Adaptation Schemes for Question's Level to be Proposed by Intelligent Tutoring Systems

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Abstract: The main challenge in developing a good Intelligent Tutoring System (ITS) is suit the difficulty level of questions and tasks to the current student's capabilities. According to state of the art, most ITS systems use the Q-learning algorithm for this adaptation task. Our paper presents innovative results that compare the performance of several methods, most of which have not been previously applied for ITS, to handle the above challenge. In particular, to the best of our knowledge, this is the first attempt to apply the Bayesian inference algorithm to question level matching in ITS. To identify the best adaptation scheme based on this groundwork research, for the evaluation phase we used an artificial environment with simulated students. The results were benchmarked with the optimal performance of the system, assuming the user model (abilities) is completely known to the ITS. The results show that the best performing method, in most of the environments considered, is based on a Bayesian Inference, which achieved 90% or more of the optimal performance. Our conclusion is that it may be worthwhile to integrate Bayesian inference based algorithms to adapt questions to a student's level in ITS. Future work is required to apply these empirical results to environments with real students.

1 INTRODUCTION

Intelligent Tutoring Systems (ITS) are based on artificial intelligence methods (Woolf, 2009) to customize instruction with respect to the student's capabilities that dynamically change over the tutoring period. To accomplish this, an ITS should contain knowledge about the student's capabilities, termed the student's model, and a set of pedagogical rules.

A critical characteristic of an ITS is the ability to challenge students, without discouraging them due to exaggerated challenges. Namely, on the one hand the system should not provide questions to the student that are too easy and leave him bored, while on the other hand, the questions should not be too hard to the point that they discourage the student from using the system. In both of these extreme cases the student will not realize its potential and therefore will not benefit from using it. Therefore, we aimed to construct an ITS that will match the hardest challenges the user can face and by doing so realize his potential.

According to state of the art, most ITS systems make use of the Q-learning algorithm for this adaptation task. Our paper presents innovative results that compare performances of several methods, most of which have not been previously applied in ITS, to handle this challenging adaptation task. In particular, to the best of our knowledge, this is the first attempt to apply the Bayesian inference algorithm in ITS for question level adaptation.

The leading principles involved in the development of an efficient ITS include keeping track of dynamically improving capabilities and knowledge, and keeping the user active and satisfied. These are achieved by considering the correctness of the answers the student provides hitherto to the ITS. In particular the ITS must choose a fixed number of questions from a pre-prepared pull of questions to present to the student. Obviously, the system is not faced with a single decision for all the questions, but instead question by question decisions, while taking into account the history of the student's answers.

To enable a comparison between the various
proposed adaption schemes, we defined a utility function to reflect the efficiency of each chosen question. Specifically we considered the following adapting schemes: (i) Q-learning (Sutton and Barto 2005, Russell & Norvig 1995, Martin and Arroyo, 2004); (ii) Virtual Learning (Vreind, 1997); (iii) Temporal Reasoning (Beck, Woolf and Beal 2000); (iv) DVRL (Azoulay-Schwartz et al., 2013); (v) Bayesian Inference (Conitzer and Garera, 2006); and (vi) Gittins Based Method (Gittins, 1989). Most of the algorithms have not been used for ITS, excluding the Q-learning method which has been used for ITS, as discussed in Section 2.

As this is a groundwork research to identify the best adaptation scheme, at this stage of the study we developed an artificial environment by simulating students with ability distributions derived from a normal distribution.

The performance of the various proposed methods were benchmarked with the optimal performance of the system assuming the user model (abilities) is completely known to the ITS. The results show that the method that outperformed the others in most of the environments we considered is based on a Bayesian Inference which achieved more than 90% of the optimal performance.

This paper is organized as follows. In Section 2 we provide a review of the current state of the art methods used for choosing questions in ITS. In Section 3 we present the ITS model, including the utility function used in this study. In Section 4 we describe the various adaption schemes, and present a detailed description of the Bayesian inference algorithm which we provide in Section 5. In Section 6 we describe the construction of the artificial environment used for the evaluations and in Section 7 we present the simulation results. Finally, we conclude and discuss directions for future work in Section 8.

2 RELATED WORK

It is well known that a student learns much better by one-on-one teaching methods than by common classroom teaching. An Intelligent Tutoring System (ITS) is one of the best instances of one-on-one teaching (Woolf, 2009) that uses technology development. A student who is supposed to learn a certain topic by means of an ITS is assumed to do so by solving problems given to him by the ITS.

The ITS evaluates a given answer by comparing it to the predefined answer as it appears in its knowledge base. The system keeps track of the user's actions, and correspondingly builds and constantly updates its student model. Moreover, it observes the topics that need more training and selects the next question accordingly.

In this paper, we consider the way the student model will be represented, how it will be used to find the student's subsequent goals, and how it should be updated according to the student's results. In the current section, we survey several ITS systems that also contain a learning process to adapt to the student's abilities.

Martin and Arroyo (Martin and Arroyo, 2004) used Reinforcement Learning agents to dynamically customize ITS systems to the student. The system clusters students into learning levels, and chooses appropriate hints for each student. The student's level is updated based on the answers they enter or the hints they ask for. Their best success was by using the e-greedy agent (e=0.1). Following Martin and Arroyo, we also used a Q-learning algorithm where the probability ε of trying a non-optimal level, was fixed at 0.1.

Iglesias et al. (Iglesias et al., 2008) proposed a knowledge representation based on RL that allows the ITS system to adapt the tutoring to students' needs. The system uses the experience previously acquired from interactions with other students with similar learning characteristics. In contrast to Iglesias et al., in our work the learning process is done individually for each student, in order to learn the level of each student.

Malpani et al. (Malpani, Ravindran and Murthy, 2009) present a Personalized Intelligent Tutoring System that uses Reinforcement Learning techniques to learn teaching rules and provide instructions to students based on their needs. They used RL to teach the tutor the optimal way of presenting the instructions to students. Their RL has two components, the Critic and the Actor. The Critic follows Q-learning, and the actor follows a Policy Gradient approach with parameters representing the preference of the choosing actions.

Sarma et al. (Sarma and Ravindran, 2007) developed an ITS system using RL to teach autistic students, who are unable to communicate well with others. The ITS aimed to teach pattern classification problems. The student has to classify the pattern (question) given. This classification is used to validate an ANN, but does not teach real children. The pedagogical module used in (Sarma and Ravindran, 2007) selects the appropriate action to teach students by updating Q-values.

Finally, Beck et al. (Beck, Woolf and Beal, 2000) constructed a learning agent that models the student behavior in the ITS. Rather than focusing on
whether the student knows particular knowledge, the learning agent determines the probability the student will answer a problem correctly and how long it will take him to generate his response. They used an RL agent to produce a teaching policy that meets the educational goals.

In this paper, we concentrate on the goal of selecting the appropriate question level, with regards to the current student model and our knowledge about its past performance. For this goal, we compared different learning schemes, most of which have not been used for tutorial systems, and we compared their results by means of simulation.

3 THE ITS MODEL

In the following section, we provide the ITS model we developed. We detail the ITS process and the utility function used to evaluate its performance.

The ITS process

We consider an ITS that provides a student with questions. In particular, a specific student should obtain a set of \( N \) questions. The simplified ITS process is presented in Figure 1, and is based on 3 steps: (1) an initialization step; (2) the process of choosing the next appropriate question; (3) examination of the student's answer and saving the information for the future steps.

The simplified template used in Figure 1 can assist in providing the details of the different schemes used in the paper as well.

The ITS utility function

Clearly, we would like the student to succeed in answering the questions correctly, but, in addition, we would like the question to be of the highest level possible.

The ITS utility function should reflect both criteria. In this study, we provide the following two possible utility functions:

For each successful question utility function \( #1 \) considers its level: as the level of a successful question is higher, the total utility is also higher.

**Utility function \( #1 \):**

\[
\sum_{\text{question } i=1\ldots N} (\text{level } i \mid Q_i \text{ was correctly answered})
\]

Utility function \( #2 \) also considers the level of questions, but also adds a constant reward for each success, and a constant penalty for each failure, in order to better represent the student's utility, which decreases when it fails to answer questions.

**Utility function \( #2 \):**

\[
\sum_{\text{question } i=1\ldots N} (\text{level } i + C \mid \text{question } i \text{ correctly answered}) - \sum_{\text{question } i=1\ldots N} (C \mid \text{question } i \text{ incorrectly answered}).
\]

Both functions consider the student's results for each of the questions, \( 1..N \). The motivation behind these two functions is as follows:

Utility function \( #1 \) considers the advances in the student's knowledge, thus it calculates the sum of successful answers, while also considering their levels. However, utility function \( #2 \) also considers the preferences of the student himself, who does not like to fail in answering queries, thus the utility function adds a positive constant for each success and reduces this constant as a penalty in cases of failure.

To summarize, utility function \( #2 \) places more importance on the success or failure event, by adding a constant value or deducting a constant value from the utility function, for each success or failure, respectively.
4 ALGORITHMS FOR ADAPTING QUESTIONS TO THE STUDENTS

In this section we provide a detailed description of the various algorithms we propose for the challenge of adapting the level of the next question that will be given to the student. Note that all the proposed algorithms aim to determine the level of the next question, and given this, the ITS in turn should choose a particular question of that level to present to the student.

In this research we implemented and tested the various algorithms we propose in order to identify which has the best performance and therefore should be integrated in the ITS for further investigation. The algorithms are as follows.

1. Simple Q-learning algorithm (Harmon and Harmon 1997, Sarma 2007)
2. Temporal difference learning (TD) (Beck, Woolf and Beal, 2000)
3. Virtual Learning algorithm (Vriend, 1997)
4. DVRL (Azoulay-Schwartz et al., 2013)
5. Gittins Indices (Gittins, 1989)
6. Bayesian Learning (Conitzer and Garera, 2006) assuming a normal distribution of the student’s level.

We proceed by providing further details for each of the above algorithms.

1. **Q-Learning**

A Q-learning learning algorithm saves a value Q for each pair of actions to be taken (in the classical Q learning method, different states are considered, but in our problem only one state exists). Given the Q values, with a probability $\varepsilon$, the algorithm explores and randomly chooses an action; and with a probability $1-\varepsilon$, the algorithm exploits and chooses the action with the highest Q value.

After the action is taken and the reward is observed, the Q value of this state and action is updated using the following formula

$$Q(a) = Q(a) + \alpha[r + \gamma \max_{a'} Q(a') - Q(a)]$$

where $\alpha$ defines the speed of convergence of the Q values, and $\gamma$ defines the discount ratio.

In our case, the actions are the possible question’s level. Each question’s level is associated with a certain Q value. Once the student provides an answer, the relevant Q value is updated according to the reward which indicates the success or failure in answering the question.

Figure 2 presents the Q-learning algorithm process, whereby the figures of the other Q-based algorithms can similarly be described.

2. **TD learning algorithm**

The TD learning algorithm is different than the Q learning algorithm in the way the future reward is calculated. In Sarsas’ algorithm, which is a version of the TD-learning algorithm, the updating rule uses the following formula

$$Q(a) = Q(a) + \alpha[r + \gamma Q(a') - Q(a)]$$

where $a'$ is the action that is supposed to be taken from this state, using the policy of choosing an action taken by TD($\gamma$).

3. **Virtual Learning algorithm**

Virtual Learning is also similar to Q learning, but the idea of virtual learning is that instead of learning only on the basis of actions and payoffs actually experienced, the algorithm can also learn by reasoning from the chosen action for other actions.

In our domain once a student succeeds in answering a question, the Q learning of the current level as well as the Q learning of the lower levels are increased. Similarly, if a student fails to answer a question the Q learning of the level of the current question as well as the Q learning of the higher levels are reduced.
4. DVRL

DVRL is similar to Virtual Learning, but once a reward is received for the student's answer, the updating phase of the Q values relates not only to the given question's level but also to the level of the neighboring questions. Specifically, once a student answers a question correctly, the Q learning of the nearest higher level also increases, and when a student fails to answer a question, the Q learning of the nearest lower level also decreases.

5. Gittins Indices

Gittins (Gittins, 1989) developed a tractable method for deciding which arm to choose, given N possible arms, each with its own history. Gittins calculated indices values which indicate the attractiveness of each arm as a function of its past success and failures.

The indices calculated by Gittins consider the attractiveness of each arm including future rewards from exploring unknown arms. The indices for each arm were calculated for the standard normal distribution, and were provided in a table that can be used to determine the optimal action for different combinations of arms' success and failures and for different values of the discount ratio over time. Moreover, in our study Gittins indices can be used to compare the success rate of different levels, while also considering the average and standard deviation of the past results for each level, in the manner suggested by (Azoulay-Schwartz, Kraus and Wilkenfeld, 2004), where Gittins indices is applied to multiple arms whereby each arm is normally distributed with any value of μ and σ. In particular, Algorithm 1 is used to choose an arm, given the sum[level] and the std[level], which are the sum of rewards and the standard deviation of rewards for each level, and given GittinsIndex[n[level]] which is the Gittins index for the number of past trials of this level for the current student.

Algorithm 1: Choose next question using Gittins indices

In other words, each level of questions is considered as an arm, and in each stage, Gittins indices are calculated for each level, and the level with the highest value of Gittins index is chosen.

Figure 3 presents the Gittins algorithm process used in our study to choose the question level in ITS.

6. Bayesian Learning

Assuming a normal distribution of the student's level, and given a particular student, the algorithm can consider different parameters of the student level distribution. Initially, the algorithm associates a constant probability for each set of parameters (μ and σ).

In each step, the algorithm considers all possible distributions of the student, and for each question's level, the algorithm calculates the expected utility of this level given all possible distribution of students, and then it chooses the level with the highest expected utility. Once a question is chosen and the student's response is observed, the probability of each distribution of the student is updated using the Bayesian rule.

Given the above list of algorithms, in the next section we describe the results obtained by our simulation, which compares the performance of the various algorithms for artificial students.
5 ADDITIONAL DETAILS ON THE BAYESIAN LEARNING ALGORITHM

In the following section we provide additional details on the Bayesian approach applied in the ITS domain. In particular, it is assumed that a distribution exists from which the student's level is drawn in any given time slot. Moreover, this distribution is associated with an unknown mean and an unknown standard deviation.

This is due to the fact that a certain student associated with a mean level of 3, for example, can sometimes succeed in answering questions from level 4, and similarly can sometimes fail in answering questions from level 2.

The Algorithm’s Details

In order to learn the student's level at each stage of the simulation, for each student the algorithm constructs a matrix of various possible mean intervals and standard deviation intervals. In particular, each row represents a certain mean interval of the student's level and each column represents a certain standard deviation interval such that each cell \((i,j)\) in the matrix stands for the probability that the given student's level is of a mean between the upper limit of the previous row and the upper limit of row \(i\) assuming the standard deviation interval of column \(j\).

In the initialization stage, each value in the matrix is associated with an arbitrary probability such that the sum of all the cells' probabilities is one. Next, at each phase where a question is chosen and the student provides an answer, each cell’s probability is updated according to the Bayesian rule. Finally, formula 5 is used to determine the next level of the question.

\[
\text{chosenLevel} = \text{Argmax}_\text{level=1..5} \sum_{\mu,\sigma} \text{prob}(\mu,\sigma) \ast \text{pwins}(\text{level} | \mu, \sigma) \ast \text{util} \text{ (level)} + (1 - \text{pwins}(\text{level} | \mu, \sigma)) \ast \text{utilFailure}).
\] (3)

Where \(\text{pwins}(\text{level} | \mu, \sigma)\) is the probability of a question \(n\) from level \(\text{level}\) to be chosen, \(\text{util} \text{ (level)}\) is the utility from a successful answer to a question from this level, and \(\text{utilFail}\) is the utility from failure to answer a question from this level.

The explanation of the formula is as follows.

1. For each possible \(\text{level}\), we review the entire table, and calculate \(\text{pwins}(\text{level} | \mu, \sigma)\), which is the probability that a student with mean level \(\mu\) and a standard deviation \(\sigma\) will be able to correctly answer a question of the current \(\text{level}\).

2. Given \(\text{pwins}(\text{level} | \mu, \sigma)\), we calculate the expected utility from a question from \(\text{level}\), if the student distribution is normal with \((\mu, \sigma)\), where \(\text{pwins}\) is multiplied by the success utility, and \((1 - \text{pwins})\) is multiplied by the failure utility.

3. Once this value is calculated for each possible level, the level that achieves the highest expected utility is chosen.

Figure 4 illustrates the Bayesian learning process when used in ITS.

A Particular Example of the Bayesian Algorithm

Table 1 includes the initial beliefs for the ITS, assuming, for simplicity, that the mean and the standard deviation are assumed to be integer values between 0..5. The table is initialized as provided in Table 1, where the probability of each pair of \((\mu, \sigma)\) is equal, and the sum of probabilities is 1.

\[
\begin{array}{cccccc}
\mu=0 & \mu=1 & \mu=2 & \mu=3 & \mu=4 & \mu=5 \\
\sigma=0 & 0.0278 & 0.0278 & 0.0278 & 0.0278 & 0.0278 \\
\sigma=1 & 0.0278 & 0.0278 & 0.0278 & 0.0278 & 0.0278 \\
\sigma=2 & 0.0278 & 0.0278 & 0.0278 & 0.0278 & 0.0278 \\
\sigma=3 & 0.0278 & 0.0278 & 0.0278 & 0.0278 & 0.0278 \\
\sigma=4 & 0.0278 & 0.0278 & 0.0278 & 0.0278 & 0.0278 \\
\sigma=5 & 0.0278 & 0.0278 & 0.0278 & 0.0278 & 0.0278 \\
\end{array}
\]
Adaptation Schemes for Question's Level to be Proposed by Intelligent Tutoring Systems

Given Table 1, the expected utility of each question's level is provided in Table 2:

<table>
<thead>
<tr>
<th>Level</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>0.686</td>
<td>1.13</td>
<td>1.31</td>
<td>1.26</td>
<td>1.01</td>
</tr>
</tbody>
</table>

For example, the utility of a question from level 2 is as follows:

$$\sum_{\mu=0.5, \sigma=0.5} p(\mu, \sigma) * \text{pwin}(2|x~N(\mu, \sigma)) * \text{util}(2) = 1.13$$

$$\sum_{\mu=0.5, \sigma=0.5} p(\mu, \sigma) * \text{pwin}(2|x~N(\mu, \sigma))^2 = 1.13$$

Where util(2)=2 is the utility from the student successfully answering a question from level 2.

Given Table 2, a question from level 3 is offered, since its expected utility is the highest.

Now, suppose that the first student's threshold was 4, thus the question is correctly answered by the student. Hence, the probability table should be updated according to the Bayesian rule:

$$\text{Prob}(2,1) = 0.0278 * \text{prob}(x \geq 3 | x~N(\mu, \sigma))/0.437 = 0.0101$$

For example, the utility of a question from level 2 is 1.13.

The ITS model is defined as follows. For each question Qi, with a difficulty level of leveli, the student will either succeed or fail to answer the question. The goal of the software is to match appropriate questions to the students to maximize both (i) the number of correct answers and (ii) the question's level presented to the student. For the evaluation of the combination of these two maximization problems we proposed utility function #1, as defined in Section 3.

$$\sum_{i=1..N} \text{(leveli} \mid Q_i \text{ was correctly answered})$$

This utility function reflects the fact that the higher a question's level and the more correct answers there are, the higher the utility obtained. Notice, however that an incorrect answer provides zero rewards.

The ITS model is defined as follows. For each new student/user that starts to use ITS, no preliminary information is assumed. Therefore, the first question presented to the new user should be arbitrarily determined by ITS. Once an answer is given by the student, the adaptation algorithm should decide the level of the next question to present based on the history of the student's answers. Once the student finishes using the ITS software the utility value for the student is measured.

Given the assumptions above we now describe the construction of the simulated students. We first assume that a student with a certain level will not always successfully answer questions of his or lower level; similarly, he will not always incorrectly answer questions of higher levels. Namely, the user's level might be dependent on several conditions and therefore may change with time.

Next, as this was groundwork research to identify the best question level adaptation scheme, we proposed an artificial environment with simulated students for the evaluation phase.

For each question Qi, with a difficulty level of Leveli, the student will either succeed or fail to answer the question. The goal of the software is to match appropriate questions to the students to maximize both (i) the number of correct answers and (ii) the question's level presented to the student. For the evaluation of the combination of these two maximization problems we proposed utility function #1, as defined in Section 3.

Given Table 3, the expected utility of each question's level is calculated again, the next question is chosen, the student's response is observed, and again, an updating step is performed which updates the probability table.

After explaining the Bayesian algorithm in detail, we proceed by describing the simulated environment used for our experimental study.

### 6 THE SIMULATED ENVIRONMENT

Next, as this was groundwork research to identify the best question level adaptation scheme, we proposed an artificial environment with simulated students for the evaluation phase.

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Given the assumptions above we now describe the construction of the simulated students. We first assume that a student with a certain level will not always successfully answer questions of his or lower level; similarly, he will not always incorrectly answer questions of higher levels. Namely, the user's level might be dependent on several conditions and therefore may change with time.

However, in order to decide whether an artificial student will answer a given question correctly or not, a threshold level must be defined for him in order to determine the level for which any harder question will be answered incorrectly and for any easier or equal level question he will answer correctly.

Second, we assume there are N questions that the
ITS will present to the student within the tutoring period. Given these two assumptions we simulated a certain student by drawing a certain mean value and a certain standard deviation from the Normal distribution for the student. Then we created a sequence of N threshold levels. Namely, given this sequence, for any question presented to this user we will compare the question's level with the threshold level to determine whether it will be answered correctly or not.

For each simulated student, we ran each of the algorithms described in section 3. Thus, we actually tested each of the proposed algorithms on the same set of randomized inputs. In the next section we present the experimental results.

7 EXPERIMENTAL RESULTS

For comparison reasons, we first implemented each of the proposed algorithms. Next, we created the artificial environment by producing a set of simulated students.

For the Q learning, TD and VL algorithms, the parameters (provided in Section 4) were set as: \( \varepsilon = 0.1, \gamma = 0.95, \sigma = 0.5 \). Note also that utility function #1 is used in most of our experiments.

We ran simulations for an ITS, that aimed to present the student with 10, 20, and 100 questions. In particular, for each number of questions (10, 20, and 100), and for each randomly created student, we ran each of the six proposed adaptation algorithms for 1000 runs. The simulation results are presented in Table 4 and in Figure 5.

Table 4: The average utility, (standard deviation) and successful rate obtained by different RL algorithms for 10, 20 and 100 questions.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>10 questions</th>
<th>20 questions</th>
<th>100 questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-Learning</td>
<td>10.826 (6.31)</td>
<td>22.876 (11.92)</td>
<td>115.479 (57.53)</td>
</tr>
<tr>
<td>TD</td>
<td>10.853 (6.31)</td>
<td>22.787 (12.03)</td>
<td>115.685 (59.32)</td>
</tr>
<tr>
<td>Virtual Learning</td>
<td>10.601 (6.26)</td>
<td>22.211 (11.93)</td>
<td>105.342 (53.32)</td>
</tr>
<tr>
<td>DVRL</td>
<td>13.368 (5.92)</td>
<td>21.435 (11.12)</td>
<td>104.543 (53.67)</td>
</tr>
<tr>
<td>Gittins Indices</td>
<td>10.158 (6.13)</td>
<td>23.105 (13.58)</td>
<td>134.143 (74.16)</td>
</tr>
<tr>
<td>Virtual Gittins Indices</td>
<td>10.825 (6.76)</td>
<td>24.074 (13.81)</td>
<td>122.72 (74.16)</td>
</tr>
<tr>
<td>Bayesian Inference</td>
<td>13.244 (8.21)</td>
<td>27.411 (15.54)</td>
<td>137.458 (73.22)</td>
</tr>
</tbody>
</table>

In each entry in the table there are three values (upper left, upper right, and bottom). The upper-left value represents the average utility achieved by the given algorithm for the particular number of questions. The upper-right value (in parentheses) represents the standard deviation of that average. The bottom value represents the ratio between the average utility obtained by the algorithm in the simulated environment and the average utility obtained by the ITS based on the student's known level of distribution at each time slot.

These results can be explained by the fact the Q-learning algorithm's decision is based on the best action achieved hitherto and a certain small probability of random choice while the Bayesian inference algorithm considers several different normal distributions of the student and therefore its performance is much more accurate.

![Figure 5: The average utility obtained by the different algorithms as a function of the number of questions.](image)

The results of the Gittins method were relatively lower than the Bayesian algorithms, probably due to the fact that the Gittins indices were calculated assuming independencies among the possible values, and this assumption does not hold in our domain, where the different levels are correlated. Nevertheless, the Gittins Indices' performance is relatively high in comparison to most of the algorithms' performances.

To summarize, when comparing the Gittins indices method and the Bayesian learning based algorithm, we can note the following points:
1. Both methods assume normal distributed arms or alternatives.
2. The Gittins indices takes into account future rewards as a result of the exploration of new arms, considering an infinite horizon scope, given a constant discount ratio, while the Bayesian inference algorithm maximizes the expected utility of the next step.
3. The Bayesian learning algorithm considers the correlation between different alternatives, while...
the Gittins based algorithm assumes independent arms. In fact, when comparing both methods, the Bayesian based algorithm outperforms the Gittins based algorithm. However, the Gittins Indices' performance is relatively higher than most of the algorithms' performances, given more than 10 steps. Moreover, the Gittins algorithm is the best one when considering high levels of improvement of the student level, as illustrated in Figure 8.

Note also that if the normal distribution does not hold, the UCB algorithm (Auer, Cesa-Bianchi and Fischer, 2002) can be applied for multi armed bandit processes. In contrast to Gittins method, the UCB algorithm considers a finite set of actions, but again, the alternative arms are assumed to be independent, while in our study, a correlation does exists, as explained in point 3 above.

Next we compare the rate of successes in answering a question as shown in Figure 6. As observed, the DVRL and the Q-learning methods achieve the highest rate of success in answering question. But, since they achieve a relatively low average of utility, one can infer that these methods tend to offer easy questions relative to the optimal level of questions that should be provided.

![Figure 6: The rate of questions that were successfully answered as a function of the number of questions in the simulation.](image)

Figure 6: The rate of questions that were successfully answered as a function of the number of questions in the simulation.

Note that the rate of success of the Bayesian can also be increased by assigning higher weights for correctly answering a question. For example, this can be done by assigning a negative utility for a question answered incorrectly, as provided in Utility function #2, which is defined in Section 3:

\[
\sum_{i=1 \ldots N} \left( \text{level}_i + C \mid \text{question } i \text{ was correctly answered} \right) - \sum_{i=1 \ldots N} \left( C \mid \text{question } i \text{ was not correctly answered} \right).
\]

According to this utility function, for each correctly answered question the student receives a positive constant C (reward) in addition to the question's level. However, for any incorrectly answered question the student receives a constant negative value C (penalty).

Consequently, next we compare all proposed algorithms when the following utility function is used to calculate the performance of the various algorithms as presented in Figure 7.

As can be seen in Figure 7, for the simulations considering the above utility function with a constant higher than 2 (i.e., the value of 2 for the reward and -2 for the penalty), and with 50 questions a tutoring session, the Bayesian algorithm achieves the highest accuracy level.

The explanation can be due to the fact that the policy of the Q learning family algorithms, and in particular the policy of DVRL, is careful, and for the most part they prefer to choose the well-known successful choice rather than explore new possibilities. Thus, they chose relatively low level questions, which achieve higher success rates but with lower total utilities.

However, as the successfulness of question answering has a higher effect on the utility function, the Bayesian learning method, which aims to maximize the utility function, also suggests low level questions to obtain higher utility values, and thus the rate of successful questions increases respectively.

Thus we can conclude that the choice of the utility function has a great impact on the algorithms' performances and further research should be conducted in educational arena to verify which utility function ideally represents the success of an ITS when taking into account the teachers' and the students' preferences that will allow the utmost efficiency of the ITS.

Another interesting matter is the effect of students who improve their abilities over time. In order to test the performance of the different algorithms in this case, we assumed that a mean threshold level for Adaptation Schemes for Question's Level to be Proposed by Intelligent Tutoring Systems

![Figure 7: The rate of questions correctly answered w.r.t. all questions that were asked, as a function of the constant C taken in the utility function (ITS with 50 questions).](image)

Figure 7: The rate of questions correctly answered w.r.t. all questions that were asked, as a function of the constant C taken in the utility function (ITS with 50 questions).
each student will increase with constant rate $\delta$ after each step of the simulation.

Figure 8 demonstrates that for runs of 50 questions, where the mean level of the student is improved each step by $\delta = 0.05$, and for more improvement, the Gittins algorithm outperforms all other algorithms, including the Bayesian inference method.

Figure 8: The algorithms’ average utility for given improvements in the student’s level during time.

However, an improvement level of $d = 0.05$ means that after 50 questions, a relative weak student (with an average level of 2) will become an almost excellent student (with an average level of 4.5), and in real environments, such an improvement does not occur so quickly.

The reason behind this lies in the fact that the Gittins based algorithm may return to non beneficial arms when their usage becomes relatively low (since the Gittins indices depends on the usage of the arm), and thus, the Gittins based algorithm does not ignore the improvement of the student, and gives ITS the ability to present harder questions as the student’s level improves. Future research is needed to include the student’s level of improvement in the model considered by the Bayesian learning algorithm.

8 CONCLUSIONS

In this paper we examine different RL algorithms that decide how to learn students’ ability and how to adapt the level of the question to the student’s ability. We examined different algorithms, including Q learning, TD, VRL, DVRL, Gittins’ indices and Bayesian inference, and we found that the Bayesian inference based algorithm achieved the best results. Moreover, the Q-learning based algorithm, named DVRL, achieved the highest success rate.

The advantage of the Bayesian inference based algorithm lie in the fact that it considers all the alternative distributions for each student, and updates its beliefs regarding all the alternatives after each step.

The conclusion from this paper is that it may be worthwhile to integrate the Bayesian inference based algorithm as a Reinforcement Learning method in future Intelligent Tutoring Systems.

However, these results are limited to artificial environments with simulated students. In future work we intend to compare the Bayesian inference algorithm using real data on the distribution of student levels. Moreover, we intend to implement the Bayesian inference based algorithm in a real ITS by practicing reading comprehension, to check its results on real students, and compare the results obtained when questions are chosen by the Bayesian inference algorithms w.r.t. results obtained from a software providing random generated questions.

An additional area of research is to define a more accurate utility function which will correctly reflect the students’ preferences from the automated software.

Another direction of future work would be to take into account the dynamic level of a student which changes during the running of the ITS, and suggest the appropriate model for the Bayesian algorithm to handle this situation. This direction is important when considering ITS which works with the same students over a long period, since during the said time period the student’s level can change. Consequently the appropriate adaptive algorithm should be considered for such cases.

REFERENCES


