Combining Test and Proof in MBAT
An Aerospace Case Study

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Abstract: In the aerospace industry, it has become possible to use formal analysis results as certification evidence thanks to the new version of the standard DO-178C and its formal methods supplement DO-333. Furthermore, formal proof has a high potential of cost reduction. On the other hand, it is not possible to replace testing completely by formal analysis, because the latter only considers more or less abstract models of the system under analysis, and can fail due to a too high complexity. But since certain verification tasks can be carried out by formal analysis with an advantage compared to testing, the question arises how both techniques, i.e., proof and test, can be combined in the best way. The European project MBAT gives answers to this question, and in this article we show how the combined approach has been applied to a relevant use case from Rockwell Collins.

1 INTRODUCTION

The aerospace industry is typically developing highly critical embedded software, and the costs for ensuring the safety of such systems can be very high. In the development of the Boeing 777, software accounted for a third of all costs, and in this third, 70% consisted in verification and validation (V&V) costs while only 30% were devoted to software development (Feron et al., 2012). Other aircraft manufacturers have similar figures.

The software specific certification regulatory document, the recently updated DO-178C, characterizes different levels of criticality from level A - the most critical - to level E - the less critical. Depending on the identified level, verification and validation activities are more or less intensive and therefore costly. This certification document has recently been updated and it also provides the formal methods supplement RTCA DO 333. This supplement explicitly enables the use of formal methods for critical embedded software.

Even if formal proofs can be used as certification evidence, they cannot completely replace testing, since a proof is always done on a formal representation which is an abstraction from certain aspects of the real implementation. Furthermore, it can be very difficult to obtain a proof, such that in practice the cost for finding a proof can be prohibitively high, and it is in general very difficult to predict with a certain reliability if the activity of searching for a proof will be successful. On the other hand, highly automated formal analysis tools are available today, and in many cases, proofs can be found with an acceptable amount of user effort. The particularity of formal proofs to cover the entireness of all possible executions of the analyzed software gives them a high potential for cost reduction compared to testing, and therefore it is preferable to use formal analysis as much as possible. But then, the question arises how test and proof can be combined such that synergies between the two techniques can be exploited in the most efficient way. It is the aim of the MBAT project to give answers to this question.

The ARTEMIS project MBAT is providing a new V&V technology in form of a Reference Technology Platform (MBAT RTP) and a methodology for its application that enables the production of high-quality and safe embedded systems at reduced costs. This is made possible by an approach in which model-based testing technologies is combined with static analysis techniques.

In this work we present an instance of the MBAT method applied to a redundancy management unit which is used on aircraft. Even if this system is relatively small in terms of code volume, its behavior is complex and presents a real challenge to formal analysis tools as well as to testing approaches. However, we show how formal analysis on model and code level can be intertwined with testing in order to obtain a
very high degree of confidence in the correct behavior of the system.

This paper is structured as follows: In section 2, we outline the MBAT methodology framework, and we describe the instance of this framework that we used for our purpose. In section 3, we present the use case on which we applied the MBAT technology, and in section 4, we present the analysis results that we have obtained. We share some practical insights in section 5, and conclude in section 6.

2 THE MBAT METHODOLOGY

The MBAT project has defined a methodological framework for the combined use of test and proof, and a Reference Technology Platform (RTP) which gives the technical base to apply this methodology. The architecture of the RTP is highly modular, such that different tools can be combined depending on the user’s needs.

In this section, we will give an outline of the methodological framework of MBAT, and then we will describe the particular instance of the MBAT methodology that we have applied on our use case.

2.1 MBAT Main V&V Flow

The MBAT main V&V flow is shown in figure 1 taken from (Nielsen, 2013). It consists of the following main steps:

1. From the requirements, a (formal) specification model is constructed that reflects the main aspects of specified behavior.
2. From this requirement model, the design and V&V flow can start; analysis, test, and design models can be derived.
3. The low level design models elaborate in detail how (as opposed to what is required) each component is going to function.
4. The relation between the produced code for a component and its design model is checked by dynamic and static analysis.
5. The integrated system is validated by means of testing (Hardware/Processor in the Loop).

2.2 Our Instance of the MBAT Methodology

In this section, we present the instance of the MBAT methodology applied to our use case. It is based on two commercial tools, and an in-house tool for model checking developed at Rockwell Collins together with several academic partners:

- **MaTeLo**: Model based test generation tool (All4tec, 2013). MaTeLo applies usage models based on Markov chains in order to generate input for the system under test. This approach is very suitable to generate realistic input sequences, and to express the ideas of a test engineer of how a system should be tested. MaTeLo is developed by the MBAT partner All4tec.

- **Astrée**: Code analysis tool (AbsInt, 2013). Astrée is based on the technique of abstract interpretation, and is able to analyze very large programs thanks to its high scalability. The focus of Astrée are mainly non-functional properties like the absence of runtime errors due to division by zero, arithmetic overflows, etc. However, the user can also specify his own properties, which enables to analyze functional properties.

An important advantage of Astrée is the availability of a qualification support kit according to the aerospace standard DO-178B, which means that its reliability can be ensured at a level high enough to produce certification evidence. Astrée is developed by the MBAT partner AbsInt.

- **In-house Model Checking Tool**: Developed by Rockwell Collins for the analysis on model level. This tool is based on modern SMT solvers, and one of its components, called Stuff (Champion et al., 2012), is able to find invariants of models with linear real arithmetic in a completely automated way. Stuff is based on recent research results in the domains of SMT solving and quantifier elimination.

Our in-house tool is able to analyze Simulink models for checking if a model satisfies its specification (Miller et al., 2010). Technically, the Simulink models are translated into Lustre, which is then used as the base for further formal analysis using different tools. However, it is also possible to process Lustre models directly, if it is convenient to use Lustre as modelling language.

The workflow we used is shown in figure 2, and can be divided into the following steps:

1. From a semi-formal specification, a Simulink model is edited, which serves as base for generating an implementation in C, either by automated code generation, or by hand.
2. Also based on the specification, a set of formal properties which need to be verified is derived.
3. An analysis and test (A&T) model is edited. In our case, we combine a MaTeLo usage model with a Lustre model of the system which is analyzed:

- The usage model is based on Markov chains, and is used to generate the input that is injected into the system under test.
- The Lustre model can be used to compute the expected output, but it serves also for model checking, and for the automated generation of invariants.

4. An analysis of the Lustre model is performed, with the objective to validate the model against the specification, but also to generate information which can be used for the code level analysis. This can be done by using and combining different analysis techniques depending on the nature of the analyzed system (Champion et al., 2013). In fact, it is often easier to find certain information on model level than on code level, for the following reasons:

- Knowledge from domain experts is available. For example, if we deal with control systems, knowledge about system invariants found by control theory methods can be exploited. This information is much more difficult to find on code level.
- The information about the structure of the system can be exploited. On code level, this structure may be partially lost.

- Powerful analysis methods are available which can find invariants automatically on systems using real arithmetic, like for example the analyzer Stuff (Champion et al., 2012) developed by Rockwell Collins and Onera.

The information obtained from the code level analysis can be invariants, or invariants which only hold if the system input is constrained in some way. The latter can still be useful to decrease the number of tests.

5. The proof information obtained from the model analysis is exploited to prove the formal properties on code level. For example, it is checked whether invariants which hold on the model also can be proven on code level. Note that this is not necessarily the case even if the implementation does not contain coding errors, since on code level, rounding errors can occur. In some cases, it may be possible that the invariants can only be proven under additional constraints, i.e. only for certain executions of the program, but not for all.

The result of this step is a partial formal proof, i.e. a proof which only holds under certain constraints. The negation of these constraints is a characterization of the cases which are not covered by the proof, and therefore need to be tested.
Note that the proof may be total, in which case no testing is necessary, or it may fail completely, in which case the property can only be tested.

6. Test are generated based on the A&T model for the cases which are not covered by the partial proof. We use a combination of test generation based on usage models following the approach of MaTeLo, and test generation based on model checking. Using only one of these techniques separately is limited for the following reasons:

- Test generation like it is done by MaTeLo, i.e. based on usage models in form of Markov chains is driven by the probabilities associated to the transitions in the model. While this approach is very useful to generate realistic test sequences, it is not suited to generate test sequences which will put the system into a state satisfying a given property.

- On the other hand, test generation based on model checking is able to generate test sequences with a precise test goal, i.e. input sequences which lead to a desired state, but the length of the test sequences it can generate is very limited due to combinatorial explosion of bounded model checking.

To overcome these limitations, we use a combined approach, which exploits the intelligence and the experience of the human user as well as the computational power of a modern computer: when he is shown a predicate on state variables, we assume that a user is in general able to understand which kind of test sequences will put the system into states which are close to states which fulfill the given predicate. Then, bounded model checking can be used to consider all possible input sequences up to a certain depth, and to check whether a sequence exists which puts the system into a state fulfilling the property.

The principles which guided the design of our method can be summarized as follows:

- Formal proof on code level should be preferred to testing, as long as the necessary effort stays within some acceptable limits.

- If a complete proof has not been achieved within these limits, the insights gained by attempting the proof should be exploited as a partial proof.

- The method must allow to use information available on higher abstraction level.

- It must be possible to exploit capabilities of modern code and model analysis tools.

- The method must allow to exploit the intuition and the intelligence of a human user.

- The effort for maintaining the different models must be kept as small as possible.

Our method is meant to be a powerful tool for V&V engineers, enabling them to concentrate their experience and intuition to non-trivial test cases, and relieving them from going too much into details.
3 USE CASE: TRIPLEX SENSOR VOTER

The use case on which we will demonstrate our approach is a triplex sensor voter, i.e. a redundancy management unit for three sensor input values. It implements a voting algorithm which is not based on the computation of an average value, but on the \textit{middleValue}(x, y, z) function, which returns the input value which is between the minimum and the maximum input values (for example, if \( y < z < x \), it would return \( z \)). Other voter algorithms which use a (possibly weighted) average value are more sensitive to one of the input values being out of the normal bounds. Furthermore, the voter contains a fault detection mechanism, which allows to detect that one of the input values is abnormally different from the other inputs, and which is then used to trigger an appropriate fault tolerance mechanism (typically the faulty input value will be ignored).

The specification of the triplex voter is given in figure 3. We only give an example for an error detection mechanism here, and we do not include the actual fault tolerance mechanism. In a complete implementation, error detection mechanisms are typically more sophisticated, and there is additional functionality to deactivate a faulty input. For confidentiality reasons, we cannot give the complete design of our triplex voter here, however, the simplified version captures the essential characteristics of the verification problem. A Simulink model of the voter is shown in figure 4.

An important requirement for the fault detection logic is that a sensor may not be declared as faulty as long as its value is within a certain tolerance range around the real physical value. For a given maximal tolerance value of a sensor, the question is how the ErrorLimit value must be chosen in order to guarantee a correct operation of the error detection. Furthermore, an implementation of the voter will typically use floating point arithmetic. Then, the question arises whether we can exclude that due to accumulation of rounding errors, the fault detection can be triggered erroneously. Also, it must be excluded that runtime errors due to an arithmetical overflow can occur, either because the design is not stable, or because rounding errors could accumulate under certain circumstances. So, we will consider the two following requirements:

- **Requirement 1**: No arithmetical overflows shall occur.
- **Requirement 2**: An input shall be considered valid as long as its error stays within the tolerated range.

Even if the voting algorithm consists only of a few equations, its dynamic behavior is not easy to predict. An attempt has been made by a control theory expert to prove the stability of the voter using the methods of control theory like Lyapunov stability analysis. However, the results were only partial and not sufficient to be acceptable as a formal proof. Therefore, we see a real need to apply analysis methods based on computer science to this system in order to guarantee its correct behavior.

In our analysis, we assumed that the maximal tolerance of the sensors is 0.5, i.e. the value reported by a sensor can be up to 0.5 units above or below the real physical value. We present the results of our analysis in the next section.

4 ANALYSIS RESULTS

In this section, we present the results obtained by the application of MBAT technologies to our use case.

4.1 Model Level Analysis

On the model level, we performed an analysis a Lustre model derived from the equations which specify the voter algorithm. Another possibility would be to generate a Lustre model automatically from the implementation model in Simulink, which might be preferable if the effort of maintaining an additional analysis model in Lustre is to be avoided.

The model level analysis is done assuming that all computations are carried out with an infinite precision, which means that rounding errors which can occur in a real implementation are not taken into account. A detailed presentation of the model level analysis of the triplex sensor voter can be found in (Dierkes, 2011).

Using our in-house tool Stuff, we can detect and prove the following invariants in an almost completely automated way: for \( X, Y \in \{A, B, C\} \),

\[
|EqualizationX| \leq 1.0
\]
\[
|EqualizationX - EqualizationY| \leq 1.0
\]
\[
|\sum_{X \in \{A, B, C\}} EqualizationX| \leq 1.5
\]

Note that these invariants are highly relevant for our requirements, since they establish an upper bound for the equalization values as well as an upper bound for their pairwise difference. However, at this stage these results are only valid on model level, since they do not take into account rounding errors which occur when floating point arithmetic is used. The next step consists of proving them also on code level, and if this is not possible, to prove them at least partially.
4.2 Code Level Analysis

On code level, the proof of invariants from model level can fail for different reasons:

- The use of floating point arithmetic changes the behavior of the implementation so much that the invariant does not hold.

Figure 3: Specification of the triplex sensor voter.

Figure 4: MATLAB Simulink model of the triplex sensor voter.
• The technique in the used analysis tool does not enable a sufficient precision.

In the first case, it is still possible that a slightly weaker form of the invariant holds. For example, if the possible values of a variable are proven to be bounded by a constant $C$ in the model, it might be possible to prove an invariant with a constant slightly bigger than $C$ on the model level. This has been done in (Dierkes and Küstner, 2012). In the second case, it might still be possible to prove the invariant for certain cases, which are expressed by an additional constraint. The cases not covered by the constraint need to be analyzed by testing.

The code level analysis was done using Astrée. Concerning the first requirement, the proof of the upper bounds of the equalization values which were found on model level fails with Astrée, however it was still possible to prove the following weaker property. As long as

$$|\text{Centering}| \leq 0.25,$$

it holds that

$$|\text{Equalization}_{X}| \leq 1.0001 \text{ for } X \in \{A,B,C\}.$$

This can be considered as a partial proof of the property that we found on model level, and it means that tests can only provide additional information if the centering value exceeds 0.25. The condition that the absolute value of the centering value must be less than 0.25 has to be found by the user, but it is easy to find since it corresponds to the saturation limit which is applied to the centering value. In other words, the condition simply says that the centering value must not be saturated. We suppose that this kind of condition would naturally be examined by a test engineer.

We can even further constrain the cases which need to be tested, because we can prove that if the system is in a state $S$ where $|\text{Equalization}_{X}| \leq 0.96$, then in every state which is reachable from $S$ in one transition it holds that $|\text{Equalization}_{X}| \leq 1.0001$.

For the second requirement, we did not succeed in proving the property with additional constraints. However, we still can prove that if the system is in a state $S$ in which for $X,Y \in \{A,B,C\}$

$$|\text{Equalization}_{X} - \text{Equalization}_{Y}| \leq 0.96,$$

then in every state which is reachable from $S$ in one transition it holds that for $X,Y \in \{A,B,C\}$

$$|\text{Equalization}_{X} - \text{Equalization}_{Y}| \leq 1.0.$$

This result is relatively weak, since it only says that if we want to test whether the pairwise difference of two equalization values can exceed 1.0, we first have to reach a state where it exceeds 0.96. However, we can still consider this as a quality criterion for tests, i.e. that the difference value must exceed 0.96, and furthermore it may be possible to get a larger bound than 0.96 by using a more precise analysis, possibly implemented in future versions of Astrée. Finally, the result was easy to obtain, and even if in the end the verification is mainly done by testing, the additional overhead stays low.

In (Dierkes, 2011), we have presented an approach in which rounding errors are over-approximated on the model level, and the resulting verification problem is treated using an SMT solver. This approach allows to obtain a significantly higher precision as tools based on abstract interpretation, but it has two drawbacks:

1. The complexity of the resulting SMT problem is very high, and the scalability of this approach is very week.
2. No qualifiable tool exists today which can translate C code into an SMT formula, and check the SMT formula for satisfiability, and probably, no such tool will be available in the near future. Results furnished by non-qualified tools can increase confidence that an implementation behaves correctly, but they cannot be used to replace tests with respect to certification. Therefore, the testing effort would not be diminished.

Note that the property that the sum of all three equalization values is bounded by 1.5 that we have proven to be invariant on model level cannot be exploited by the current version of Astrée, since it cannot be represented with sufficient precision by any abstract domain implemented into this tool under its current version.

### 4.3 Test Generation

For the first requirement, the test objective is to find an input sequence which leads to a state in which one of the equalization values, let’s say $\text{Equalization}_{A}$, is greater than 1.0001. We know from formal analysis that this is only possible if a state is reached which fulfills the predicate

$$P = \text{Equalization}_{A} > 0.96 \text{ and } \text{Centering} > 0.25.$$

Therefore, our first test objective is to find an input sequence which leads to a state satisfying $P$.

For a human test engineer, an intuitive way to reach a state which satisfies $P$ would be first to make $\text{Equalization}_{A}$ as large as possible. This can be obtained by setting $\text{Input}_{A}$ to 0.5 and the two other inputs to $-0.5$ for a large number of cycles, which would set $\text{Equalization}_{A}$ to a value close to 1.0. Then, in order to make the centering value maximal, $\text{Input}_{B}$
can be set to 0.5 for a certain number of steps. It is not easy to see how many cycles each of the two phases should last, but a good strategy would be to try a large number of different possibilities, by using a probability-based test model.

A MaTeLo test model used for the generation of test input for the voter is shown in figure 5. The numbers associated to the transitions of the model indicate the probability that the corresponding transition occurs. In this way, it is possible to generate a large number of test case by a relatively simple formalism.

The test sequences derived from the test model drive the system into a state that “almost” fulfills the desired property in many cases. For example, a state with EqualizationA = 0.955 and Centering = 0.257 was reached. However, among the several hundreds of input sequences we generated using MaTeLo, none succeeded in fulfilling $P$ itself.

In order to increase our confidence that no state fulfilling $P$ can be reached, we used the final states of the test runs as initial states for bounded model checking up to a depth of 10, i.e. we considered all states which are reachable by the model from the final states by at most 10 transitions. But still, it was not possible to satisfy $P$. We therefore concluded that with a very high probability, states satisfying $P$ are not reachable, and as a consequence, that the first requirement is met by the implementation.

Since the formal analysis results for the second requirement where much weaker, its verification is mainly based on testing, but still taking into account the requirement that the test sequences must lead to a state in which the difference between two equalization values is greater than 0.96. Using our combined test generation technique, we did not find any test sequences which falsified the property.

5 PRACTICAL ASPECTS

In a development process, it is difficult and time consuming to maintain several different models, especially if frequent specification changes occur. On the other hand, a T&A model which is independent from the design model increases the probability to discover errors, since it represents an interpretation of the requirements which is irrespective of the design.

In our approach, the T&A model consists of a test model linked to an analysis model. Since the test model is used for the generation of tests for cases which are not covered by a proof, its size depends on the width of the proof: if the latter covers many cases, the test model can be relatively small, and therefore easy to maintain. However, if the proof fails, the test model must be more extensive. However, the effort of designing a test model is justified by the time gain it enables compared to the classical testing approach.

Concerning the analysis model contained in the T&A model, two cases can occur:

1. If the analysis model is only used to generate proof information, it can be derived from the design model. This can be done automatically, like in the Rockwell Collins tool which translates Simulink to Lustre. The reason is that information which is used to guide a proof on code level can help to obtain a proof, but it cannot lead to “false” proof. Therefore, errors in the design model cannot lead to erroneous proofs.

2. If the analysis model is used as a test oracle, it must be developed independently from the design model, which clearly requires an additional effort. However, the use of declarative languages, which only says what needs to be computed, but not how it is to be computed in detail, should limit this effort.
6 CONCLUSIONS

We have presented the application of an approach for combined model-based testing and formal analysis to a non-trivial use case from the aerospace domain. This approach has been developed within the European research project MBAT. Even if the code volume of the use case is relatively small, its dynamic behavior and reachable state space have a high complexity, which require a considerable validation and verification effort if only testing is used. We have shown how the testing effort can be reduced by using highly automated analysis tools in order to replace certain tests, and to direct testing towards corner cases which are difficult to deal with by the available formal analysis tools. In our analysis, we used the commercially available tools Astrée (formal code analysis) and MaTeLo (model based test generation), as well as an in-house-tool for model checking.

The principles which guided our approach are that formal proof on code level is used as much as possible, and that it should be guided by information obtained at model level. However, formal analysis is done under the constraint that a certain amount of effort is not exceeded, which means that a certain amount of person hours should not be passed, and the partial proof results obtained by then should still be usable to decrease the amount of testing that needs to be done. Furthermore, the testing should allow to combine the experience and the intuition of a human test engineer with the computational power of a machine.

In this article, we concentrated on the technical aspect of combining proof and testing techniques. Reliable figures about the cost reduction which can be achieved are not yet available, but future investigations will include the measurement of such figures.

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