compare the profit of neighbours who have used
each strategy in the previous period and then copy the one, which resulted in higher average profit.

There are of course other rules which we would have been justified in adopting such as the least profitable rule (agents never copy the strategy used by the least profitable neighbor), but for simplicity we have chosen the two basic rules discussed above.

Indeed, these two appear particularly adapted to a model with boundedly rational agents and they have been studied in the literature. See for example, Selten and Ostmann (2001), for the max rule and Ellison and Fudenberg (1995) and Schlag (1998) for the average rule.

We restrict our attention to mimicking rules with limited memory. In particular, in this paper we have decided to focus on the memory length of one that is the profit realized in the previous period and not on the cumulative profits or on the weighted average of past-recorded profits. Choosing the cumulative profits in the performance measure does not allow for the fact that the most profitable agent may not have always used her current strategy. This make it difficult to determine which specific strategy worked most effectively. We avoid this difficulty therefore by only considering only one lag to measure agents’ performances.

We conduct simulations for the two mimicking rules in three different network structures: a regular network (rewiring probability $p=0$), a random network (rewiring probability $p=1$) and a small world graph (rewiring probability $p=0.1$) to investigate if and how the network structure influences agents’ market share. We follow Panchenko et al (2013) in choosing $p=0.1$ to generate a small world network.

We start with a market in which half of the investors are chartists and the other half are fundamentalists. It should be noted that the simulations assume that the fundamental value is at all times constant.

We analyze the evolution of the distribution of forecasting strategies for $N = 100$ agents. We study two cases: $K=4$ (Each agent is connected to a small fraction of the entire network: each one has 4 neighbors) and $K=10$ (Each agent has 10 neighbors). Since there are only two strategies: a chartist strategy and a fundamentalist strategy, we look at one strategy: the chartist strategy.

In each case, a total of 100 of simulations were conducted, each with 1000 periods of trading. We ignore the first 250 periods and compute the fraction of chartists and the market price for the last 750 periods of simulations. The network is regenerated for each simulation and we check whether each time the graph is connected.

We also study an additionally stochastic version of our model. We assume that agents, with a small probability, fail to employ the forecasting strategy that these mimicking rules specify, and use the opposite. The interaction between chartists and fundamentalists leads to the evolution in their market share; the proportion of each group in the market is based on the choice of each investor of how to update her forecasting strategy.

### 5.3.1 The Deterministic Model

#### 5.3.1.1 Each Agent Has 4 Neighbors (K=4)

We now discuss the relationships between the network structure, the mimicking rules and the distribution of the forecasting strategies. The aim is to show the impact of the mimicking rule and the network topology on the distribution of the two groups in the market and to demonstrate the effect of the latter on price volatility.

Figure 4 shows some simulated times series of the evolution of chartists’ fraction for all networks when the most profitable rule is adopted. We observe that the network topology influences the speed with which the system gets absorbed in one extreme or the other. The regular graph is highly clustered large world. Because of this, the information transmission between agents who are not neighbors is very slow. A small world network represents both a low diameter and a high clustering coefficient.

When the rewiring probability increases, both the clustering coefficient and the diameter of the network decrease thereby speeding up the spreading of information. We notice that the stabilization is more rapid than in the regular network.

The chance that both types of investors co-exist in the market for a long time decreases significantly in the small world network and disappears completely in the random network. This demonstrates that information transmission is much faster in the random network.

However, few of these observed results are representative cases. To better understand the properties of the distribution of the forecasting strategies generated by the three considered networks, we show scatter graphs summarizing the joint evolution between the coefficient of variation of prices and the mean number of chartists computed for the last 750 periods for each simulation (See Figure 5.)
Figure 4: Some simulated times series of the evolution of chartists’ fraction for various network topologies when the most profitable rule is adopted.

Histograms show the distribution of the chartist’s fraction over simulations. For the regular network model, the histogram of the fraction of chartists shows that in half of the simulations the fraction of chartists or fundamentalists reaches an extremity within a relatively short period of time (less than 250 periods). The other half of the simulations indicates that stability comes later as shown in Figure 5. A smaller persistence of the concurrent presence of both investing types is demonstrated in less than 10% of the simulations in the case of the small world network. In the case of the random graph, the system clings on to situation of only one forecasting strategy in a very short timeframe.

Fundamentals are more likely to dominate the market (61%) than chartists (39%) for the regular network. A similar finding is observed for the other two network topologies. Prices volatility increases along with the proportion of chartists. The coefficient of price variation in some cases can rise to 135%. When the proportion of chartists is high in the market, the price of the asset may become unstable and deviate continuously from its fundamental value.

In the case of the average rule, the network structure does not affect the distribution of forecasting strategies. Neither chartists nor fundamentalists dominate. A simultaneous existence of these two groups is demonstrated for all network topologies. The share of each type of agents fluctuates around the half the number of agents (See Figure 6.)

These results show a sharp difference between the two mimicking rules when agents are connected to a small fraction of the network. For the most profitable rule, everyone is looking for the most successful neighbor and copying exactly his strategy. On the contrary, when the average rule is followed, even if in a neighborhood one of the two strategies is most profitable, on average investors following these strategies may make the same amount of profits and this does not give the same results.

Contrary to the case of the most profitable rule, the observed time series are representative outcomes.
of the evolution of the fractions of the two types of agents (See Figure 7.) The coefficient of price variation is between 14.23% and 31.19%. The interval of the volatility’s degree is close to that in the case of absence of imitation due to the fact that both types of investors exist in similar proportions.

**Average Rule**

![Figure 7](image1)

**5.3.12 Each Agent Has 10 Neighbors (K=10)**

This section also illustrates the impact of mimicking rules and network structures on chartists’ distribution for the two different updating rules and for three different network structures with the difference that agents are connected to a greater fraction of the network. Each agent has 10 neighbors.

Figure 8 shows that increasing the number of neighbors has an impact on the speed with which all individuals end up taking the same strategy. Similar results were found for the small world and the random networks. The system gets absorbed in one of the two extremes fairly quickly. This can be explained by the structural properties of the small world network and the random network and more precisely by their small characteristic path length. On the other hand, for the regular network, the stabilization is slower but still much rapid than in the case of k=4.

**Most profitable Rule k=10**

![Figure 8](image2)

Fundamentalists appear to dominate the market in over 60% of simulations whatever the network topology. The coefficient of price variation, as expected, is high when chartists predominate. When the average rule is followed, the distributions of agents produced when each investor has 4 neighbors are very similar to those produced when each agent has 10 neighbors. Both types of investors exist in similar proportions for all network topologies. This can clearly be attributed to the fact that profits made by chartists’ group are much the same as those realized by fundamentalists’ group.

**5.3.2 The Stochastic Model**

So far, we have seen that a sharp difference exists between the most profitable rule and the average rule. Indeed, without the noise and when the most...
profitable rule is adopted the chance that both types of investors exist simultaneously in the market for a

\textit{Average Rule k=10}

Figure 9: The price dispersion and the share of agents for different network topologies when all agents adopt the average Rule. Cloud of points representing the joint evolution between the coefficient of variation of prices and the mean number of chartists for all simulations. Left: Regular network. Middle: Small world network. Right: Random network.

This subsection presents a doubly stochastic version of our model. We assume that agents, with a small probability (0.1), (a "trembling hand"), fail to employ the forecasting strategy that these mimicking rules specify, and use the alternative strategy.

5.3.21 Each Agent Has 4 Neighbors (K=4)

In the presence of noise in the mimicking rule, the average rule and the most profitable rule lead to the similar average fraction of two groups in the market for all network structures. When the noise is small and when agents are mimicking their local neighbors, this provided periods of simultaneous existence of both trading strategies whatever the mimicking rule and whatever the network topology introduces the price dispersion and the share of agents over simulations for the two different updating rules adopted and for three networks topologies: the regular network, the small world network, and the random network. The mean number of chartists for all networks structures and for the two updating rules is near the average. The price volatility for the three different network structures does not exceed 28.2%. The values are also close across the different topologies of the network.

There is, however, difference in the volatility of the fraction of agents between the two rules even in the presence of noise in the mimicking behavior. Figure 11 graphs the coefficient of variation of chartists’ proportion averaged over all simulations

\textit{Most profitable Rule}

Figure 10: The price dispersion and the share of agents for different network topologies for the two mimicking rules. Cloud of points representing the joint evolution between the coefficient of variation of prices and the mean number of chartists for all simulations. Left: Regular network. Middle: Small world network. Right: Random network.

relative to the volatility of chartist’s proportion. Indeed, when the most profitable rule is used there is greater volatility. This is because each individual plays the action of the best neighbor. We also show that this volatility increases when there are more random links, as the spread of the influence of the lucky chartist in the network is higher. But if we are looking in terms of average payoffs this is no longer observed.

5.3.22 Each Agent Has 10 Neighbors (K=10)

Figure 11: Difference in the volatility of the fraction of chartists between the most profitable and the average rule.
5.3.22 Each Agent Has 10 Neighbors (K=10)

In this subsection we extend the analysis to the neighborhood of size ten. Let us consider Figure 12, which shows considerable similarity between the two neighborhood cases. We thus note that the distributions of agents produced by the two updating rules when each investor has 4 neighbors are the same as those produced when each agent has 10 neighbors. The average rule and the most successful rule lead to the same distribution of the two groups in the market. We demonstrate a concurrent existence of both trading strategies with sometimes a more pronounced presence of one of the two groups. Profits generated by chartists and fundamentalists groups sometimes rise and sometimes fall accordingly. Moreover, an increasing relationship between the number of neighbors and volatility of fraction of chartists has been illustrated as shown in Figures 11 and 13.

**Most profitable Rule:**

![Graph](image1)

**Average Rule:**

![Graph](image2)

Figure 12: The price dispersion and the share of agents for different network topologies for the two mimicking rules. Cloud of points representing the joint evolution between the coefficient of variation of prices and the mean number of chartists for all simulations. Left: Regular network. Middle: Small world network. Right: Random network.

6 EXPECTED OUTCOME

Our results show a sharp difference between the outcomes of two mimicking rules, the most profitable and the average rules, when agents are connected to a small fraction of the network (k=4 and k=10). When the most profitable rule is adopted, the network topology influences the speed with which all agents end up taking the same strategy.

When the rewiring probability increases, both the clustering coefficient and the diameter of the network decrease thereby speeding up the spreading of information. Information transmission also increases, as the number of neighbors is higher.

![Graph](image3)

Figure 13: Difference in the volatility of the fraction of chartists between the most profitable and the average rule. Fundamentalists are more likely to dominate the market than chartists. There is a greater chance that the profit of the best performing fundamentalist is higher than the profit of the best performing chartist and that the average profit of fundamentalists exceeds the average profit of chartists. When the proportion of chartists is high in the market, the price of the asset may become unstable and deviate systematically from its fundamental value. On the other hand, In the case of the average rule, neither chartists nor fundamentalists dominate. A simultaneous existence of these two groups is demonstrated.

The difference in the outcome between two mimicking rules, however, disappears when the noise in the mimetic behavior is larger. In the presence of noise in mimicking rule, the average rule and the most successful rule lead to the similar average fraction of two groups in the market whatever the network structure. There is, however, difference in the volatility of the fraction of agents between the two rules even in the presence of noise in the mimicking behavior. The most profitable rule experiences greater volatility, and this volatility increases when there are more random links, as well as the number of neighbors each agent has.

In addition, we notice that price volatility increases monotonically with an increase in the proportion of chartists. Interaction between these two types of investors involving endogenous modification of strategies according to their performance leads to unstable prices. Asset prices
may move away for considerable periods from fundamentals and then return abruptly. This is due to the link between the profitability and the fraction of the different strategies, which engenders a self-reinforcing contagion process.

REFERENCES


