A Mixed-Integer Linear Program for Routing and Scheduling Trains through a Railway Station

Lijie Bai¹, Thomas Bourdeaud’huy², Besoa Rabenasolo¹ and Emmanuel Castelain¹

¹LM2O - Laboratoire de Modélisation et de Management des Organisations, École Centrale de Lille, Cité Scientifique, Villeneuve d’Ascq, France
²LAGIS - Laboratoire d’Automatique, Génie Informatique et Signal, École Centrale de Lille, Cité Scientifique, Villeneuve d’Ascq, France

Keywords: Train Scheduling, Train Routing, Mixed-Integer Linear Program, Robust Timetable.

Abstract: This paper studies a train routing and scheduling problem faced by railway station infrastructure managers to generate a conflict-free timetable which consists of two parts, commercial movements and technical movements. Firstly, we present the problem and propose a discrete-time mixed-integer linear mathematical model formulation. Due to the computational complexity of integer programming methods, we need to improve the calculation performance. On one hand, we consider the problem in continuous-time domain which decrease the computational size. The integrality of the scheduling variables is proved. On the other hand, the redundant constraints are cut off by probing the potential conflicts between trains and movements. The full practical problem is large: 247 trains consisting of 503 movements per day should be considered. The proposed approach can solve an instance made of 60 trains and 121 movements representing 385 minutes of traffic within less than 2 minutes.

1 INTRODUCTION

In most countries, rail network is a busy system with increasing patterns of train services that require accurate scheduling and routing to adapt to the limited infrastructures. The traditional process to generate a timetable for a railway network is divided into several stages (Watson., 2001). First, a draft timetable is generated by train activities managers (national, regional, freight) based on the traffic frequencies, the volume of traffic, the rough layout of the railway network between the railway stations together with the desired lines and their connection requirements (Schrijver and Steenbeek, 1994) (Serafini and Ukovich., 1989). Then, station operators need to check whether the draft timetable is feasible within the railway station while satisfying capacity, safety and customer service (Kroon and Zwaneveld, 1995) (Zwaneveld et al., 1996). At the same time, schedules for the trains through the railway station are generated by including all the required technical operations such as carriage preparation, maintenance, etc. So far, the conflicts of proposed train times, lines and platforms are found and resolved by hand. Most of the studies focus on the problem of railway network with a global point of view (D’Ariano et al., 2007) (D’Ariano, 2008) (Caimi, 2009). Nevertheless the routing and scheduling problem in large, busy, complex train stations is also a complex issue with respect to time and space.

This paper studies a train routing and scheduling problem faced by railway station managers to generate a conflict-free timetable which consists of two sets of circulations. The first set is made of commercial circulations given by several administrative levels (national, regional, freight) over a large time horizon (typically one year before the effective realization of the production). The other set corresponds to technical circulations added by the railway station managers to prepare or repair the trains. The routing problem is the problem of assigning each of the involved trains to a route through the railway station and to a platform in the station. Thus, routes and platforms in the station are here the critical resources of the system. The scheduling problem is to adjust the timetable of technical circulations to guarantee on-time arrivals and leavings of all the commercial circulations. A conflict-free timetable with acceptable commercial circulations and needed technical circulations is generated. Commercial circulations with unsolvable conflicts will return to their original activity managers with suggestions for the modification of the arrival and leaving times.
(Carey, 1994b) proposes a mixed integer program to find the paths of trains in a one-way track system. The numerical example provided in Carey’s paper has 10 nodes, 28 links, and 10 train services and requires a significant amount of time to be solved. In another article, (Carey, 1994a) extends the model from one-way to two-way tracks system. The resulting model is also a mixed integer program, which is easier to solve than his earlier model, but this newer study does not provide testing results. (Kroon et al., 1997) consider computational complexity of the problem of routing trains through railway stations. They show that the problem is NP-complete if each train has three or more routing possibilities. (Zwaneveld et al., 2001) describe the routing problem of trains through a railway station with the given arrival and leaving times of trains and the detailed layout of the railway station. In section 4, we give practical improvements to our model is proposed as a mixed-integer linear program. (Carey, 1994) extends the model from one-way to two-way tracks system. The subset of trains is defined by a set of ordered switches \( s^p \), with the cardinal number \( S^p \). Switches of a path are always described from railway station to the outside. For each path, we consider two special switches \( s_1^p \) (internal switch) and \( s_2^p \) (external switch). The set of lines is defined by \( L = \{ l_1, l_2, \ldots, l_l \} = \{ l_i \}_{i \in [1, L]} \). \( L \) denotes the cardinal of the set of lines \( L \). We make a distinction between internal and external lines. Passengers board or get off the train on the platforms in front of internal lines. They are denoted by the set \( L^{\cap} \). External lines are located at the entrance of the railway station. They are denoted by the set \( L^{\cup} \). Internal and external lines can be connected together using the set of switches, through a small railway network inside the railway station. Every line \( l \in L \) is connected to a unique “entrance” switch denoted as \( \zeta(l) \in S \), while a switch may be connected to multiple lines.

### 2 PROBLEM FORMALIZATION

**Definition 1 (Railway Station).** A railway station \( R = (S, L, P) \) is defined by a set of lines \( L \) on which trains follow some paths in a set \( P \), defined using switches in the set \( S \).

**Switches \( (s_k) \).** The set \( S = \{ s_1, s_2, \ldots, s_S \} = \{ s_k \}_{k \in [1, S]} \) designates a set of switches. The cardinal number of \( S \) is denoted as \( S \).

**Lines \( (l_i) \).** The set of lines is defined by \( L = \{ l_1, l_2, \ldots, l_L \} = \{ l_i \}_{i \in [1, L]} \). \( L \) denotes the cardinal of the set of lines \( L \). We make a distinction between internal and external lines. Passengers board or get off the train on the platforms in front of internal lines. They are denoted by the set \( L^{\cap} \). External lines are located at the entrance of the railway station. They are denoted by the set \( L^{\cup} \).

**Paths \( (p_c) \).** The set of paths is denoted by \( P = \{ p_1, p_2, \ldots, p_P \} = \{ p_c \}_{c \in [1, P]} \). \( P \) denotes the cardinal number of \( P \). A path \( p \in P \) consists of a set of ordered switches \( p = \{ s_1^p, s_2^p, \ldots, s_p^p \} = \{ s_k^p \}_{k \in [1,p]} \) with the cardinal number \( S^p \). Switches of a path are always described from railway station to the outside. For each path, we consider two special switches \( s_1^p \) (internal switch) and \( s_2^p \) (external switch). The set \( P \) reflects the topology of the railway station, and some sequences of switches are not valid paths.

The subset of paths that connect the internal line \( l_i \in L^{\cap} \) and the external line \( l_e \in L^{\cup} \) is denoted by \( P^{(l_i, l_e)} = \{ p_{c, i} \}_{c \in [1,P]} \). The subset of internal lines \( l_i \) reachable from an external line \( l_e \in L^{\cup} \) is denoted by \( L^{\cap}_{L^{\cup}} \).

The traffic in the railway station is defined by a set of trains. Each train may be composed of several “technical” movements and “commercial” movements. A commercial movement denotes a circulation of a train taking passengers onboard. A technical movement denotes a circulation without passengers, corresponding to the locomotive only or to empty wagons.

**Definition 2 (Movement).** Let \( R = (S, L, P) \) be a railway station. The set of considered movements is defined as \( M = \{ m_1, m_2, \ldots, m_M \} = \{ m_j \}_{j \in [1, M]} \). \( M \) is the cardinal of \( M \).

A movement \( m \in M \) is defined by its type, its reference time, its actual time interval, its internal line (generally unknown to be determined), its external line (given) and its path (to be determined).

**Lines of a Movement \( (l_m^{\cap} \) and \( l_m^{\cup} \).** The internal line (resp. external line) of a movement \( m \in M \) is denoted by \( l_m^{\cap} \in L^{\cap} \) (resp. \( l_m^{\cup} \in L^{\cup} \)).

The subset of movements going through an external line \( l \in L^{\cup} \) (i.e. for which \( l_m^{\cup} = l \)) is denoted...
by \( M'. \)

**Paths of a Movement** \((p_m)\). Let \( m \in M \) be a movement. The path of the movement \( m \) is denoted by \( p_m \in P \). Since this path should describe a circulation between lines \( \ell_m^\alpha \) and \( \ell_m^\beta \), we have obviously:

\[
s_{1}^{p_m} = \zeta(\ell_m^\alpha) \text{ and } s_{2}^{p_m} = \zeta(\ell_m^\beta) \tag{1}
\]

which restricts the number of possible paths for the movement \( m \).

**Actual Time Interval of a Movement** \( ([\alpha_m, \beta_m]) \). Let \( m \in M \) be a movement. The actual time interval of the movement \( m \) is defined by \( [\alpha_m, \beta_m] \) with \( \alpha_m, \beta_m \in \mathbb{N} \) and \( \alpha_m < \beta_m \). In this paper, we consider that the movement occupies all corresponding resources (i.e. the switches) over its actual time interval. In our case study, the length of this time interval is fixed to \( S = 5 \) minutes, so we have \( \beta_m = \alpha_m + S \).

**Type of a Movement**. Four types of movements are defined depending on their commercial or technical nature, and their direction (entering or leaving the railway station).

In the following paragraphs, the technical movements are denoted by a semi-arrow \( \hookrightarrow \): the commercial movements are denoted by a full arrow \( \rightarrow \): a train leaving the railway station is denoted by \( \bigcirc \): a train entering the railway station is denoted by \( \blacklozenge \) (the full circle \( \bigcirc \) being a mnemotechnic way to denote the railway station).

We divide thus the set of movements \( M \) into four subsets such that:

\[
M = M^\bigcirc \cup M^{\bigcirc} \cup M^{\blacklozenge} \cup M^{\blacklozenge}.
\]

**Reference Times** \( (\alpha_m^{ref}, \beta_m^{ref}) \). We define reference times \( \alpha_m^{ref} \) and \( \beta_m^{ref} \) depending on the type of the considered movement. These reference times constrain the possible values for the actual time interval of a movement, allowing to advance or postpone some technical movements in order to free the railway network for other commercial circulations. In this study, we consider that the adjustment should not last more than \( L = 10 \) minutes.

- A technical movement \( m \in M' \) entering the railway station is associated to a reference time \( \beta_m^{ref} \) denoting the latest termination date of this movement such that:

\[
\forall m \in M^{\bigcirc}, \exists \beta_m^{ref} \in \mathbb{N} \text{ s.t. } \beta_m^{ref} = \beta_m - L \tag{2}
\]

- A technical movement \( m \in M' \) leaving the railway station is associated to a reference time \( \alpha_m^{ref} \) denoting its earliest starting date such that:

\[
\forall m \in M^{\bigcirc}, \exists \alpha_m^{ref} \in \mathbb{N} \text{ s.t. } \alpha_m^{ref} + L \geq \alpha_m \geq \alpha_m^{ref} \tag{3}
\]

- A commercial movement \( m \in M \) entering the railway station should arrive exactly at the reference time \( \beta_m^{ref} \) such that:

\[
\forall m \in M^{\bigcirc}, \beta_m = \beta_m^{ref} \tag{4}
\]

- A commercial movement \( m \in M \) leaving the railway station should leave exactly at the reference time \( \alpha_m^{ref} \) such that:

\[
\forall m \in M^{\bigcirc}, \alpha_m = \alpha_m^{ref} \tag{5}
\]

A set of technical and commercial movements can define a train whose properties are inherited from its movements.

**Definition 3** (Train). Let \( R = (S, L, P) \) be a railway station. The set of trains is denoted by \( T = \{t_1, t_2, \ldots, t_r\} \subseteq \{t_1, \ldots, T\} \). The cardinal number of \( T \) is denoted by \( T \). Every train \( t \in T \) consists of a set of movements \( M' = \{m_1', m_2', \ldots, m_{t_1}'\} \subset M' \). We denote by \( M' \) the cardinal number of \( M' \).

**Internal Lines of a Train**. Each movement of a train must be executed on the same internal line, denoted by \( \lambda_t \in L \) such that:

\[
\forall t \in T, \exists \lambda_t \in L \text{ s.t. } \forall m \in M', \ell_m^{\lambda_t} = \lambda_t \tag{6}
\]

**Actual Time Interval of a Train** \( ([A_t, B_t]) \). Let \( t \in T \) be a train and \( \lambda_t \in L \) its internal line. The train \( t \) occupies the line \( \lambda_t \) during the interval \([A_t, B_t]\), such that:

\[
A_t = \min_{m \in M'} \lambda_m \tag{7}
\]

\[
B_t = \max_{m \in M'} \lambda_m \tag{8}
\]

Obviously, every movement of the train occurs during this interval of time:

\[
\forall t \in T, \forall m \in M', [\alpha_m, \beta_m] \subset [A_t, B_t] \tag{9}
\]

We partition the set of movements of a train \( t \) according to the types of movements defined above:

\[
M' = M^\bigcirc M^{\bigcirc} M^{\blacklozenge} M^{\blacklozenge}.
\]

Obviously, the constraints (2) to (5) must be applied to the movements of a train. Furthermore, we need additional constraints to ensure the safety of trains movements.

**Lines Occupation Constraint**. A line can not be occupied by two trains at the same time:

\[
\forall t, t' \in T \text{ s.t. } \lambda_t = \lambda_{t'}, [A_t, B_t] \cap [A_{t'}, B_{t'}] = \emptyset \tag{10}
\]
Switches Occupation Constraint. Two movements using paths containing a common switch cannot be scheduled during the same time interval:

$$\forall s \in S, \forall m, m' \in M \text{ s.t. } s \in S_p \cap S_{p'} \left[ \alpha_m, \beta_m \cap [\alpha_m', \beta_m'] = \emptyset \right] \ (11)$$

In the next section, we propose a mixed-integer linear program to solve the allocation problem described above.

## 3 MIXED-INTEGER LINEAR PROGRAMMING MODEL

Hereafter, the function $\delta(Q)$ is an indicator such that $\delta(Q) = 1$ if the condition $Q$ is valid, otherwise 0.

### 3.1 Parameters

- $C$ is a sufficiently large constant.
- $L$ is the adjustable time interval of the technical movements.
- $c_{m,i}^{ref}$ is the reference starting time of the movement $m$.
- $\beta_{m,i}^{ref}$ is the reference ending time of the movement $m$.
- $S$ is the time allocated to a movement. In our context, $S = 5$ minutes.
- $Y_{m,i}$ identifies the movements belonging to trains, $Y_{m,i} = \delta(m \in M^t)$.
- $Y_{s,p}$ identifies the switches composing a path $p$. $Y_{s,p} = \delta(s \in p)$.
- $Y_{l,m}^{LBM}$ identifies the external line of the movement. $Y_{l,m}^{LBM} = \delta(l = l_m)$.
- $Y_{p,p'}^{LBM}$ identifies the conflict of switches between two paths. $Y_{p,p'} = \delta(p \cap p' \neq \emptyset)$.

### 3.2 Variables

In the practical situation, the arrival and leaving times of trains are measured in minutes. The scheduling decision variables are thus defined as integers with units of minutes, characterizing a discrete-time scheduling problem.

- $\alpha_m$ is actual starting time of the movement $m$.
- $\beta_m$ is actual ending time of the movement $m$, $\alpha_m + S = \beta_m$.
- $A_t$ is the starting time of occupation of the railway station by the train $t$.
- $B_t$ is the ending time of occupation of the railway station by the train $t$.

All the scheduling decision variables have values that fit the length of one day (1440 minutes). The routing decision variables are defined as binary variables.

- $X_{t,m}^A$ identifies the first movement of trains. $X_{t,m}^A = \delta(A_t = \alpha_m)$.
- $X_{t,m}^B$ identifies the last movement of trains. $X_{t,m}^B = \delta(B_t = \beta_m)$.
- $X_{t,m}^{LBM}$ identifies the internal lines of trains. $X_{t,m}^{LBM} = \delta(l = l_m)$.
- $X_{t,m}^{LBM}$ identifies the internal lines of movements. $X_{t,m}^{LBM} = \delta(l = l_m)$.
- $X_{p,m}^{LBM}$ identifies the path of movements. $X_{p,m}^{LBM} = \delta(p = p_m)$.
- $X_{t,m}^{Order}$ identifies the time order of two trains using the same line. $X_{t,m}^{Order} = \delta(t$ circulates before $t')$.
- $X_{m,m}^{Order}$ identifies the time order of two movements using two paths with the same switch(es). $X_{m,m}^{Order} = \delta(m$ circulates before $m')$.

### 3.3 Constraints

The constraints (1) to (11) are expressed as linear constraints with the parameters and variables defined above.

#### Time Interval of a Train

According to equations (7)/(8), the time interval of a train covers all the movements of the train, which can be formulated in a classical way as below:

$$\forall t \in T, \forall m \in M^t, \quad A_t \leq \alpha_m \leq A_t + C \cdot (1 - X_{t,m}^A) \ (12)$$

$$\forall t \in T, \forall m \in M^t, \quad \alpha_m \leq B_t \leq \beta_m \leq \beta_m + L \cdot X_{t,m}^B \ (13)$$

$$\forall t \in T, \forall m \in M^t, \quad B_t \leq B_m \leq B_m + C \cdot (1 - X_{t,m}^B) \ (14)$$

#### Time Constraints

The constraints (2) to (5) are expressed as follows:

$$\forall m \in M, \quad \beta_m - L \leq \beta_m \leq \beta_m^ref \ (16)$$

$$\forall m \in M, \quad \alpha_m^ref + L \geq \alpha_m \geq \alpha_m \ (17)$$

$$\forall m \in M, \quad \beta_m = \beta_m^ref \ (18)$$

$$\forall m \in M, \quad \alpha_m = \alpha_m^ref \ (19)$$
Allocation of Lines. For the movements passing on a given external line \( l_e \), we allocate an internal line \( l_t \in L_{i \rightarrow e}^c \) which is reachable from the line \( l_e \). This property and equation (6) can be expressed as follows:

\[
\forall t \in T, \forall m \in M_t^e, \forall l_t \in L_{i \rightarrow e}^c, X_{i \rightarrow e, m} = X_{l_t \rightarrow l_t}^{c} \tag{20}
\]

\[
\forall l_t \in L_{i \rightarrow e}^c, \forall m \in M_t^e, \sum_{l_t \in L_{i \rightarrow e}^c} X_{l_t \rightarrow l_t}^{c} = 1 \tag{21}
\]

Allocation of Paths. According to equation (1), the choice of paths for a movement can be expressed as below:

\[
\forall l_t \in L_{i \rightarrow e}^c, \forall m \in M_t^e, \forall l_e \in L_{e \rightarrow i}^c, \sum_{p_{e \rightarrow i}(i \rightarrow e)} X_{p \rightarrow e}^{PM} = X_{l_t \rightarrow l_t}^{M} \tag{22}
\]

Compatibility of Lines. The constraints of occupation of lines (10) indicate that two trains cannot occupy a same line at the same time. This rule is expressed as follows:

\[
\forall t, t' \in T, t \neq t', \forall l \in L_{i \rightarrow e}^c, B_{l} \leq A_{l} + C \cdot (3 - X_{l_t \rightarrow l_t}^{c} - X_{t \rightarrow t'}^{c} - X_{t \rightarrow t \rightarrow t'}^{c} - X_{t \rightarrow t \rightarrow t'}^{c}) \tag{23}
\]

The constraint (23) indicates that if two trains \( t \) and \( t' \) are allocated to the same line \( l \) in the railway station and if the train \( t \) circulates before \( t' \), then the term \( 3 - X_{l_t \rightarrow l_t}^{c} - X_{t \rightarrow t'}^{c} - X_{t \rightarrow t \rightarrow t'}^{c} - X_{t \rightarrow t \rightarrow t'}^{c} = 0 \). We have then \( B_{l} \leq A_{l} \). Otherwise this term is larger than zero, and the constraint (23) is relaxed.

Compatibility of Switches. The constraint of occupation of switches (11) indicates that two movements \( m \) and \( m' \) cannot pass the same switches at the same time. Such constraint is enforced as above.

\[
\forall m, m' \in M, m \neq m', \forall p, p' \in P, p \neq p' \leq \beta_{m} \leq \alpha_{m} + C \cdot (4 - X_{p \rightarrow m}^{PM} - X_{p' \rightarrow m}^{PM} - X_{m, m'}^{OrderM} - X_{p, p'}^{PM}) \tag{24}
\]

\[
\forall m, m' \in M, m \neq m', X_{m, m'}^{OrderM} + X_{m, m'}^{OrderP} = 1 \tag{25}
\]

Objective Function. The objective we focus on is to minimize the lines’ occupancy rate, which can be expressed as follows:

\[
\min_{t \in T} \sum_{i \in I} (B_{i} - A_{i}) \tag{26}
\]
• each column contains at most two non-zeroes;
• the set $N$ of row indices of $A$ can be partitioned into $N_1 \cup N_2$, so that in each column $j$ with two non-zeroes we have $\sum_{i \in N_1} a_{i,j} = \sum_{i \in N_2} a_{i,j}$.

For the matrix $A^T$, we have $\sum_{i \in N_1} a_{i,j} = \sum_{i \in N_2} a_{i,j} = 0$. According to the proposition above, $A^T$ is a totally unimodular matrix.

**Proposition 2.** If $A$ is TU then $A^T$ also TU.

**Theorem 1.** (Hoffman and Kruskal, 1956) Let $A$ be an integral $m \times n$ matrix, the polyhedron $P(A, b) = \{x : x \geq 0, Ax = b\}$ is integral for all integral vectors $b \in \mathbb{Z}^n$ if and only if $A$ is totally unimodular.

Based on Hoffman and Kruskal’s theorem, every vertex, solution, the $n$-vector $x$, is integral. The integrality of the scheduling decision variables is guaranteed. So the scheduling decision variables $\alpha_m, \beta_m, A'_m$, and $B'_m$ are defined in the continuous-time domain.

### 4.2 Reduction of Model

To improve the calculation performance, we seek to reduce the number of constraints. We design an indicator as the probe of potential conflicts between movements $C_{m,m'}^{ref \text{FM}}$, and between trains $C_{i,j}^{ref \text{T}}$. In this way, the constraints are created only for the movements and trains with potential conflicts. The undesired constraints are cut off. Four additional parameters need to be created as below.

$\alpha_m^{Early}$ is the earliest departure time of the movement $m$.

$\beta_m^{Late}$ is the latest arrival time of the movement $m$.

$A'_m^{Early} = \min_{m' \in M^T} \alpha_m^{Early}$

$B'_m^{Late} = \max_{m' \in M^T} \beta_m^{Late}$

The possible time interval of technical movements is $[\alpha_m^{Early}, \beta_m^{Late}]$. The possible time interval of trains is $[A'_m^{Early}, B'_m^{Late}]$. These parameters can be precalculated using the given problem instance. In this case, for all $m \in M^T$, we have $[\alpha_m^{Early}, \beta_m^{Late}] = [\alpha_m^{ref}, \beta_m^{ref} + L]$. For all $m \in M^T$, we have $[\alpha_m^{Early}, \beta_m^{Late}] = [\alpha_m^{ref} - L, \beta_m^{ref}]$.

$C_{m,m'}^{ref \text{FM}} = S(\alpha_m^{Early}, \beta_m^{Late}) \cap \{[\alpha_m^{Early}, \beta_m^{Late}] \neq \emptyset\}$ indicate the potential time conflict of two movements.

$C_{i,j}^{ref \text{T}} = S([A'_i^{Early}, B'_i^{Late}] \cap [A'_j^{Early}, B'_j^{Late}] \neq \emptyset)$ indicate the potential time conflict of two trains.

So $C_{m,m'}^{ref \text{FM}} = 1$ is added as a condition in the equation (23) and (24), $C_{i,j}^{ref \text{T}} = 1$ is added as a condition in the equation (25) and (26).

The numerical experiments in section 4.3 show that the number of constraints decreases considerably.

### 4.3 Numerical Experiments

The computational study is conducted using AMPL and CPLEX version 12.5. The hardware architecture is x86-64, with Intel i5-2520M CPU at 2.5GHz and 8GB memory RAM.

We compare the original model and the improved model using a real railway station with 18 switches, 15 internal lines and 10 external lines. There are 247 trains 504 movements per day. In the rush hours, trains are up to 3 trains running at the same time and up to 10 trains staying at the platforms.

Once the variables and constraints are sent to the solver, the problem will be adjusted by CPLEX to solve which eliminates the redundant constraints and variables. The whole problem is divided into small size problems in chronological order. So we have 50 groups of 5 trains, 24 groups of 10 trains, 16 groups of 15 trains, 12 groups of 20 trains, 9 groups of 25 trains and 8 groups of 30 trains. The draft timetable that defines the problem instances includes the parameters reference times of commercial movements and technical movements without any feasibility checking.

We try to solve the problems with three different models that are described in Section 3 and Section 4: discrete-time model (DT), continuous-time model (CT) and reduced continuous-time model (RCT). The results are separately presented in Table 1, Table 2 and Table 3. In each group, the complexity of the problems is different. The problem instance solved in the minimum or the maximum solve time is presented in the tables(rows labeled Min and Max respectively).

The problem with the average solve time is to be constructed using the solve information of the whole groups.

Compared with the discrete-time model in Table 1, the continuous-time model has the same amount of variables and constraints, but the solve time decreases by 17.5% on average. The discrete-time model has 9 problems unsolved, and the continuous-time model has 5 problems unsolved. The solutions are all integral as we have proven in the section 4.1. So the improvements of continuous-time model are qualified.

Compared with the complete model, the reduced continuous-time model drops 22.1% variables and 66.2% constraints on average. The solve time decreases by 45.7% compared with the discrete-time model and decreases by 30.6% compared with the continuous-time model.

The infeasibility case is caused by the conflicts between the technical movements and the commercial movements. The adjustable time interval for technical movements $L = 10$ minutes in equations (16) and (17) is too tight to ensure the existence of solution.
When the value of $L$ is increased, one can find an optimal solution but the solving time can also be greatly increased. For example, setting $L = 30$ helps to find solutions to three cases previously labeled infeasible at the expense of solve time 330 seconds instead of the average time of 12 seconds. Further experiments are necessary so that the value of $L$ is adjusted in order to get the best tradeoff between the solution feasibility and the solving time.

### 5 CONCLUSIONS

This paper describes a mixed-integer linear program for routing and scheduling trains through a railway station to find a conflict-free schedule, given the detailed information of commercial movements. Considering the time representation, we compare the continuous-time and discrete-time movements. The continuous-time mathematical model satisfies our computational requirement and decreases the problem size. Furthermore, to speed up the calculation, we try to get the best tradeoff between the solution feasibility and the solving time.
to cut off the redundant constraints and concentrate on the potential conflicts. Based on the numerical experiments, the improvements of the reduced continuous-time model are qualified. For the moment, we can solve example up to 60 trains, 121 movements during 385 minutes. The solve time of the first feasible solution is 97.8438 seconds. The solve time depends on the testing example.

To solve problems of larger size, we propose to use the decomposition methods (Benders, 1962) (Binato et al., 2001) (Cordeau et al., 1975). All trains are divided into groups in chronological sequence. The group solved is considered as the valid constraints of shared resources for the succedent groups. The adjacent groups have common trains as a buffer, i.e. the group size is 40 and the buffer group size is 20. The partitioning procedures are followed until the end of the problem. This method can be used to solve the real-time train routing and scheduling problem.

REFERENCES


