Using Clustering Method to Solve Two Echelon Multi-Products Location-Routing Problem with Pickup and Delivery

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Abstract: In this paper, we consider the Location Routing Problem in two-echelon network with Multi-Products, and Pickup and Delivery (LRP-MPPD-2E). The objective of LRP-MPP-2E is to minimize simultaneously the location and routing costs, considering many realistic non-tackled constraints in the literature. The first echelon deals with simultaneously selection of processing centers from a set of potential sites and the construction of the primary tours such that each primary tour starts from the main depot, visits the selected processing centers and returns to the main depot. The second echelon aims at assigning customers to the selected processing centers and defining the secondary tours. Each secondary tour, starts at a processing center, visits a set of customers, through one or several processing centers, and then returns to the first processing center. We develop a Hybrid Clustering Algorithm (HCA) with the objective of constructing Global-Clusters such that each Global-Cluster represents the set of clients associated to one feasible secondary tour, then Cplex Solver calculates the feasible tour within Global-Cluster. The HCA is compared with a Nearest Neighbour heuristic (NNH), which actually is the only available method for this problem, and with a Clustering-NNH in which Cplex solver is used to improve each secondary route obtained by NNH. Computational experiments are conducted to evaluate the performances of proposed approaches.

1 INTRODUCTION

By the emergence of complex logistic networks, the enterprises need more flexible and efficient decision methods to manage the involved flows. The location routing problem (LRP) and its variants are the models of the literature that addressing issues related to these complex logistic networks. The LRP allows combining the strategic decisions related to the selection of potential sites with the tactical and operational decisions related to the assignment of customers to the selected potential sites and the construction of routes in order to serve all customers demands. The objective of the LRP is to minimize the total cost including routing costs, vehicle fixed costs, and potential site operating costs. Many authors showed that ignoring routing in the location problem might lead to sub-optimal solution (Prins et al., 2006a).

A wide variety of application fields are concerned by LRP and its variants, which explain a growing number of LRP studies considered in the literature. Some review of LRP models, approaches and applications could be found in many studies (Min et al., 1998); (Nagy and Salhi, 2007); (Duhamel et al., 2010); (Derbel et al., 2012); (Borges et al., 2013). Exact approaches such as mixed integer linear programming, branch-and-bound are proposed (Laporte and Nobert, 1988); (Labbe et al., 2004); (Contardo et al., 2013); (Saraiva de Camargo et al., 2013); (Hashemi and Seifi, 2013). For large instances of LRP problems heuristic and meta-heuristic approaches are developed in the literature, such as: nearest neighbour method (Rahmani et al., 2013b), simulated annealing, (Wu et al., 2002); (Yu et al., 2010); (Doulabi and Seifi, 2013); (Albareda-Sambola et al., 2005); (Mousavi and Tavakkoli-Moghaddam, 2013); (Ghaffari-Nasab et al., 2013); (Fazel et al., 2013), memetic algorithms (Prins et al., 2006b), greedy randomized adaptive search procedure (Prins et al., 2012); (Duhamel et al., 2009), variable neighborhood search algorithms, (Melechovsky and Prins, 2005); (Schwengerer et al., 2012); (Jarboui et al., 2013), and ant colony optimisation (Ting et al., 2013). According to (Mehrjerdi and Nadizadeh, 2013),
heuristic algorithms applied to LRP in literature could be divided into sequential, iterative, hierarchical and clustering methods. In sequential methods, first the total sum of distances from the potential sites to all customers is minimised. A set of potential sites is selected then the vehicle routing problem (VRP) is solved. In iterative methods, the routing and the location problems are solved iteratively. In hierarchical heuristics, the location of the potential sites is solved as the principal problem then the routing is considered as the secondary problem. The clustering methods, proceed by partitioning the customers into clusters, one cluster per potential site or one per vehicle route then solve the routing problem for each cluster and find the best location of potential sites.

The potential of cluster analysis to solve the LRP problems has been recognized by several authors (Bruns and Klose, 1995); (Barreto et al., 2007). However, few studies have considered the clustering approaches for the LRP, such as: (Özdamar and Demir, 2012); (Barreto et al., 2007); (Mehrjerdi and Nadizadeh, 2013); (Guerrero and Prodhon, 2013). To the best of our knowledge, the clustering methods have never been applied to the LRP in a two-echelon network. Our contribution, in this paper, is to develop a clustering approach to a general and complex LRP in a two-echelon network that was proposed in (Rahmani et al., 2013a).

The rest of this paper is organized as follows. Section 2 presents the considered LRP problem and its specific constraints. Section 3 explains briefly the nearest neighbour method that was already applied to the studied problem. Section 4 gives the details of the proposed clustering approach. Experimentation and concluding remarks are discussed in the sections 5 and 6, respectively.

2 PROBLEM DESCRIPTION

A new complex LRP, named LRP-MPPD-2E (2 Echelon Multi-products Location-Routing problem with Pickup and Delivery) has been defined (Rahmani et al., 2013a). The proposed model was inspired by a real problem encountered in the context of the distribution of shoes (Carrera et al., 2010). The goal is to locate processing centers (intermediate stores, relays, logistic platforms) to optimize the distribution of different kinds of shoes from a central platform to final stores. The proposed model combines two families of realistic constraints that have not been considered simultaneously in LRP literature: multi-products constraint and pickup and delivery constraints.

In LRP-MPPD-2E, two levels are considered: at the first level, tours are constructed from a main depot to a set of active processing centers that must be selected, and at the second level, a set of vehicles...
visit customers from the selected processing centers. We denote primary and secondary tour the tours constructed at the first and the second level, respectively. The LRP-MPPD-2E is an extension of the LRP-2E (for recent studies on LRP-2E, see (Nguyen et al., 2012), (Boccia et al., 2010). A concise formulation and heuristic approaches based on neighbourhood strategy was proposed in (Rahmani et al., 2013a).

The LRP-MPPD-2E is modelled as an undirected and valued graph \( G = (N, A, l) \). \( N \) refers to the set of nodes, where \( N = N_0 \cup N_e \), in which \( N_0 \) and \( N_e \) represent the sets of the potential processing centers nodes (considered as depots in the case of LRP), and the customers, respectively. Node \( 0 \) is considered as a depot. \( A \) is the set of edges and \( l \) refers to a function that associate a positive cost (time) to each arc (typically travel time). At depot there is a set \( V_1 \) of homogeneous fleet of (primary) vehicles. Each primary vehicle has a limited capacity \( CV_1 \) and a fixed cost \( FV_1 \). Another set \( V_2 \) of homogeneous fleet of (secondary) vehicles is available at the processing center’s sites. Each secondary vehicle has a limited capacity \( CV_2 \) and a fixed cost \( FV_2 \) (we consider the general case when \( CV_1 \) is different from \( CV_2 \)). Each potential processing center has an opening cost.

Each client asks for one or several type of products denoted c-products, known in advance and could be satisfied. In each processing center, pickup and delivery operations are performed. Primary products, denoted h-products, are delivered from main depot to active processing centers. Each active processing center can receive only one type of h-products. The h-products are transformed into final products, denoted c-products. Each processing center should provide exactly one secondary c-product.

We consider two types of vehicles as explained above. The primary vehicles should pick up the h-products from the main depot and deliver them to the active processing centers, which have been opened, such as each processing center is visited only once in each primary tour. When once satisfying the demand of processing centers, the secondary vehicles can pickup c-products, which are available in the processing centers, and continue their trips in a way that each customer and processing center is visited at most once by each secondary trip. The secondary trips start from an active processing center, which will represent the departure node, serve several customers, can visit one or several processing centers and must end up at the departure node. We assume that products have the same size, the splitting demand of customers for a given c-product is not allowed, and that each processing center can provide exactly one type of c-product.

The goal of LRP-MPPD-2E is to determine the location of active processing centers, the assignment of customers to the opened processing centers and the construction of the corresponding primary and secondary tours with a minimum total cost. The total cost includes the opening cost of processing centers, the exploitation cost of vehicles and the sum of edges costs traversed by vehicles. An illustrative example of the two-echelon model is given in Figure 1.

3 NEAREST NEIGHBOUR HEURISTIC (NNH)

In this section, we explain briefly the neighbour nearest heuristic, named NNH, proposed in (Rahmani et al., 2013a), which is actually the only proposed method for LRP-MPPD-2E.

For the primary routes a constructive approach, based on two steps is used. In the first step, a processing center is selected according to some criteria in order to initialize the route. Then the nearest neighbour strategy is used in the second step to complete the tours. Both steps are repeated until all activated processing centers could satisfy the customer demands. To construct the secondary routes all inactive processing centers are ignored, then an open processing center is selected according to some criteria. To compute the current route a nearest neighbour strategy is used. This process is repeated until all customer requests are satisfied. A neighbour candidate (active processing center or client) is inserted in the tour if all constraints are satisfied, otherwise a second neighbour candidate will be checked, until neither processing center nor client can be inserted in the tour. In that case a new secondary tour is started. This process is repeated until all demands of customers are satisfied.

4 HYBRID CLUSTERING ALGORITHM (HCA)

In this section, a Hybrid Clustering Algorithm - HCA is proposed for the LRP-MPPD-2E.

The hybrid-clustering algorithm - HCA, is a non-trivial extension of a greedy clustering method proposed by (Mehrjerdi and Nadizadeh, 2013) for a
classical LRP with fuzzy demands. The proposed HCA algorithm proceeds in five steps (see Figure 2). In the first step, customers are clustered using an algorithm based on nearest neighbour, such that each cluster should involve only clients that request the same product (Figure 2a). In the second step, the gravity center of each cluster is calculated. This allows to select a set of potential processing centers (Figure 2b). In the third step, clusters are merged as well as possible in order to create the Global-Clusters (GC) in which only one vehicle will be exploited. That means each Global-Cluster represents one feasible secondary tour (Hamiltonian tour). This merging step considers the distance between the gravity centers of the clusters as well as the route time limit, (Figure 2c). In order to ensure the feasibility of the solution in each Global-Cluster, the merged clusters should not have any common client, because the exploited vehicle for each Global-Cluster must visit only once each customer and each processing center. The clusters are allocated to the opened processing center(s) in the forth step, considering the distance between the processing centers and the gravity center of the clusters as well as the capacity of the processing centers (Figure 2d). Finally, in the fifth step, Cplex
sparser is used to find a feasible secondary tour in each Global-Cluster (Figure 2e). Details of the HCA’s steps are given below.

### 4.1 Clustering the Customers

The goal of the first step of the HCA is the clustering of the customers. The customers are separated into different groups considering their intra-distance, the sum of their customer’s demands, the vehicle capacity, the time route limit, and an estimation of the route travel time given in formula (1) in which \(N_c, cl\) and \(N_0, cl\) present the number of clients and processing centers, respectively in cluster \(cl\). \(DMaxC\) is the maximum distance between two clients in cluster \(cl\). The maximum distance between the processing centers and the clients in cluster \(cl\) is denoted by \(DMaxPC\).

\[
T = ((N_c, cl - N_0, cl) \times DMaxC) + 2 \times (N_0, cl \times DMaxPC) \tag{1}
\]

The T value associated to a cluster \(cl\), is an overestimation of a route starting from a processing center, visiting all the customers assigned to the cluster \(cl\), and ending at the starting processing center.

More precisely, for each c-product \(p\), a set of non-clustered customers (NCC\(p\)) is initialised by all customers \(j\) such as \(Q_{jp} > 0\), where \(Q_{jp}\) indicates the quantity of the product \(p\) asked by the customer \(j\). At first, a customer is selected randomly from a set NCC\(p\), then the nearest customer to the last selected customer of the current cluster is chosen from NCC\(p\). Therefore the clusters are formed for a single. The nearest customer is selected as follow: when a new customer \(j\) for product \(p\) is selected, \((j\) is the closest customer regarding the distance to the current customer in the cluster \(cl\)) it has this customer to the last selected customer of the current cluster is chosen from NCC\(p\). Therefore the clusters are formed for a single.

In this step, the clusters are merged in order to create a set of Global-Clusters (GC) in which represents one feasible secondary route. Since the assigned vehicle to each Global-Cluster must visit customers and processing centers only once, then the merged clusters should not have any common client.

At first, a cluster \(cl\) will be selected randomly, and then a sorted list of the not merged clusters \(cl’\) is constructed according to the distance between the gravity centers of \(cl\) and \(cl’\). The first cluster in the list is added into the current Global-Cluster (GC) if the value of \(T\) Calculated by equation (1), with \(N_0, cl\) equal to the number of merged clusters in GC did not exceed the time route limit (Figure. 2c). This procedure is repeated until that no cluster can be added to the current Global-Cluster. In that case, either the process stops because all the clusters are merged or the process is restarted to search for a new Global-Cluster.

### 4.2 Processing Center (PC) Selection

In the second step of the HCA, the method of (Mehrjerdi and Nadizadeh, 2013) for establishing the processing centers is used. This method is based on a gravity center criterion as illustrated by Equation (2), in which \((X_{cl}, Y_{cl})\) is the coordinates of the gravity center of the cluster \(cl\) and \((x_i, y_i)\) is the coordinates of customer \(i\), where \(n_{cl}\) is the number of customers assigned to cluster \(cl\).

\[
(X_{cl}, Y_{cl}) = \left( \frac{\sum_{i=1}^{n_{cl}} x_i}{n_{cl}}, \frac{\sum_{i=1}^{n_{cl}} y_i}{n_{cl}} \right) \tag{2}
\]

For each processing center, we calculate the sum of the distances between this potential site and all the gravity centers.

The potential sites are re-indexed in non-decreasing order according to their Euclidean distance to the gravity center of the clusters. If the current opened top-ranked potential site is not able to fulfil all the remaining customers’ demands, the next potential site of the sorted list is selected to be open. This procedure is repeated until all the clusters are covered. Therefore, each selected processing center will be assigned to one or more cluster and each cluster is covered by one or more processing centers.

### 4.3 Merging the Clusters into Global-Cluster

In this step, the clusters are merged in order to create a set of Global-Clusters (GC) in which represents one feasible secondary route. Since the assigned vehicle to each Global-Cluster must visit customers and processing centers only once, then the merged clusters should not have any common client.

The HCA’s steps are given below.
4.4 Assigning Clusters to Processing Center(S)

In the forth step of the HCA, the clusters are respectively allocated to the processing center that were ranked and opened in the processing center selection step. Each processing center serves as many clusters as possible according to its capacity. Note that we can’t allocate two clusters cl₁, cl₂ to the same processing center when they were merged in the same GC. Because a vehicle cannot visit a processing center twice in a given route. In order to allocate the clusters to the processing center, the Euclidian distance between the gravity center of each cluster and the opened processing center is calculated. Then the unassigned clusters are ranked in an ascending order based upon the distance of their gravity centers to the processing center. The top-ranked cluster clᵢ will be allocated to the top-ranked processing center pr: 1) if the processing center pr has enough capacity to cover the total demands of the cluster clᵢ, and 2) if this processing center pr is not already affected to a cluster clⱼ such as clᵢ and clⱼ belong to the same Global-Cluster. The allocation process to the processing center pr is finished when there is not enough capacity to allocate a new cluster. In that case, the allocation procedure is repeated for the next top-ranked processing center until all clusters are allocated.

4.5 Routing Problem

In the fifth and last phase of the HCA, the routing problem is solved for each Global-Cluster (GC) with the relevant processing centers and assigned clients. Actually, each Global-Cluster is served by exactly one vehicle and the vehicle is not allowed to visit any node two times. Cplex solver is used to create one secondary route per one Global-Cluster. The routing between the selected processing centers for the first level (primary tours) is obtained by a vehicle routing nearest neighbour heuristic.

5 COMPUTATIONAL EXPERIMENTS

In this section, we review the performance of the HCA, Iterative HCA (IHCA) and Clustering-NNH method in comparison with NNH (Rahmani et al., 2013a, briefly presented in section 3). In Iterative HCA, the HCA algorithm is executed 10 times with a new client, chosen randomly, to initialize the clustering step. In Clustering–NNH, firstly NNH is applied in order to create the routes, and then each secondary route is considered as a Global-Cluster and a Cplex solver is used for each Global-Cluster to improve the secondary routes. We note that the routing in primary level is kept like in NNH.

Since our problem is not considered in the literature, we have adapted a known LRP-2E instance from (Prodhon et al., 2012) to our problem (Rahmani et al., 2013a). These instances are grouped in four subsets with the following features: number of customers n ∈ {20, 50, 100, 200}, uniform distribution demands in interval [11, 20], number of satellites-depots m ∈ {5, 10}, with their opening costs, β={1,2,3} is manner of customers distribution, for instance, β = 1 means a uniform distribution of customers.

In order to adapt these instances to our problem,
we have considered the following hypotheses:

- Each satellite-depot corresponds to a processing center in our problem.
- We consider 3-distinct products: one h-product \( \{p_0\} \) and two others c-products \( \{p_1, p_2\} \).
- Each client asks for products, \( p_1 \) or \( p_2 \) or both products with equal probability.
- The capacity CV1 must be greater than the quantity of all h-product demands.

We added the h-product demand for each processing center, such as the demand of each h-product is equal to 1/5 of c-product availability in this processing center.

The proposed heuristic was coded in C++ and we evaluated its performance on a PC with Intel (R) Core (TM) Solo CPU 1.40 GHz, 2GB of RAM. The routing steps of HCA use a Cplex solver version 14.0. Cplex Solver is unable to find any optimal solution. However, a feasible solution is obtained when we limit the computation time to one hour. Table 2 shows the improvement result details of proposed heuristics.

In Table 1, the first column indicates the problem

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Table 2: Evaluation of the Clustering Method against Cplex Lower Bound.

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The proposed heuristic was coded in C++ and we evaluated its performance on a PC with Intel (R) Core (TM) Solo CPU 1.40 GHz, 2GB of RAM. The routing steps of HCA use a Cplex solver version 14.0. Cplex Solver is unable to find any optimal solution. However, a feasible solution is obtained when we limit the computation time to one hour. Table 2 shows the improvement result details of proposed heuristics.

In Table 1, the first column indicates the problem
name “coord \( n - m - \beta \cdot 2E \)”. In columns 2 and 3, the best result of the neighbour nearest heuristic (NNH) is reported. The results of Clustering-NNH are gathered in columns 4 and 5, the results of HCA are represented in columns 6 and 7, and the results of IHCA are represented in columns 8 and 9. Column “Time” and “Cost” present the computation time (in seconds) of NNH and the obtained value of the total cost, respectively. The percentage of “Gap_NNH” indicates the improvement of the Clustering methods in comparison with NNH solution. It is calculated as \((\text{NNH Cost} - \text{Clustering_NNH}) / \text{NNH Cost}\) for column 5, \((\text{NNH Cost} - \text{HCA Cost}) / \text{NNH Cost}\) for column 7, and \((\text{NNH Cost} - \text{IHCA Cost}) / \text{NNH Cost}\) for column 9.

In Table 2, a comparison with the lower bound LB provided by Cplex is given. This LB is obtained by solving a mixed linear formulation of LRP-MPPD-2E (Rahmani et al., 2013a). The value of LB is given in column 2. Columns 3, 4, 5, and 6 present the gap between the lower bound (LB), and heuristic solution. It is calculated as \((\text{Cost} - \text{LB}) / \text{Cost}\).

Results in table 1 show that HCA outperforms NNH and the Clustering NNH for \( n < 200 \) except for the third instance. For \( n \geq 200 \), it is not possible to evaluate the performance of HCA, because Cplex cannot generate any solution after 1 hour processing time (see Table 2). The maximum and minimum gap (improvement) between the HCA solution and the NNH solution are 27.76% and 1.13%, respectively. Furthermore, The HCA methods outperforms the Clustering_NNH, since HCA is able to solve 14 instances while Clustering_NNH only solves 10 instances. Clustering_NNH doesn’t succeed to improve all results of NNH (only 6 instances are outperformed). The average improvement of HCA in comparison with NNH is 7.99%, against 2.61% of improvement is obtained by the clustering-NNH. Note that we limited the computation time of HCA to one hour; however we noticed that the solution is obtained on average after only 10 minutes.

The results of HCA are enhanced by IHCA from 7.98% to 8.99%. Results also show that IHCA is able to resolve some instances when HCA don’t succeed to find any solution (instance 3).

### 6 CONCLUSIONS

Some studies from the literature on routing problems confirm the interest of the clustering technique for the location routing problem but only a few papers deal with the application of the clustering techniques for a classical location routing problems. In this paper, we have developed a Hybrid Clustering Algorithm (HCA) for a more complex location routing problem, considering many realistic non-tackled constraints in the literature. The studied problem, named LRP-MPPD-2E, has been proposed recently in (Rahmani et al., 2013a). The authors proposed a Nearest Neighbour Heuristic (NNH) to solve the problem. Computational results show that the HCA outperforms the result of NNH. In addition, HCA works better than another clustering technique, in which the secondary tours of NNH are used to form the Global-Clusters. Iterative HCA, which is a randomised version of HCA, outperforms all the methods.

In further researches, we aim to improve HCA with metaheuristic techniques and an iterative process. For example, to improve the computation time of HCA, we can use a metaheuristic approach instead of Cplex to solve the routing problems. The primary tours can easily be improved by more sophisticated heuristic like the one that is used in this paper. It would be also interesting to develop more efficient lower bound. Another perspective is to generalize the LRP-MPPD-2E to deal with some other realistic constraints such as splitting of demand and the possibility to provide several types of e-products per processing center.

### REFERENCES


Contardo, C., Cordeau, J. F., Gendron, B. 2013. A computational comparison of flow formulations for
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Prins, C., Prodhon, C., Wolfler Calvo, R., 2006a. Solving the capacitated location-routing problem by a GRASP complemented by a learning process and a path relinking. 4OR 4,221–238.


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