Evaluation of Energy Efficiency of Aggregation in WSNs using Petri Nets

Ákos Milánkovich, Gergely Ill, Károly Lendvai, Sándor Imre and Sándor Szabó
Department of Networked Systems and Services, Budapest University of Technology and Economics, Budapest, Hungary

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Abstract: Energy efficiency is one of the key issues of wireless sensor networks. Aggregation of packets may increase significantly the lifetime of batteries in exchange for some variations in delay. In this paper we have investigated how to determine the optimal amount of packets gathered for aggregation that minimizes the energy consumption of the whole multi-hop network assuming predefined boundary conditions for the delay. To achieve this goal, appropriate models were created to calculate the energy consumption and delay, where we exploited the modelling capabilities of generalized stochastic Petri nets. Using these models, the impact of aggregation was analysed for various test cases. We examined how a network behaves in case of ideal, low and high BERs and investigated how different FEC coding schemes influence the energy consumption. Based on these results, we evaluated the properties of aggregation. We will show, that in case of a good quality radio channel (with low BER) it is not recommended to use FEC codes to optimize for energy consumption. In case of high aggregation numbers and high BER without the use of FEC the consumed energy converges to infinity. The simulation results show that using the delay as a constraint can narrow down the search for the minimal energy consumption of aggregation number vectors.

1 INTRODUCTION

Sensor networks and their applications are getting an increasingly important role in everyday life. With their help, we can solve various challenges, such as the development of agricultural monitoring and smart metering systems. In these systems, the devices used as nodes often operate in small-scale energy source (e.g., alkaline cell, battery). As a result, during the development of such systems energy efficient operation is extremely important.

In addition, unlike traditional protocols used in the Internet, the protocols used in sensor networks are not particularly sensitive to latency, because in the vast majority of cases, it is irrelevant when the data arrives within a certain time T interval to the data centre. This fact allows the devices to build up measurement or other useful information, and not send them immediately, but with some delay, treated in larger units. Exploiting this, the majority of the headers of the packets brought together can be saved, and only appear once in a larger sized packet. Therefore, the number of bits sent is reduced, which entails energy saving for the complete network.

To model the behaviour of a complete multi-hop network, first a model of a chain can be constructed, which can be extended to an arbitrary network topology. Using the model presented in this paper, the aggregation number vector can be determined solving an optimization problem, which minimizes energy consumption in the network.

The structure of this paper is the following: Section 2 presents some related studies; Section 3 introduces the mathematical model based on Petri nets. In Section 4 the model for energy consumption is constructed and the methods used for the simulations are described. Section 5 shows and analyses the results, and finally, Section 7 concludes the observations.

2 RELATED WORK

In our previous work (Lendvai et al., 2012), we have analysed the optimal packet size for energy efficient communication in delay-tolerant sensor networks using aggregation of the payload and considering the SNR (Signal to Noise Ratio) and BER (Bit Error
Rate) of the channel. In our following work (Lendvai et al., 2013), we extended our results for a FEC (Forward Error Correction) enabled channel.

Other studies introduced some other aspects of aggregation. For example, in (Feng et al., 2011) some methods can be found on how to avoid data loss in case of faults in the aggregation tree. The amendment scheme includes localized aggregation tree repairing algorithms and distributed rescheduling algorithms. Yu et al. (Yu et al., 2011) investigated the security aspects of aggregation by detecting false temporal variation patterns. Compared with the existing schemes, the scheme decreases the communication cost by checking only a small part of aggregation results to verify the correctness in a time window. Shoaib and Song (Shoaib and Song, 2012) deals with particle swarm optimization (PSO) used to optimize process of multi-objective data aggregation in vehicular ad-hoc network.

In this paper, we focus on aggregation without modifying the data itself; instead, we use bulk sending and analyse its energy efficiency.

3 PETRI NETS

This section describes the mathematical representation of Petri nets, which will be used in our model construction.

The simple Petri net (Peterson, 1981) is a directed, weighted bipartite graph. The elements of one vertex class is called Places ($P$) and the other class is called Transitions ($T$). In the directed graph, all edges connect a place and a transition. A positive integer, which is called the edge weight is assigned to the edges. The state of a Petri net can be described by a function that assigns a non-negative integer to each place. This is called token distribution, and the numbers represent the number of tokens at the places. Formally a Petri net is a $\mathbb{PN} = (P, T, E, W)$ structure, where

- $P = \{p_1, p_2, \ldots, p_k\}$ is the finite set of places,
- $T = \{t_1, t_2, \ldots, t_k\}$ is the finite set of transitions,
- $E \subseteq (P \times T) \cup (T \times P)$ is the set of edges, and
- $W: E \rightarrow \mathbb{Z}^+$ is the weight function.

3.1 Stochastic Petri Nets (SPN)

The SPN (Marsan, 1990) is a simple extension of Petri nets. A random firing time (delay) is assigned to transitions, which can be characterized by negative exponential probability distribution function. In addition, the firing semantics is altered as follows:

A transition can fire at time $t + d$, if
- it became enabled in time $t$,
- $d$ delay was drawn according to the corresponding distribution function,
- it has been enabled during $[t, t + d]$ time interval.

The transitions have a unique parameter, called rate. The $\lambda_i \in \mathbb{N}^+$ rate is the parameter of a $T_i$ transition’s delay’s negative exponential distribution. Such transitions are graphically marked by empty rectangles opposed to the general, immediately firing transitions. The drawn $d_i$ delay times are formulated as:

$$P(d_i \leq t) = 1 - e^{-\lambda_i t}$$
$$P(d_i > t) = e^{-\lambda_i t}$$

Next, let us discuss what happens if more than one transition is enabled as well. In this case, the firing transition will be the one, who’s drawn time delay expires first, therefore the enabled transitions are competing and the decision is based on probability. After one of the enabled transitions fired, a new marking is formed. In this case, the question may arise, should we to draw a new delay value. There are two possible solutions: a new draw can be made, or we use the remaining delay values. The solutions are indifferent, as the delay time has exponential distribution and the Markov property (Durrett, 2010) holds. As a result the remaining firing time is statistically independent of the elapsed time since the transition became enabled. In addition, for the enabled transitions the remaining firing time remains exponentially distributed, no matter how long they have been enabled.

The generalized stochastic Petri nets (GSPN) (M.Ajmone Marsan, 1995) are the extensions of stochastic Petri nets. The GSPN contains the following enhancements compared to the SPN:

- immediate firing transitions (dealing with logical dependencies),
- priorities between transitions,
- inhibitor edges,
- and guard conditions.

4 MODELING AND SIMULATION

In this section first the model for energy consumption is constructed using Petri nets. Then the methods and software used for the simulations are described. The section introduces the analysed FEC codes and a method for delay calculation.
4.1 Energy Consumption Model

To determine the energy consumption, the modelling power of GSPN-s was used, which can describe the overall network behaviour. The model developed by the authors the Places in the Petri net represent the packet storage queues of the nodes, while Transitions simulate the sending of packets. During the creation of the model, we assumed that each node spends zero time for processing packets. In addition, the nodes send the aggregated packet immediately, when they collected as many packets as their specific aggregation number. In addition, each node generates packets itself, generated according to an exponential distribution. According to previous description, the transitions either have an exponential delay, or fire immediately. We also assumed, that the nodes in the network are possess the same parameters, so the network is homogeneous. The weight of the edges in the graph created by the model is the same as the actual node’s aggregation number, with the default of one.

The developed model actually describes a single chain network topology. This is sufficient, because in any network topology a route is required in order to achieve communication between two nodes, which route consists of a chain of nodes.

To illustrate our model, let us take a chain topology consisting of five nodes. Consequently, assume, that the sensor data reaches the data centre (sink) in five hops. The graphical appearance of the model can be seen in Figure 1.

In Figure 1, the nodes of the network are marked by different colours. It can be seen, that apart from the first node and the data collector unit (called sink), all the other nodes are composed of two parts. The first part consists of the places and transitions responsible for the data coming from the node’s own sensors. The second group represents the data coming from the previous node. The own arrivals (i.e. the measured values read from the local sensors) are modelled by transitions with exponential distribution, which are illustrated by rectangles containing ‘T’. The arrival of the neighbour nodes is represented by the immediately firing transitions. The other transitions of the nodes immediately fire, when the predefined number of tokens is available.

The vector of aggregation numbers required to achieve minimum power consumption of the chain can be calculated with the previously presented model. To determine how much energy consumption a combination of aggregation numbers represent for the entire chain topology, we have to calculate the stationary distribution of the system, i.e., the amount of packets sent to each node in percentage distribution. Using this result, we can determine the total power consumption of the chain, if the energy consumption parameters of the nodes are known.

4.2 PetriDotNet

PetriDotNet (PetriDotNet, 2011) is a software, that runs on Windows and Mac OS X with a graphical user interface and can be used for editing, simulate and analyse Petri nets.

The software was developed by the Department of Measurement and Information Systems of BUTE. It was created with the aim of being easy to use in education. Their aim is to implement the latest verification and model checking algorithms for this easily extendable framework, and make it available to a larger user community. The program is able to:

- Saturation-based symbolic state-space generation, representation, and fixed-point computation based CTL model checking
- Transform the model checking problem to a linear programming task making it capable of examining infinite state space Petri nets
- Management and analysis of complex data structures is under development. The program is able to determine the long-term behaviour of Petri nets with its “Large Scale Statistics” module. The user can view the percentage distribution of the firing of transitions.

4.3 Simulation of Energy Consumption

The determination of the equilibrium distribution was carried out by the previously presented PetriDotNet software, because according to our tests, it was the fastest and it generated the best output results. To determine the optimal aggregation number vector, a preparation was needed, so that PetriDotNet calculates the long-term behaviour of the appropriate Petri net. This task was performed by Matlab (Mathworks, 2013).

As shown in Figure 2, MATLAB was used to generate all possible permutations of the aggregation number vector and the associated .pnml Petri net descriptions. Matlab sequentially calls (can be parallel) the PetriDotNet software with different aggregation number combinations. The PetriDotNet software’s Large Scale Statistics module was used to simulate 1,000,000 firings to determine the long-term behaviour. Using the output of the application and a predefined consumption function was given to calculate the total consumption of the chain. The developed consumption function in case of one
package is submitted (fire), for a single node in ideal case (when the radio channel is perfect), is (Milańkovich et al., 2012):

\[ E_{\text{node}} = (k_1 + k_2)(\omega + n\varphi) + k_3 \]  

(1)

In a real system to solve the multi-variable equation the values of \( k_1, k_2, k_3, \varphi, \omega \) parameters have to be defined according to (Milańkovich et al., 2012). Among these parameters \( k_1, k_2, k_3 \) describe the used hardware and \( \varphi, \omega \) are protocol specific. The amount of \( \omega \) overhead, and the \( \varphi \) payload and the size of the ACK is needed to be known. The formula described above, is a general solution to calculate the energy consumption, and can be applied to arbitrary hardware and communication protocol.

Using formula (1) the energy usage of the whole chain is the following:

\[ E_2 = \sum \varphi_i E_{\text{node} i} \]  

, where \( N \) the length of the aggregation chain and \( \varphi_i \) denotes the amount of packets sent (firings in the model) for the \( i \)th node.

### 4.4 Simulation of Delay

PetriDotNet has been used for calculating the equilibrium distribution, but could not be used to determine the amount of delay. To solve this problem, we created a simulation written in C++. The implemented program works according to the algorithm shown in Figure 3.

Initially, the aggregation number permutations, which were generated previously by Matlab have to be read. Then cyclically we do the following: the nodes are matched with their corresponding aggregation number of the current scenario. Then, aggregated messages are sent to the next neighbours in the chain in case the number of packets reached the aggregation number. The sending time of the messages is recorded, and the simulation goes, until the iteration number is reached. By the last message sending the total delay of the chain can be determined, and saved. Then the next scenario for aggregation vector is evaluated.

All the nodes aggregate their messages in a
specific recurring pattern. The length of these sequences depends on how many nodes preceded the current node and how many messages were aggregated by the previous nodes and the current one. In these sequences it can happen, that the amount of received messages reach the amount of the aggregation number, in this case the nodes send the messages immediately. However, it can also occur, that the nodes have to wait for more messages from their previous neighbour to reach their aggregation number and send the aggregated messages to the next node. Considering this fact, the delay fluctuates periodically. These fluctuations can be summed throughout the entire chain, and exert synergistic or antagonistic effects.

Given by the consumption formula described above (2), results on delay by the simulation algorithm, and binding it with the aggregation numbers, the best (minimal energy consuming) aggregation number configuration can be determined under the given boundary conditions.

4.5 Using FEC

In this section, we examine how the system behaves in the presence of packet loss (including packet errors can not be corrected) and FEC (Forward Error Correction) coding. We examined the energy consumption of the entire network and the amount of delay under these conditions. In this analysis, we need to introduce a new variable, which is denoted by \( \eta \). This variable shows how much energy is needed to encode one bit with FEC codes. The \( \eta \) values and other attributes of the applied FEC schemes are summarized in Table 1. The values were determined according to (Lendvai et al., 2013).

In this paper the following block codes were chosen for analysis: Hamming (255,247) (Lin, 2004), Reed-Solomon (511,501) (Bhargava, 1999), and BCH (511,502) (Ray-Chaudhuri, 1960), where the first number represents the output block length and the second number refers to the input length of the block code.

Table 1: Properties of applied FEC codes.

<table>
<thead>
<tr>
<th>FEC</th>
<th>Complexity</th>
<th>Type</th>
<th>Correctable bits (( t ))</th>
<th>( \kappa_4 ) [1 bit]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamming (255,247)</td>
<td>low</td>
<td>block</td>
<td>1</td>
<td>5.0522 \times 10^{-9}</td>
</tr>
<tr>
<td>Reed-Solomon (511,501)</td>
<td>high</td>
<td>block</td>
<td>5</td>
<td>5.4344 \times 10^{-7}</td>
</tr>
<tr>
<td>BCH (511,502)</td>
<td>high</td>
<td>block</td>
<td>4</td>
<td>9.0037 \times 10^{-5}</td>
</tr>
</tbody>
</table>

Based on the previous facts (Peterson, 1981), the formula of energy consumption for sending ad receiving a packet between two nodes is changed compared to formula (1) described in the previous chapter:

\[
E_{FECC} = (\kappa_1 + \kappa_2) \left( \omega + N \frac{N_d}{N_c} \right) + \kappa_4 N + \kappa_5 \]

, where \( N \) is the output of the FEC encoder used, and \( K \) is the length in bits of the useful portion of the FEC code. Other symbols used in the formula (3) are identical to those presented in section 4.3. If the values of \( N \) and \( K \) are both set to one and \( \kappa_4 \) is set to zero, then the result is the same as if no FEC was applied.

The packets sent over the noisy radio channel arrive erroneously at the receiver side. The rate of errors is expressed by the BER (Bit Error Rate), which is the number of bit errors divided by the number the total sent bits. The FEC codes are able to repair the errors to a certain extent, so that the package can be restored. The value of PER (Packet Error Rate) is the probability of failure of a package. The use of FEC codes reduce the probability of PER.

Of course, this probability depends on the error correction capability of the applied FEC code and the BER of the channel. The following computations use the value of PER, which is defined by BER using the following equation:

\[
PER_{FECC} = 1 - \left( 1 - p \right)^t \left( \sum_{i=0}^{t} \binom{N}{i} p^i (1-p)^{N-i} \right) \]

, where \( t \) denotes the error correcting capability of the FEC, and \( p \) is the BER.

The formula (5) and (6) jointly determines how many packets needed to be sent and re-sent in total for a successful reception. The \( m \) in the formula in the total number of packets we want to send and \( \Phi \) specifies how many packets are actually sent for the successful reception of \( m \) packets.

\[
min \{ k : \left[ m \ PER_{FECC}^k \right] < 1, k \in \mathbb{Z}^+ \} \]

\[
\Phi = \sum_{i=0}^{k} m \ PER_{FECC}^i \]

With formulas (5) and (6), the total consumption of the chain topology can be determined, which is as follows:

\[
E_{total} = \sum_{i=1}^{n} \Phi_i E_{FECC_i} \]
, where \( n \) in this case marks the number of nodes in the chain.

The presence of packet loss and the use of FEC affects the delays occur on the chain. Let us imagine that one node in the chain has incorrectly received an aggregated packet and was not able to restore it. Then it cannot send a positive acknowledgment to the node in before of it, so the previous node retransmits the packet when its timer expires. The probability of receiving the packet incorrectly repeatedly will be less and less, so it will eventually become a successful reception. Delay suffered because of faulty reception and retransmission due to specific boundary conditions is negligible, even if packet loss occurs in the chain repeatedly.

5 RESULTS

In this section, we report and analyse the results obtained by simulations with the developed model with different parameters. To determine the energy consumption, a sensor network protocol developed by the authors was used. The calculation method of the parameters can be found in (Lendvai et al., 2013). The parameters of the used devices (TI CC1101 (Texas Instruments, 2011) radio module, and Atmel AVR XMEGA A3 (Atmel corporation, 2010) microcontroller) are determined by their datasheets; therefore, the values of the constants in the formula are the following:

\[
\kappa_1 = 2.339 \cdot 10^{-6} \frac{J}{\text{bit}},
\]
\[
\kappa_2 = 13.742 \cdot 10^{-9} \frac{J}{\text{bit}},
\]
\[
\kappa_3 = 93.387 \cdot 10^{-6} \frac{J}{\text{bit}},
\]
\[
\omega = 288 \frac{\text{bit}}{\text{packet}},
\]
\[
\varphi = 80 \frac{\text{bit}}{\text{packet}}.
\]

During the simulations, we have investigated the five-node-long chain shown in Figure 1.

In the simulations, the delays are in the order of hours, because the delay resulting of the additional packet loss – at worst a few minutes – is considered negligible.

The following section describes various test cases. Table 2 and Table 3 presents the first and last three of aggregation number vectors in case we optimised for energy efficiency. Table 3 shows the corresponding aggregation numbers of the cases with the most and least delays. In these scenarios, the radio channel was considered ideal (i.e. causing no bit errors). The columns N1-N5 represent the aggregation numbers on the nodes of the chain.

Table 2: The aggregation numbers of the best and worst energy consumption.

<table>
<thead>
<tr>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>N4</th>
<th>N5</th>
<th>E [J]</th>
<th>Delay [h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>427.02</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>428.43</td>
<td>1.9</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>429.83</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Table 3: Aggregation numbers of the best and worst delays.

<table>
<thead>
<tr>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>N4</th>
<th>N5</th>
<th>E [J]</th>
<th>Delay [h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>475.6</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>495.02</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>495.02</td>
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<td>5</td>
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<td>1.9</td>
</tr>
</tbody>
</table>

Figure 4 and Figure 5 show the diagrams of the simulation results of scenarios with no bit errors on the radio channel. The values of energy consumption have been scaled so that they can be displayed on a common chart with the values of delay. The simulations inspected a five-node-long chain, where the aggregation numbers were integers between zero and five, which equals the inspection of 3125 test cases. To help the interpretation of the presented charts, the linear regression of the delay is also drawn.

Figure 4: The characteristics of energy consumption and delay of various aggregation vectors in ascending order of energy consumption.

The values of Figure 4 are the scenarios of aggregation number vectors in ascending order of energy consumption, while the values of Figure 5
are shown in ascending order of delay. In terms of energy efficiency the aggregation number vector (5,5,5,5,5) performed best, however, in terms of delay the best vector was (1,1,1,1,1).

The simulations of Figure 6 were conducted on a homogeneous radio channel with a BER value of $5 \cdot 10^{-5}$ for every link. The used aggregation numbers were low integers between zero and five. It can be seen, that FEC schemes cause additional energy consumption compared to baseline. Among FEC schemes, the ones with longer code length produce higher consumption values.

Figure 7 shows the simulation results for energy consumption without FEC and with some FEC schemes on a radio channel with homogeneous BER ($5 \cdot 10^{-5}$). This figure shows clearly, that if the value of BER increases, the importance of FEC grows in terms of energy efficiency. If BER is increased two orders of magnitude, then the scenarios using FEC produce a much more efficient result. The diagrams of Figure 7 seem to contradict the expectations, as the scenarios using FEC do not follow the trends of the base scenario without FEC.

The aggregation number vectors are low in case of low energy consumption without FEC when we simulated on a worse quality radio channel, because the length of the aggregated packet is less, so that the PER is also less according to formula (4), which results in a lower energy consumption. On the contrary, in case of higher aggregation number vectors the packets must be resent more frequently due to packet errors, so the energy consumption increases. However, in case of using FEC, the higher aggregation number vectors produce better energy efficiency (i.e. lower consumption values). This can be explained as the following: the use of FEC decreases the PER, so the number of resent packets also decreases, which results in a lower energy consumption. The more we aggregate the beneficial effects of FEC codes increase.

Figure 8 shows the results of a simulation conducted with a BER value of $5 \cdot 10^{-4}$, and the aggregation numbers were the permutations of the following set: \{30,21,12,5,10\}. According to the charts, the best performing FEC scheme was the Hamming code.

Figure 9 shows the energy consumption of the test cases with different aggregation number vectors, but identical delays (in this case 0.9 hours). It can be seen, that using the delay, as a boundary condition there is no exact solution for minimal energy consumption, because the different aggregation numbers require different amounts of energy. Practically the aggregation number vector with the minimum energy can be selected for a given delay.
6 CONCLUSIONS

According to our investigations on packet aggregation in wireless sensor networks we found, that in real systems there is a conflict between energy consumption and delay, therefore finding the optimal value can be a question of trade-off.

We also concluded, that in case of a good quality radio channel (with low BER) it is not worthy to use FEC codes in case optimizing for energy consumption. The reason is that the additional energy needed for coding and the overhead of the code word length is present in the system. On the contrary, in case of bad quality channels (with higher BER) the use of FEC is reasonable to decrease energy consumption as the energy needed for retransmission due to packet errors can be spared.

The use of FEC in case of higher BER and aggregation number vectors is also beneficial, because the PER of longer packets decreases, which results in lower energy consumption. The energy consumption in case of high aggregation numbers and high BER without the use of FEC converges to infinity.

The simulation results show that the delay as a constraint can narrow down the search for the minimal energy consumption of aggregation number vectors.

Further research will focus on the application of the presented model in routing algorithms for sensor networks.

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