A Revisited Model for the Real Time Traffic Management

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Abstract: The real-time traffic management allow to solve unexpected disturbances that occur along a railway line during the normal development of the traffic. The original timetable is restored through the rescheduling process. Despite the increase of real-time decision support tools for trains dispatchers that enable a better use of rail infrastructure, real-time traffic management received a limited scientific attention. In this paper, we deal with the real time traffic management for regional railway networks, mainly single tracks, in which a centralized traffic control system is installed. The rescheduling problem is presented as a Mixed Integer Linear Programming Model which resolution allows to carry out the rescheduling process in a very short computational time.

1 INTRODUCTION

A railway system is a complex system with many interacting processes that depend on technical devices, human behavior, external environment, and therefore contains many risks of disturbances. The usual method how railways manage their traffic performance is through a carefully designed plan of operations, defining several months in advance routes, orders and timing for all trains. This process, called off-line timetabling, is followed by a real-time traffic management which consists in managing disturbances that may occur during the ordinary functioning of the network.

Once a delayed train deviates from its original schedule, it may propagate its delay to other trains due to infrastructure, signaling or timing conflicts. Major disturbances may influence the off-line plan of operations that should be subject to short-term adjustments in order to minimize the negative effects of the disturbances. Possible traffic control actions include changing dwell times at scheduled stops, changing train speeds along lines, or adjusting train orders at junctions, stations and passing points. Other control actions involve major modifications such as changing train routes or even canceling scheduled train journeys. The main goal of the real-time dispatching is to minimize trains delays, while satisfying the traffic regulation constraints, and ensuring compatibility with the current position of each train, see (D’Ariano, 2008).

In this paper we deal with real-time traffic control problem for a regional single-track railway where an operating system called Centralized Traffic Control (CTC) is installed. The CTC provides a centralized control for signals and switches within a limited territory, controlled from a single control console. The command is carried out by the Train Dispatcher (TD).

The train dispatcher observes the status of the territory – i.e. occupation of line sections, location of trains, etc. – in a continuous manner and collects information; meanwhile, he communicates with the upper level decision-makers and the staff in the territory in order to exchange decisions taken. In case of an unplanned event and emergency he takes a decision and makes necessary actions in accordance with the rules and regulations pre-defined by the railway authority, see (Ismail, 1999).

The TD may benefit from appropriate decision support system, such as scheduling algorithms, to perform a real-time simulation and evaluation of traffic under disturbances in order to quickly reschedule train movements and to reduce delays from a global perspective.

It is important to find a good compromise between the solution quality, the time horizon of the traffic prediction, and the computational effort. If a short time horizon is adopted, only few trains, and few conflicts, can be detected and solved with short computation times. On the other hand, a longer time horizon
leads to a larger number of trains running in the system, in order to eliminate completely the propagation of the disturbance. There is a tradeoff between the size of the time horizon of traffic prediction (bigger time horizon meaning better quality) and the computational time. In fact, in a small time horizon the real-time dispatching does not take into account conflicting trains outside the time horizon. On the other hand, a conflict arising far in the future may not be as relevant as a closer conflict, since other unforeseen events could still affect the further conflict, see (D’Ariano, 2008). In a small time horizon, the computational time is smaller because data are limited.

Usually the train dispatcher reschedules the involved trains, depending on the known duration of the disturbance. He bases his decisions on his own knowledge, resolving a conflict at a time when it occurs, and then manually rebuilds the timetable, with a considerable waste of time and no certainty that its decisions will lead to an optimal solution.

Building on the formalism given in (Dotoli et al., 2013), we present a model that solves the rescheduling problem for regional passenger transport networks with stations of equal importance, where the CTC system is installed. We formulate the problem as a Mixed Integer Linear Programming Problem (MILP).

In the original model the new timetable after the disturbance is obtained by minimizing train delays in all the stations programmed in their path, while considering constraints regarding travel times, stop times at stations, safety standards and network capacity. The model is applied to a limited time horizon that is chosen by the analyst. In order to solve conflicts that may occur in the rescheduled timetable after the time horizon, an iterative heuristic algorithm is applied. The heuristic algorithm solves a conflict at the time when it occurs; priority is given to the train with the highest traveling time, namely the longest distance.

The revised model solves all conflicts that arise along the railway line after the occurrence of the disturbance; the heuristic algorithm is therefore no longer applied. The rescheduled timetable is established in a shorter time, then discomfort for passengers is restricted and the quality of the transport service is increased.

To show its effectiveness, we study the problem in a particular section of a railway network located in Southern Italy, see (FSE - Ferrovie del Sud Est, 2013). The FSE network is constituted by single tracks with few double track segments and in some stations only one train can stop or pass through.

The paper is organized as follows. In Section 2 we present the problem formalization. In Section 3 the mathematical model for the resolution of the problem is proposed. In section 4 we present the application of the model to the case study of the FSE railway network. Finally, Section 5 contains some concluding remarks and suggestions for further research.

2 PROBLEM FORMALIZATION

2.1 Initial Scheduling

Definition 1 (Railway Network). A railway network is defined by a set of segments on which trains runs.

Segment \( b_i \): A segment \( b_i \) is a railway section between two points. We define by \( B = \{b_1, b_2, \ldots, b_B\} \) the set of segments. \( B \) denote the cardinality of the set \( B \). The set of segments is partitioned into the subset \( B^r \) corresponding to segments into a station, and \( B^s \) corresponding to the subset of rail connections outside stations.

Track \( v_j \): Let \( b \) be a segment \( \in B \). We define by \( V^b = \{v^b_1, v^b_2, \ldots, v^b_s\} = \{v^b_j\}_{j \in [1, V^b]} \) the set of parallel tracks in \( b \). The set of all tracks in the railway network is denoted by \( V \). \( V \) and \( V^b \) denote respectively the cardinality of \( V \) and \( V^b \) for a given segment \( b \). Given a track \( v \in V \), we denote by \( b^v \) its corresponding segment.

Circulations in a railway network are defined by a set of trains. Train’s path is made of an ordered set of movements.

Definition 2 (Trains and Movements). We assume that the train’s length is compatible with the length of all tracks that compose the railway line. Trains are thus defined as follows.

Train \( t_k \): The set of trains using the railway network is denoted as \( T = \{t_1, t_2, \ldots, t_T\} = \{t_k\}_{k \in [1, T]} \). \( T \) denotes the cardinality of \( T \).

Train Direction \( d^f \): Each train is defined by a direction parameter expressing the position of its
destination station. Let \( t \) be a train \( \in T \). We denote by \( d^t = 0 \) the direction of a train which head goes to the north (a.k.a. “even trains”) of the railway line, and \( d^t = 1 \) of a train running to the south (a.k.a. “odd trains”).

**Movement (\( \mu \)):** A movement indicates the request for a track by a train. Let \( t \) be a train \( \in T \). We define by \( M^t = \{ \mu^t_1, \mu^t_2, \ldots, \mu^t_{\#} \} = \{ \mu^t_p \}_{p \in [1, M^t]} \) the ordered set of movements of train \( t \). \( M^t \) denotes the cardinality of \( M^t \). \( \mu^t_1 \) and \( \mu^t_{\#} \) denote respectively the first and the last element in \( M^t \). We define by \( M \) the set of all movements on the railway line and by \( M^{\text{last}} \) the set of the last movements of each train.

Train movements are defined by several parameters.

**Movement Direction:** All movements \( \mu \) of a train \( t \) share the same direction, as their denoted as \( \delta^\mu \).

\[
\forall t \in T, \forall \mu \in M^t, d^t = \delta^\mu \tag{1}
\]

**Track and Segment of a movement (\( b, v \)):** Each movement \( \mu \) of a train is scheduled in an unique segment \( b, v \in B \). We define by \( M^b \) the set of movements scheduled in the same segment \( b \in B \). Each movement \( \mu \in M \) must be scheduled in a track of the segment \( b \), denoted as \( v \), according to the following constraint:

\[
\forall t \in T, \forall \mu \in M^t, \forall v \in V^b \tag{2}
\]

**Reference Schedule Times (\( \alpha^\mu_{\text{ref}}, \gamma^\mu_{\text{ref}}, \delta^\mu_{\text{ref}} \)):** Each train movement \( \mu \) is associated to reference times corresponding to its initial schedule. Let \( \forall t \in T \) be a train and \( \mu \in M^t \) one of its movements. We define three reference times \( \alpha^\mu_{\text{ref}}, \gamma^\mu_{\text{ref}}, \delta^\mu_{\text{ref}} \in \mathbb{N} \), where:

- \( \alpha^\mu_{\text{ref}} \) is the starting time of \( \mu \) as established in the initial schedule, expressed in minutes taking as reference a time \( T_0 \in \mathbb{N} \).
- \( \gamma^\mu_{\text{ref}} \) is the duration of \( \mu \) (in minutes) if it occurs in a rail connection, i.e. the minimum running time defined in the initial schedule. This quantity is equal to 0 if the movement occurs in a station. Formally:

\[
\forall t \in T, \forall \mu \in M^t, b, v \in B^t \Rightarrow \gamma^\mu_{\text{ref}} = 0 \tag{3}
\]

- \( \delta^\mu_{\text{ref}} \) is the duration of a movement \( \mu \) (in minutes) if it occurs in a station, i.e. the minimum stopping time defined in the initial schedule. This quantity is equal to 0 is the movement occurs in a rail connection. Formally:

\[
\forall t \in T, \forall \mu \in M^t, b, v \in B^s \Rightarrow \delta^\mu_{\text{ref}} = 0 \tag{4}
\]

Using such notations, the time interval during which a movement \( \mu \) reserves its track can be expressed as \( [\left[ \alpha^\mu_{\text{ref}}, \alpha^\mu_{\text{ref}} + \gamma^\mu_{\text{ref}} + \delta^\mu_{\text{ref}} = \gamma^\mu_{\text{ref}} \right] \]. Two consecutive movements must be scheduled according to these intervals, thus we have:

\[
\forall t \in T, \forall \mu \in [1, M^t],
\alpha^\mu_{\text{ref}} = \alpha^\mu_{\text{ref}} + \gamma^\mu_{\text{ref}} + \delta^\mu_{\text{ref}}
\]

In order to reduce the size of the initial problem we could use only one variable \( \zeta = \gamma + \delta \) to represent movements duration. However, even if such formulation would reduce the number of initial variables, the size of the problem after the presolve phase would remain unchanged, since modern solvers are able to detect such redundant variables. For clarity, we decided thus to keep using two different variables \( \gamma \) and \( \delta \) to represent movement duration respectively in a rail connection and in a station.

**Definition 3 (Security Constraints):** Since several trains run at the same time on a railway network, several constraints must be verified to ensure the security of circulations.

**Track Occupation Constraints:** A track cannot be occupied by two trains at the same time. Such restriction can be expressed formally by constraining any pair of movements using the same track to be scheduled on disjoint timing intervals:

\[
\forall t_1, t_2 \in T, \forall \mu_1 \in M^{t_1}, \forall \mu_j \in M^{t_2},
\nu_{\mu_1} = \nu_{\mu_j} \Rightarrow [\left[ \alpha_{\mu_1}, \alpha_{\mu_1} + \gamma_{\mu_1} + \delta_{\mu_1} \right] \cap \left[ \alpha_{\mu_2}, \alpha_{\mu_2} + \gamma_{\mu_2} + \delta_{\mu_2} \right] = \emptyset \tag{6}
\]

**Safety Times (\( \Delta_m, \Delta_f \)):** The safety time is the separation time that has to elapse between two movements \( \mu_1, \mu_j \) on the same track (i.e. between a train leaving one track and another one entering the same track). We denote by \( \Delta_m \in \mathbb{N} \) the safety time required if trains meet and \( \Delta_f \in \mathbb{N} \) if one train is following the other one. \( \Delta_m \) and \( \Delta_f \) are expressed in the same time units as \( \gamma \) and \( \delta \). These time delays must occur between the end of the first movement (denoted as \( \beta^\mu_1 \)) and the start of the other one (denoted as \( \beta^\mu_2 \)). Formally, safety time constraints can be expressed as follows:

\[
\forall b \in B, \forall \mu_1, \mu_j \in M^b
\nu_{\mu_1} = \nu_{\mu_j} \Rightarrow \left\{ \begin{array}{l}
\gamma_{\mu_1} \geq \gamma_{\mu_1} + \Delta_f \\
\gamma_{\mu_2} \geq \gamma_{\mu_2} + \Delta_f \\
\delta_{\mu_1} \geq \delta_{\mu_1} + \Delta_m \\
\delta_{\mu_2} \geq \delta_{\mu_2} + \Delta_m
\end{array} \right. \tag{7}
\]

\[
\forall b \in B, \forall \mu_1, \mu_j \in M^b
\nu_{\mu_1} = \nu_{\mu_j} \Rightarrow \left\{ \begin{array}{l}
\gamma_{\mu_1} \geq \gamma_{\mu_1} + \Delta_f \\
\gamma_{\mu_2} \geq \gamma_{\mu_2} + \Delta_f \\
\delta_{\mu_1} \geq \delta_{\mu_1} + \Delta_m \\
\delta_{\mu_2} \geq \delta_{\mu_2} + \Delta_m
\end{array} \right. \tag{8}
\]
Note such constraints reinforce the constraint (6) since they induce a separation delay between the time intervals of two movements on the same track.

A rail transport service on a railway line is graphically represented by a cartesian graph, representing the safety constraints.

**Graphic Timetable:** The graphic timetable is a cartesian graph that represents the situation of a railway schedule for a day, see (Vicuna, 1989). The diagram shows all movements, scheduled and safety times. The time line is plotted on the x axis. The railway line (space) is plotted on the y axis. Inside stations, tracks are represented by dashed lines parallel to the x axis. Trains are represented by an oblique broken line which orientation indicates train direction. For instance, train lines are oriented from bottom to up for even trains that travel from south to north (i.e. to the station on the upper end of the y axis).

Figure 1 represents the graphic timetable of three trains \( t_1, t_2 \) and \( t_4 \). Train \( t_1 \) is directed to the South, while \( t_2 \) and \( t_4 \) travel in the opposite direction. The railway line is made of 5 segments: single-tracked stations \( b_1 \) and \( b_3 \), double-tracked station \( b_2 \) and single-tracked rail connections \( b_2 \) and \( b_3 \). Track sets for the five segments are defined by: \( V^{b_1} = \{ v_1 \}, V^{b_2} = \{ v_1, v_2 \}, V^{b_3} = \{ v_1, v_2 \} \). Train’s paths are defined by sets: \( M^1 = (\mu^1_1, \mu^1_2, \mu^1_3) \), \( M^2 = (\mu^2_1, \mu^2_2, \mu^2_3) \), \( M^4 = (\mu^4_1, \mu^4_2, \mu^4_3) \). All trains cross station \( b_3 \) that is single-tracked. As shown, safety times are applied when two trains occupy the same track of a segment. \( \Delta_\gamma \) is the safety time that has to elapse between the end of the movement \( \mu^2_2 \) and the beginning of \( \mu^2_4 \), where \( t_2 \) and \( t_4 \) travel in the same direction. \( \Delta_\gamma \) is the safety time that has to elapse between the end of \( \mu^2_4 \) and the beginning of \( \mu^2_1 \), where \( t_1 \) and \( t_4 \) travel in opposite direction.

### 2.2 Disturbances Issues

When a disturbance occurs along the railway network, that compromises the normal traffic operation, a rescheduling process must be accomplished taking into account time constraints imposed by the initial schedule.

**Definition 4** (Disturbance). A disturbance denotes the deviation of a train \( t_d \in T \) from its original schedule due to an unforeseen situation, concerning one of its movements \( \mu_d \in M^d \).

**Disturbance Duration and Reference Time:**

When a disturbance occurs, a *disturbance reference time* \( T_d \) is defined as the first time on which the disturbance has an impact on the schedule of the others trains, i.e. the reference ending time of the disturbed movement. If only one disturbance affects one movement \( \mu^d_{t_d} \in M^d \) of a train \( t_d \) along the railway line, \( T_d \) is defined as:

\[
T_d = \beta^\text{ref}_{\mu_d} + \Delta_d
\]

We denote by \( \Delta_d \) the disturbance duration expressed in minutes. The impact of a disturbance on the movement is expressed by an increase of the value of parameter \( \delta \) or \( \gamma \) depending on the nature of the segment on which the disturbance occurs. For instance, if \( b^{t_d} \in B^\gamma \), \( \delta_{\mu_d} = \delta^\text{ref}_{\mu_d} + \Delta_d \).

Conversely, if \( b^{t_d} \in B^\gamma \), \( \gamma_{\mu_d} = \gamma^\text{ref}_{\mu_d} + \Delta_d \).

If two independent disturbances affect two different trains along the line, \( T_d \) is defined as the minimum final time of movements affected by the perturbation. Let \( t_{d_1} \) and \( t_{d_2} \) be two trains affected by the disturbance. Let \( \mu_{d_1} \in M^{d_1} \) and \( \mu_{d_2} \in M^{d_2} \) be the perturbed movements. \( T_d \) is defined as:

\[
T_d = \min \{ \mu_{d_1}^{\text{ref}}, \mu_{d_2}^{\text{ref}} \}
\]

Figure 2 represents a railway line made of three station \( (b_1, b_3 \) and \( b_5) \) and two single-tracked rail connections \( (b_2 \) and \( b_4) \). A disturbance occurs in \( b_2 \) and affects only the movement \( \mu^2_2 \in M^2 \). Reference time \( T_d \) coincides with \( \beta^\text{ref}_{\mu^2_2} \). The dashed line shows the movement \( \mu^2_1 \) after the end of the rescheduling process. The rescheduled crossing time \( \delta^\text{ref}_{\mu^2_1} \) is equal to the reference time \( \delta^\text{ref}_{\mu^2_2} \) increased by the disturbance duration \( \Delta_d \). The effective path of movement \( \mu^2_1 \) interferes with the

![Figure 1: Graphic Timetable.](image-url)

![Figure 2: Disturbance Duration and Reference Time.](image-url)
Effective Schedule Times: In order to take into account the effect of the disturbance on the subsequent movements on the railway network, we introduce for any movement its effective scheduled times denoted by $\alpha^{\text{eff}}$, $\beta^{\text{eff}}$ and $\gamma^{\text{eff}} \in \mathbb{N}$. Obviously, movements which final date $\beta$ is scheduled before $T_d$ are not altered. Formally:

$$\forall t \in T, \forall \mu \in M^s, \beta^{\text{ref}}_\mu \leq T_d,$$

Effect Time Constraints: These new scheduled times must allow to absorb the perturbation by delaying the initial reference times, according to their respective segment types, following constraints (3) and (4). Formally, these previous equations become:

$$\forall t \in T, \forall \mu \in M^s, \beta^{\text{ref}}_\mu > T_d,$$

$$b_\mu \in B^t \Rightarrow \left\{ \begin{array}{l} \alpha^{\text{eff}}_\mu \geq \alpha^{\text{ref}}_\mu, \\
\beta^{\text{eff}}_\mu \geq \beta^{\text{ref}}_\mu, \\
\gamma^{\text{eff}}_\mu \geq \gamma^{\text{ref}}_\mu \\
\epsilon^{\text{eff}}_\mu = 0 \end{array} \right. \ (10)$$

$$b_\mu \in B^c \Rightarrow \left\{ \begin{array}{l} \alpha^{\text{eff}}_\mu \geq \alpha^{\text{ref}}_\mu, \\
\beta^{\text{eff}}_\mu \geq \beta^{\text{ref}}_\mu, \\
\gamma^{\text{eff}}_\mu \geq \gamma^{\text{ref}}_\mu, \\
\epsilon^{\text{eff}}_\mu = 0 \end{array} \right. \ (11)$$

Effective schedule time must also obviously follow the sequencing constraint (5) and safety constraints (6), (7) and (8).

Time Horizon ($H$): The time horizon $H$ is the term planning in which the rescheduling operations are carried out. It consists of a given number of timetable minutes in which a given number of trains are scheduled on the railway line. Movements after the time horizon are not taken into account in the rescheduling process, even if they belong to trains of which first movements belong to the time horizon.

Of course, depending on the density of traffic at the time of the disturbance, and its duration, the number of disturbed movements can vary considerably. In this paper, we consider all the scheduled movements of the day but one of our objective functions can be designed to minimize the number of rescheduled trains.

In the following section, we present the Mixed Integer Linear Programming Model that allows to solve the rescheduling problem, i.e. to give a value to each effective scheduled time while respecting the safety constraints and optimizing practical criteria.

### 3 Mathematical Modeling

#### 3.1 Decision Variables

We introduce additional variables and constants used to express the problem in a linear way.

$X_{\mu,v} \in \{0,1\}^{M \times V}$ is the variable that identifies the track $v$ on which a movement $\mu$ occurs. $X_{\nu,v} = \varphi(v = \nu)$ where the function $\varphi(C)$ is the indicator that $\varphi(C) = 1$ if the condition $C$ is verified, 0 otherwise.

$X_{\mu,\nu}^{\text{before}} \in \{0,1\}^{M \times M}$ is the variable that characterizes the chronological order of two movements $\mu$, $\nu$ if they use the same segment. $X_{\mu,\nu}^{\text{before}} = \varphi(\mu = \nu)$ is scheduled after $\mu$).

$X_{\mu}^{\text{delay}} \in \{0,1\}^{T}$ is the variable that specifies if a train $t$ deviates from its original schedule and is therefore delayed. $X_{\mu}^{\text{delay}} = \varphi(\beta^{\text{eff}} > \beta^{\text{ref}})$.

$X_{\mu}^{\text{delay}} \in \{0,1\}^{T}$ is the variable that specifies if a movement $\mu$ deviates from its original schedule and is therefore delayed. $X_{\mu}^{\text{delay}} = \varphi(\beta^{\text{eff}} > \beta^{\text{ref}})$.

$B \in \mathbb{N}$ is a sufficiently large positive constant.

$H \in \mathbb{N}$ is a parameter that defines the size of the time horizon.

By definition, the previous decisions variables are subject to constraints characterizing their physical sense.

• Any movement can only be scheduled on one track of its segment, consequently:

$$\forall \mu \in M, \sum_{v \in V^\nu} X_{\mu,v} = 1 \ (12)$$
• Two movements scheduled on the same segment must be ordered:

\[ \forall b \in B, \forall \mu_i, \mu_j \in M^b, \quad X_{\mu_i}^{before} + X_{\mu_j}^{before} = 1 \]  

(13)

Safety constraints presented above must be expressed using those variables in linear way.

3.2 Linearization of Safety Constraints

Security constraints (6), (7) and (8) are expressed through the use of additional variables and constraints in order to obtain a linear formulation.

3.2.1 Tracks Occupation Constraints

Constraint (6) specifies that a track \( \nu \) cannot be occupied by several movements at the same time. Constraints (7) and (8) express a separation delay must elapse between two movements occupying the same track. These conditions can be expressed by the following equations:

\[ \forall b \in B, \forall \mu_i, \mu_j \in M^b s.t. \ d_\mu_i = d_\mu_j, \forall v \in \mathcal{V}^b, \]  

(14)

\[ \beta_{\mu_i}^{\text{eff}} - \alpha_{\mu_j}^{\text{ref}} + \Delta \leq B \cdot (3 - X_{\mu_i}^{before} - X_\mu_{\nu v} - X_{\mu_j}^{\text{before}}) \]  

(15)

Note for any pair of movements \( \mu_i, \mu_j \), two instances of the previous equations (14) and (15) are considered in the mathematical model depending of the order of movements: \( (\mu_i, \mu_j) \) or \( (\mu_j, \mu_i) \).

The previous equation expresses that if two movements \( \mu_i, \mu_j \) occurs on the same track, and if \( X_{\mu_i}^{before} = 1 \), then \( \mu_i \) must end before the start of \( \mu_j \). If \( X_{\mu_i}^{before} = 0 \) or \( \mu_j \) do not occur in the same track, equation (14) and (15) are trivially verified.

The disjunction operator \( \lor \) in equations (7) and (8) is taken into account by the boolean variable \( X_{\mu_i} \) before that denotes the two possible alternatives.

3.3 Objective Functions

The optimization problem compares four alternative objective functions defined as follows:

\[ Obj_1 : \min \sum_{\mu \in M} (\beta_{\mu_i}^{\text{eff}} - \beta_{\mu_i}^{\text{ref}}) \]  

(16)

\[ Obj_2 : \min \sum_{\mu \in M} (\beta_{\mu_i}^{\text{ref}} - \beta_{\mu_i}^{\text{ref}}) \]  

(17)

\[ Obj_3 : \min \sum_{\mu \in M} X_{\mu_i}^{\text{delay}} \]  

(18)

\[ Obj_4 : \min \sum_{\mu \in M} X_{\mu_i}^{\text{delay}} \]  

(19)

- \( Obj_1 \) minimizes the delay of the traffic (i.e. the sum of the delays of all the movements trains in the railway line).

- \( Obj_2 \) minimizes the total final delay of the traffic (i.e. the final delays when trains arrive at their final destination, or rather the last stop considered within the rescheduling time horizon).

- \( Obj_3 \) minimizes the number of delayed trains.

- \( Obj_4 \) minimizes the number of delayed movements for each train.

When \( Obj_3 \) or \( Obj_4 \) is used, we introduced five additional constraints (20) to (24) as follows:

\[ \forall t \in T, \beta_{\mu_i}^{\text{eff}} - \beta_{\mu_i}^{\text{ref}} \leq B \cdot X_{\mu_i}^{\text{delay}} \]  

(20)

\[ \forall t \in T, \beta_{\mu_i}^{\text{eff}} - \beta_{\mu_i}^{\text{ref}} > 1 + B \cdot (X_{\mu_i}^{\text{delay}} - 1) \]  

(21)

\[ \forall \mu \in M^t \Rightarrow \alpha_{\mu}^{\text{eff}} \leq H \]  

(22)

\[ \forall \mu \in M^t \Rightarrow \delta_{\mu}^{\text{eff}} \leq H \]  

(23)

\[ \forall \mu \in M^t \Rightarrow \chi_{\mu}^{\text{eff}} \leq H \]  

(24)

Constraints (20) and (21) specify that if the rescheduled ending time of the train coincides with its reference time, the train is not delayed and \( X_{\mu_i}^{\text{delay}} = 0 \). Conversely, if one movement of a train is delayed, the last movement is necessarily delayed according to equations (10), (11) and equation (21) implies that \( X_{\mu_i}^{\text{delay}} = 1 \).

Constraints (22), (23) and (24) mean that movements scheduled in the analyzed time horizon, after the rescheduling process must start within the same time horizon. This constraint prevents that the trains are postponed for a long time or even suppressed by moving outside the considered time horizon.

When \( Obj_4 \) is used, constraints (20) (resp. (21)) are replaced by constraints (25) (respectively (26)) that refers to all movements on the line.

\[ \forall \mu \in M^t, \beta_{\mu_i}^{\text{eff}} - \beta_{\mu_i}^{\text{ref}} \leq B \cdot X_{\mu_i}^{\text{delay}} \]  

(25)

\[ \forall \mu \in M^t, \beta_{\mu_i}^{\text{eff}} - \beta_{\mu_i}^{\text{ref}} > 1 + B \cdot (X_{\mu_i}^{\text{delay}} - 1) \]  

(26)

The full model is given in Figure 3.
Let \((\mathbb{B}, \mathbb{V})\) be a railway network, \(T\) a set of trains with their schedules and \((\mu_d, \Delta_d, T_d)\) the characteristics of a disturbance occurring on the system. The mixed integer linear programming model MILP is defined by:

\[
\begin{align*}
\text{Obj}_1 & \quad \text{Minimize} \quad & \sum_{\mu \in M} \beta_{\mu}^{\text{eff}} - \beta_{\mu}^{\text{ref}} = 0 \quad (27) \\
\text{Obj}_2 & \quad \text{Minimize} \quad & \sum_{\mu \in M} \beta_{\mu}^{\text{eff}} - \beta_{\mu}^{\text{ref}} = 0 \quad (28) \\
\text{Obj}_3 & \quad \text{Minimize} \quad & \sum_{\mathbb{B}} X_{\mu, v} \quad (29) \\
\text{Obj}_4 & \quad \text{Minimize} \quad & \sum_{\mu \in M} X_{\mu, v} \quad (30)
\end{align*}
\]

Subject to:

\[
\begin{align*}
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } b_\mu \in \mathbb{B}, \quad \beta_{\mu}^{\text{eff}} - \beta_{\mu}^{\text{ref}} = 0 \quad (31) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } b_\mu \in \mathbb{B}, \quad \gamma_{\mu}^{\text{eff}} = 0 \quad (32) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } \mu \in \mathbb{M}, \quad \alpha_{\mu}^{\text{eff}} = \alpha_{\mu}^{\text{ref}} \quad (34) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } \mu \in \mathbb{M}, \quad \gamma_{\mu}^{\text{eff}} = \gamma_{\mu}^{\text{ref}} \quad (35) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } \mu \in \mathbb{M}, \quad \delta_{\mu}^{\text{eff}} = \delta_{\mu}^{\text{ref}} \quad (36) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } \mu \in \mathbb{M}, \quad \alpha_{\mu}^{\text{eff}} > \alpha_{\mu}^{\text{ref}} \quad (37) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } \mu \in \mathbb{M}, \quad \gamma_{\mu}^{\text{eff}} > \gamma_{\mu}^{\text{ref}} \quad (38) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } \mu \in \mathbb{M}, \quad \delta_{\mu}^{\text{eff}} > \delta_{\mu}^{\text{ref}} \quad (39) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } \mu \in \mathbb{M}, \quad \alpha_{\mu}^{\text{eff}} \geq \alpha_{\mu}^{\text{ref}} \quad (40) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } \mu \in \mathbb{M}, \quad \gamma_{\mu}^{\text{eff}} \geq \gamma_{\mu}^{\text{ref}} \quad (41) \\
\forall t \in T, & \quad \forall \mu \in M' \text{ s.t. } \mu \in \mathbb{M}, \quad \delta_{\mu}^{\text{eff}} \geq \delta_{\mu}^{\text{ref}} \quad (42) \\
\forall \mu \in M, & \quad \sum_{v \in \mathbb{V}_\mu} X_{\mu, v} = 1 \quad (45)
\end{align*}
\]

Constraints (31) - (32) express that if a movement occurs in a station (resp. in a rail connection) the effective running time (res. the effective stopping time) is null.

Constraint (33) specifies that each train movement is directly succeeded by the next one, that means that when a train leaves a track, it instantly begins to occupy the next one.

Constraints (34), (35) and (36) ensure that movements scheduled completely before the occurrence of the disturbance remain unchanged.

Constraints (37) to (39) (resp.(40) to (42)) enforce the restrictions related to planned stops (respectively to planned running times) and the consequent earliest

Figure 3: Mixed Integer Linear Programming Model.
possible departure time.

Constraint (43) (resp. (44)) means that the running time of perturbed movements (respectively the stopping time) is increased according to disturbance duration.

Constraint (45) means that each movement has to use exactly one track of its segment.

Constraint (46) implies that two movements scheduled on the same segment must be ordered.

Constraints (47) - (48) mean that if several movements have to use the same track of a segment, a safety time ($\Delta_f$ or $\Delta_m$) must elapse between the end of the first movement and the beginning of the second one.

Constraints (49)-(50) (resp. (51)- (52)) denote if a train (respectively a movement) is delayed. Constraints (53) to (55) enforce the beginning and the duration of a movement within the time horizon. These constraints are active only when $Obj_3$ (respectively $Obj_4$) is applied.

4 NUMERICAL EXPERIMENTS

The presented model is applied to Ferrovie del Sud Est (FSE), the largest public transport company operating in the Apulia region of Southern Italy. We analyze the railway ring connecting Mungivacca and Putignano stations (Figure 4), where a CTC system is installed. The operation system is installed in Mungivacca station that is independent, not controlled by the CTC, as is Putignano, whereby these stations are not studied here. In particular, we refer to the line 1 of the railway ring – passing through Conversano – that is single tracked except for the line connecting Noicattaro to Rutigliano, that is double tracked. Moreover, Grotte di Castellana is a single track station, not chaired by an operator. 24 even trains and 22 odd trains run on the railway line during a day. We assume a safety time $\Delta_w = 3 \text{ min}$ for two trains traveling in opposite directions and a time $\Delta_f = 1 \text{ min}$ for trains in the same direction. In attempt to evaluate the optimality of the algorithm, we used IBM CPLEX 12.5 installed and run on an Intel Core 2 Duo 1.83 GHz CPU and 3 GB RAM, under Windows with the model formulated in AMPL.

4.1 One Disturbance on the Line

We consider a real data set referring to a train going from Putignano to Mungivacca that stops along the line that connects Castellana G. and Conversano due to a disturbance occurring at 7:50 am. That same disturbance event is used for all the experiments but with different disturbance sizes $\Delta_d$ and solved with different time horizons $H$. Various disturbance times are been considered, ranging from 10 to 50 minutes. A time horizon of $H$ minutes means that movements that should have started (according to the initial timetable) $H$ minutes or more after the instant at which the disturbance occurs are not considered in the computation. Time horizons are expressed in minutes and take values equal to 30, 60, 90, 120, 180, 240, 300, 360, 420, 480, 540, 600 and 1440. The main aspects considered to present results are $Obj_1$, $Obj_2$, $Obj_3$, $Obj_4$ that correspond respectively to sum of delays for all train movements, delay of the last movement of each train, number of rescheduled trains and number of rescheduled movements. All operational times are given in minutes: CT refers to computational time given in seconds. $N$ and $V$ refers to number of variables and constraints before and after the presolve phase.

Figure 4: Ther railway ring used for the scenarios.

4.1.1 Overall Analysis

When only one disturbance occurs on the railway line, results from experiments using the four objective functions for all different disturbance size ($\Delta_d$) and time horizons ($H$) are presented in Table 5.

The methodology is applied to a real case study in which the occurrence of short-term disturbance on the railway line is frequent, this is not a trivial problem. The model provides a proactive approach to solve, in real time, problems that occur on the railway line.

The computational time (CT) for all time horizons and disturbance sizes is of the order of a few second. Comparison between different time horizons is done in order to demonstrate that the model is able to quickly solve even considersables problems that take into account all trains on the railway line. In fact, the highest value of CT (equal to 69.77 seconds) is
obtained using $\text{Obj}_1$ for $H = 1440$ and $\Delta_d = 45$ minutes. This means that if a disturbance lasting 45 minutes occurs along the line, in just over a minute the train dispatcher can obtain the rescheduled timetable for the 24 hours following the occurrence of the fault. The speed of resolution is a very important factor for the presented problem, since the main objective of the real-time traffic management is to quickly establish a new timetable, in order to minimize the inconvenience for passengers.

Compared to the previous methodology used in (Dotoli et al., 2013) there is an improvement due to a reduction of total delay, number of rescheduled trains and computational time. The objective of this model is the minimization of delay of all movements. In Table 1 we compare results of the rescheduling process obtained with the application of the actual and the previous methodology (AM, PM) when $\Delta_d = 30$ minutes and $H = 180$ minutes. We noticed that $\text{Obj}_1$ is unchanged for all subsequent values of time horizon. This means that the delay caused by the disturbance is absorbed within the 180 minutes after its occurrence. For the previous methodology, we present values obtained for $H = 30$ minutes in addition to values obtained with the application of the heuristic algorithm after the time horizon. CT refers only at the optimization procedure.

By analyzing values we observe that with the actual methodology the new timetable is computed in 0.72 seconds. Three trains are involved in the rescheduling process for a total delay of 608 minutes. Previous methodology required 10.74 seconds to obtain the new timetable within a 30 minutes window after the occurrence of the fault. 10 trains are involved in the rescheduling process and total delay is equal to 665 minutes. Actual methodology allows to obtain the optimal solution with an exact approach, without the application of the heuristic algorithm which does not always provide optimal results. We should also take into account that solvers used by the two methodology are different. MATLAB with GLPK used by the previous methodology is replaced by IBM CPLEX in the actual one. Resolution methods used by the two solvers are different as well as their performance. The previous methodology has obtained an improvement compared to the current practice used by the train dispatcher; the actual methodology provides a further amelioration. This is in line with objectives of the real-time traffic management.

4.1.2 Comparison between Objective Functions

We compare values obtained using the four objective functions for $H = 1440$ and $\Delta_d = 50$ minutes, shown in Table 2. We analyze the time horizon of 1440 minutes because the complexity of the problem is high due to the presence, in the rescheduling process, of all trains movements on the railway line until the end of the day.

Lowest values in terms of total delay are obtained using $\text{Obj}_1$. Comparing results obtained with the first and the second objective function we notice that although the value of $\text{Obj}_2$ is the same, $\text{Obj}_1$ changes. In particular, $\text{Obj}_1$ obtained while $\text{Obj}_2$ is greater.

The reason is simple: in this case, minimizing the delay of the last movement of a train, the second objective function increases the number of its delayed movements. This means that although values of $\text{Obj}_3$ are unchanged using the first and the second objective function; values of $\text{Obj}_4$ varies. In general, $\text{Obj}_2$ increases the arrival time at intermediate stations, in order to minimize the delay at the last station of trains path. In this case, there are no differences between values of $\text{Obj}_1$ and $\text{Obj}_2$ obtained using $\text{Obj}_3$ and $\text{Obj}_4$.

Extending the analysis to all time horizons, we notice that in some cases (e.g. $H = 180$ and $\Delta_d = 50$ minutes) there is a difference between the two values, due to the fact that these objective functions does not take into account the exact delay of trains. Thus, any solution showing the same number of delayed trains is optimal, whatever the value of $\text{Obj}_1$ and $\text{Obj}_2$. Multicriteria objective functions should be used to obtain an unique optimal solution. The minimization of the number of delayed trains (respectively movements) may imply an increase of the total delay of rescheduled trains.

Extending the analysis to values obtained in all time horizons ($H$) and for all disturbance size ($\Delta_d$) we notice that minimizing $\text{Obj}_1$ provides better results in terms of $\text{Obj}_1$ and $\text{Obj}_2$. Minimizing $\text{Obj}_4$ provides better values in terms of $\text{Obj}_3$ and $\text{Obj}_4$.

4.1.3 Impact of Constraints (22), (23) and (24)

In order to prove the necessity of constraints (22), (23) and (24) when using $\text{Obj}_3$ and $\text{Obj}_4$, we present a simple railway line made by 7 segments on which circulate 5 trains, represented in Figure 5. Railway line is made by 4 single-tracked connection segment ($b_1, b_3, b_5$ and $b_7$) between 3 double-tracked stations ($b_2, b_4$ and $b_6$). Trains $t_1$ and $t_3$ are directed to the South, while $t_2, t_4$ and $t_6$ travel in the opposite direction.

Table 1: Comparison between actual and previous methodology.

<table>
<thead>
<tr>
<th></th>
<th>Our Methodology</th>
<th>Dotoli et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Obj}_1$</td>
<td>608</td>
<td>665</td>
</tr>
<tr>
<td>$\text{Obj}_3$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>CT</td>
<td>0.72</td>
<td>10.74</td>
</tr>
</tbody>
</table>


Table 4: Two independent disturbances.

<table>
<thead>
<tr>
<th>Segment</th>
<th>FirstDisturbance</th>
<th>SecondDisturbance</th>
<th>TwoDisturbances</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ob j1</td>
<td>516 61 1 9 13.43</td>
<td>308 61 3 26 17.98</td>
<td>824 122 4 35 31.41</td>
<td>871 132 2 14 18.89</td>
</tr>
<tr>
<td>Ob j2</td>
<td>516 61 1 9 19.08</td>
<td>308 61 3 26 17.63</td>
<td>824 122 4 35 29.58</td>
<td>871 132 2 14 9.16</td>
</tr>
<tr>
<td>Ob j3</td>
<td>516 61 1 9 10.42</td>
<td>308 61 1 5 9.81</td>
<td>852 132 2 14 18.89</td>
<td>871 132 2 14 10.12</td>
</tr>
<tr>
<td>Ob j4</td>
<td>516 61 1 9 10.42</td>
<td>308 61 1 5 9.81</td>
<td>852 132 2 14 18.89</td>
<td>871 132 2 14 10.12</td>
</tr>
</tbody>
</table>


Table 2: Comparison between the four objective functions with $H = 1440$ and $\Delta_d = 50$.

<table>
<thead>
<tr>
<th>Ob j1</th>
<th>Ob j2</th>
<th>Ob j3</th>
<th>Ob j4</th>
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<tbody>
<tr>
<td>829</td>
<td>854</td>
<td>878</td>
<td>878</td>
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<tr>
<td>81</td>
<td>81</td>
<td>91</td>
<td>91</td>
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<td>3</td>
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<td>1</td>
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<tr>
<td>33</td>
<td>39</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>CT</td>
<td>30.07</td>
<td>16.86</td>
<td>6.56</td>
</tr>
</tbody>
</table>

Table 3: Results from experiments using Ob j3 without and with additional constraints.

<table>
<thead>
<tr>
<th>Ob j1</th>
<th>Ob j2</th>
<th>Ob j3</th>
<th>Ob j4</th>
<th>CT</th>
</tr>
</thead>
<tbody>
<tr>
<td>10400</td>
<td>10000</td>
<td>1</td>
<td>5</td>
<td>0.01</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>1</td>
<td>5</td>
<td>0.01</td>
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</table>

In the example, $t_1$ is the only delayed train. By introducing additional constraints, values of Ob j1 (and consequently Ob j2) are lower because the system is forced to reschedule movements within the time horizon. The number of rescheduled trains remains unchanged.

4.2 Two Disturbances on the Line

When two independent disturbances occur on the railway line, the rescheduling process of the first disturbance does not influence the rescheduling process of the second one. We consider the same railway line presented in Section 4 and we suppose that two independent disturbances occur at $T_d = 09:00$ a.m. respectively in the line that connects Rutigliano to Conversano stations (segment $b_9$) and Conversano to Castellana G. (segment $b_{11}$). We consider a time horizon $H = 1440$ minutes and a disturbance size $\Delta_d = 50$ minutes.

First, we solve the problem considering only the disturbance that occurs in the segment $b_9$ and that affects a train directed from Putignano to Mungivacca station.

Then, we solve the problem considering only the disturbance that occurs in the segment $b_{11}$ and that affects a train directed from Mungivacca to Putignano station.

Finally, we solve the problem considering the two disturbances at the same time. We compare results with those given by the sum of values obtained solving the two problems separately.

Table 4 presents values of Ob j1, Ob j2, Ob j3, Ob j4 and CT for the four scenarios.

By comparing values obtained from simultaneous resolution with those obtained from the sum of individual resolutions of disturbances, i.e. values presented in the third and the fourth block of Table 4, we notice that the simultaneous resolution provides a better result in terms of computational time for all objective functions. However, the sum of individual resolutions provides an improvement in terms of Ob j1 and Ob j4 using the four objective functions. In particular by applying Ob j1, there is a reduction of the number of rescheduled movements and by applying Ob j2, there is also a reduction of total delay. By
<table>
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**Table 5: Results of analysis using $Q_{ih}, Q_{ij}, Q_{ih},$ and $Q_{ij}$ for all $H$ and $A.$**
using Obj$_3$ and Obj$_4$, there is a decrease of values of Obj$_1$. However, it is interesting to note that when multiple independent disturbances occur on the line, it is possible to decompose the problem in independent subproblems. In this way, the train dispatcher can give priority to the rescheduling of trains which paths include stations where a higher level of service is required or that have to comply connections with other trains. One could expect that two disturbances would be more difficult to solve but, according to the first experiments, this is not the case. More particularly, the time needed to solve the first disturbance is greater than the time needed to solve both, perhaps due to number of embedded variables. More explicit, we are in progress to verify that.

5 CONCLUSIONS AND PERSPECTIVES

In this paper, we propose a formalization of the rescheduling real-time problem for a regional single-tracked railway network in which a CTC control system is installed. We propose a mathematical model that operates as a decision support system for the train dispatcher. The main goal is to find a decision support system for the train dispatcher that is able to restore normal traffic conditions after the occurrence of a disturbance and to provide an adequate level of service to passengers. We analyze four alternative objective functions in order to find the optimal solution that is a good compromise between total delay, number of rescheduled trains and computational time.

There are many perspectives for this work:

- study other resolution methods most suitable to the complexity of the problem, such as constraints programming, able to reproduce a set of possible solutions.

REFERENCES


