Keywords: fMRI Time Series, Classical Statistics, Bayesian Inference, Group Analysis.

Abstract: This paper suggests one method to process fMRI time series based on Bayesian inference for group analysis. The method uses multilevel divided by session, subject and group as pair comparison to reinforce posterior probability in group analysis from single subjects as priors. And also it combines classical statistics, i.e., t-test to obtain voxel activation at subject level as prior for Bayesian inference at group level. It effectively solved computation expensive and complexity. And it shows robust on Bayesian inference for group analysis.

1 INTRODUCTION

In the past decades, functional Magnetic Resonance Images (fMRI) technology has been obtained greatly attention all over the world, especially in brain science field. Most researches have explored brain principles from the structural to effective connectivity. Especially for clinical, fMRI would provide more help for diagnosis and curing brain diseases, e.g., Alzheimer’s disease, depression, schizophrenia, sclerosis and non-communicative brain damaged patients (Margulies et al., 2010).

Functional MRI is a non-invasive technique for studying brain activities (Lindquist, 2008). It analyses blood oxygen level dependent (BOLD) hemodynamic response to identify brain activation by stimulus. It characters hemodynamic response function (HRF) to measure brain spatial distribution based on BOLD signals about neural activity by vascular hemodynamic changing. The goal of fMRI analysis is to detect, in a robust, sensitive and valid way, those parts of the brain that show increased intensity at the points in time that stimulation was applied (Smith and Dphil, 2004). They include functional segregation, functional connection and effective connectivity.

Most analysis methods of fMRI data are divided into two categories: model driven and data driven. For model driven, commonly it uses traditional statistics methods to measure fMRI data time series. For data driven, it is based on image density to compute distance, similarity or features, e.g., Cluster analysis, Independent Component Analysis (ICA) and self-organization mapping etc. Statistics methods are based on a general linear model (GLM) model to estimate parameter for each voxel and compute p-value, under null hypothesis and obtain p-value probability distribution mapping. And then it maps the probability of each voxel for whole brain to make statistics parameter mapping (SPM). Due to issues on classical method, for instance, it never rejects alternative assumption meaning activation always occurred, and has false positive ratio (FDR) for multiple comparison problems. On the contrary, alternative method is Bayesian which can give the probability that the effect is greater than some threshold under voxel activation to avoid above issues.

In Bayesian theory, the posterior distribution captures all information inferred from the data about the parameters. As such (Woolrich, 2012) it proposed the first Bayesian group inference approach using a hierarchical model. Bayesian uses high-level estimation as prior and then enable posterior inferences of the parameters in low-level. Then inference is based on the posterior distribution of the parameters from given the data.

This paper suggests a multilevel Bayesian inference for group analysis based on hierarchical model. The multilevel group method is proportional to multiple levels according to session level, subject level and group level with comparing individual
subjects as selected prior. We use classical statistics and Bayesian 1st level to compare variances to inspect prior for individual subjects. Through different subjects, it passes the estimated parameters from session level parameters in one subject as prior to compute posterior of next subject. For group level analysis, it uses the effects of single subject as prior to provide next subject analysis based on Bayesian posterior probability. This can reduce computation cost and complexity.

For the paper structure, section II describes Bayesian inference theory and estimation in multilevel group analysis. Section III shows an fMRI case analysis with lower level of individual subject as prior and passing statistics value to higher level of group. In the last part we specify Bayesian methods for fMRI dynamic analysis in the future.

2 BAYESIAN METHODS

Bayesian statistics approach is to use conditional or posterior inference based upon the posterior distribution of the activation by observed data. A fully Bayesian statistics approach as the first paper considered the full posterior probability distribution was appeared in 1998 (Woolrich, 2012).

In (Friston et al., 2002a, 2002b), it describes Bayesian on hierarchical linear model to form first level recursively. And it combines hierarchical model with classical and Empirical Bayesian, called all in one (Woolrich et al., 2004), to show two methods based on the same principle by covariance components and EM.

For group analysis based on Bayesian, most methods rely on prior selection. Usually prior is from temporal or spatial perspectives, or both of observed data. Temporal prior is commonly designed by hierarchical model divided into session level, subject level and group level under two levels. In (Woolrich et al., 2004, Beckmann et al., 2003), they use two levels and fully Bayesian framework, passing summary statistics from first level to second level. And also in (Neumann and Lohmann, 2003) it gives different relation between subject level and group level according to Bayesian principles guided by (Box and Tiao, 1992). It passed a random subject as prior to estimate parameters for other subjects. In (G’omez-laberge et al., 2011) it uses Bayesian to cluster analysis which proposes a Bayesian hierarchical model to describe the correlation structure of the observed voxel clusters. For spatial prior, some use regions or areas (Lei et al., 2009) in Brain to characterize the spatial features of the HRF over the regression coefficients (Penny et al., 2003). And in (Ahn et al., 2011), it demonstrated that hierarchical Bayesian analysis outperforms conventional maximum likelihood estimation in recovering true parameters no matter individual or group analysis.

As (Woolrich, 2012) showing the all procedures of Bayesian in fMRI analysis, Bayesian methods become popular method as statistics inference about activation voxels and group analysis. Through the above analysis on group methods based on Bayesian, we combine Bayesian with hierarchical linear model to estimate parameters from observed data by EM algorithm. And about prior selection, we suggest that prior is selected from comparing different individual subjects analysed by classical method and Bayesian level.

2.1 Model

For groups analysis, we may construct different levels from session, cluster, subject and group perspectives. As shown in Figure 1, we can divid data into hierarchical levels by the session, subject and group levels.

We accept hierarchical linear model to construct parameters among groups including session-level and group-level. According to the hemodynamic response with observed data under stimulus, the hierarchical linear model is defined for individual subject as below Equation (1).

\[
\begin{pmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_n
\end{pmatrix} = \begin{pmatrix}
  x_{11} & x_{12} & \cdots & x_{1p} \\
  x_{21} & x_{22} & \cdots & x_{2p} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{n1} & x_{n2} & \cdots & x_{np}
\end{pmatrix} \begin{pmatrix}
  \beta_1 \\
  \beta_2 \\
  \vdots \\
  \beta_p
\end{pmatrix} + \begin{pmatrix}
  \epsilon_1 \\
  \epsilon_2 \\
  \vdots \\
  \epsilon_n
\end{pmatrix}
\]

(1)

The equation is consisted by three parts: observed data Y which includes each voxel time series with n scans, design matrix X which has contrast regression coefficients with interest and error. And also it uses \( \beta \) to describe amplitude as parameters of explanatory.
In group analysis, these subjects have the same scanning environment and also have similar background, i.e., age, gender, education, health. Through these similarities of group, we assume that they have similar contrast regression of interest effects. It shows Hierarchical linear model as below equation (2) for group analysis:

\[
\begin{pmatrix}
    x_1^{(1)} \\
    y_2^{(1)} \\
    \vdots \\
    y_n^{(1)} \\
    \vdots \\
    x_1^{(m)} \\
    y_2^{(m)} \\
    \vdots \\
    y_n^{(m)}
\end{pmatrix}
= \begin{pmatrix}
    \beta_1^{(1)} \\
    \beta_2^{(1)} \\
    \vdots \\
    \beta_L^{(1)} \\
    \vdots \\
    \beta_1^{(m)} \\
    \beta_2^{(m)} \\
    \vdots \\
    \beta_L^{(m)}
\end{pmatrix}
\begin{pmatrix}
    1 \\
    x_1 \\
    \vdots \\
    x_{np}
\end{pmatrix}
\]

(2)

The equation (2) describes one group with m subjects, single subject with n scans and each subject with different estimated parameters and errors.

For fMRI data, Bayes directly obtains posterior distribution of parameters combined prior with observed data under unknown parameters and easily to compute the probability of parameters by Bayesian rules. For prior unknown, the estimation processing is referred to as empirical Bayes (Ashburner et al., 2003). And inference is based on the posterior distribution of the parameters given the data (Morris, 1983; Casella, 1985). According to the Bayesian inference based on hierarchical linear model, the procedure of computation in details is shown as Figure 2.

These priors can be estimated from given the data and we have multiple subjects of the effect interested explanatory variants. Bayesian uses high-level estimate as prior and then enable posterior inferences about the parameters in low-level by Bayesian rule.

### 2.2 Bayesian Rule

According to the two levels model, we use Bayesian rule to reduce posterior probability distribution by prior distribution. Bayesian is to calculate the posterior distribution by prior information and some new observed data on the first level. By Bayes’ rule, the posterior of data y is given by equation (3):

\[
p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}
\]

Where \(p(y|\theta)\) is marginal likelihood or evidence and \(p(\theta)\) as prior. As \(p(y)\) be known, Bayesian rule becomes the equation (4):

\[
p(\theta|y) \propto p(y|\theta) \cdot p(\theta)
\]

All marginal likelihood functions have the same distribution as prior distribution fitting to normal distribution. At first, according to the prior distribution as normal distribution \(\theta \sim N(\mu, \tau^2)\), it gives \(p(\theta)\) and \(p(\theta|y)\) likelihood functions as below (5).

\[
p(y|\theta) = \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^n \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \theta)^2\right\}
\]

And about prior with normal distribution is shown in (6):
\[ p(\theta) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\theta - \mu)^2}{2\sigma^2}\right) \quad (6) \]

Putting together, we obtain the \( p(\theta | y) \) probability density function in (7). In details reduction, it is specified at (Box and Tiao, 1992).

\[
p(\theta | y) = \frac{(\sigma_0^{-2} + \sigma_i^{-2})^{1/2}}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(\sigma_0^{-2} + \sigma_i^{-2})(\theta - \bar{\theta})^2\right] \quad (7)
\]

With the mean ad variance are shown as below (8).

\[
\bar{\theta} = \frac{1}{\sigma_0^{-2} + \sigma_i^{-2}}(\sigma_0^{-2}\theta_0 + \sigma_i^{-2}y) \quad (8)
\]

\[
\sigma^2 = \frac{1}{\sigma_0^{-2} + \sigma_i^{-2}}^{-1} \quad (9)
\]

Combining the hierarchical linear model with Bayesian rule in group, it has basic formulation as below (9).

\[
p(\theta | y_1, y_2, ..., y_m) = p(y_1, y_2, ..., y_m | \theta)p(\theta) \propto p(\theta) \prod_{i=1}^{m} p(y_i | \theta) \quad (9)
\]

This reduction is from (Bradley, 1996). Thus, it combines all formulations into multilevel in group analysis to show posterior and prior relation as (10).

\[
p(\theta | y_1, y_2, ..., y_m) = p(\theta | y_1, y_2, ..., y_m) \propto p(y_1, y_2, ..., y_m | \theta)p(\theta) \quad (10)
\]

For prior selection, some suggest spatial prior (Penny et al., 2005) and some use wavelet coefficients as prior (Sanyal and Ferreira, 2012). As like Stephan (Neumann and Lohmann, 2003) described, “Today’s posterior is tomorrow’s prior” which we use the rule as one subject parameters as prior for next subject in group analysis to decrease computation cost and complexity.

### 2.3 Estimation

We use an empirical Bayes methodology to estimate the hyperparameters which are shared by all subjects. Parametric empirical Bayes can be formulated classically in terms of covariance component estimation (e.g. within subject vs. between subject contributions to error) (Morris, 1983; Casella, 1985). Through \( p(\theta | y) \), we estimate posterior mean and posterior covariance. To estimate the covariance components, many different computation methods are used, for example, some use point estimation, some use maximum a posterior probability (MAP) with MCMC under numerical integration unavailable. In (Friston et al., 2002b), it uses EM algorithm to estimate error and prior covariance. It has two basic steps in EM algorithm as equation (11).

For two steps, one is E-step and the other is M-step.

\[
E-step: \quad Q(\theta | \theta^{(i)}) = E(\log(f(y|\theta)) | y, \theta^{(i)}) \quad (11)
\]

\[
M-step: \quad \theta^{(i+1)} = arg\max(\theta^{(i)}) \quad (11)
\]

E-step computes likelihood function according to \( i \)th effect or initial value by the first subject and M-step makes likelihood function maximum to obtain new parameters. And iteratively it obtains estimator through the two steps iteratively until convergence.

### 2.4 Inference

This section describes the construction of posterior probability maps that enable conditional or Bayesian inferences about regionally-specific effects in neuroimaging. All the procedure is focused on posterior probability computation. At the same time, Bayesian inference requires prior known or unknown estimated from given data. This posterior density can be computed, under Gaussian assumptions, using Bayes rules.

Posterior probability maps (PPMs) are images of the probability or confidence that activation exceeds some specified threshold, given the data. PPMs require the posterior distribution of a contrast of conditional parameter estimates by given the data (Ashburner et al., 2003). It will make mean as Bayesian estimator to compute \( p \) by the equation (12).

\[
P = 1 - \Phi \left( \frac{y - \beta^{(i)}}{\sqrt{\text{error}^2}} \right) \quad (12)
\]

\( \Phi(\cdot) \) is the cumulative density function of the unit normal distribution (Friston and Penny, 2003).

An image of these posterior probabilities constitutes a PPM. According to the \( p \)-value, it will map PPMs to show the activation distribution about voxels on confidence 95%. The probability that activation has occurred, given the data, at any particular voxel is the same (Friston and Penny, 2003).

At the first level of the hierarchy, it corresponds to the experimental effects at session-level and obtains the probability of voxel activation. And at
the second level of the group, it comprises the effects over subjects through the first level of the individual subject effects. We describe the Bayesian inference procedure shown in Figure 3.

[Figure 3: Bayesian inference with PPMs procedure.]

3 EXPERIMENT

3.1 Data Collection

In this experiment, we choose the dataset which consists of 24 contiguous slices, $64 \times 64 \times 24$ in each volume with $2 \times 2 \times 2$ mm$^3$ voxels in thickness 5mm in whole brain BOLD response acquired using 3.0T fMRI system. For block design, it includes blocks of 6 scans with 12 blocks by removing the first 6 scans in TR 2s. We design the task with the condition for successive blocks alternated between rest and visual picture stimulation from the beginning of rest.

3.2 Preprocessing

During scanning for fMRI data, although usually subject is required to fix in a frame to avoid motion to reduce images artifacts, due to machine heating effects, physical effects as cardiac and respiration, and moving from subjects, these images from scanning include some noises. Some noises from machine heating with high frequency are eliminated by high frequency filters and some artifacts from motion can be corrected by preprocessing.

The key issues of preprocessing in SPM are mainly involved: (1) realignment: It completes motion correct by align images according to the first image in the each session and align other sessions according to the first session; (2) coregistration: Match images from same subject but different modalities by coregistration. It supplies mean images in data to register structural image solving consistence between functional images and structural images; (3) segmentation: It segments structure T1* image to grey matter, white matter and CSF. And it obtained some parameters for normalize functional images; (4) normalization: Make results from different studies compared by aligning them to standard space it can deal with different Talairach problems. It normalizes functional images onto template images, for example, EPI template; (5) smoothing: Through removing lower frequency noises, it extends larger spatial SNR in spatial overlap by blurring over minor anatomical differences and registration errors; Smoothing can average neighbouring voxels suppresses noise and increase sensitivity to effects of similar scale to kernel.

For our experiment, we choose realignment and normalize to reduce motion artifacts and make data being consistence. And also we use classical inference which needs smoothing as preprocessing to improve SNR; we separate data without smoothing for Bayesian 1st level.

3.3 Results

Efficient computation at the second-level requires full access to the first-level parameter estimates and associated covariance. This involves both the variances of the parameter estimates and the covariance between different parameters.

PPMs show posterior probability p value about activation in group analysis. According to the activation, is given the results in PPMs which plot a map of effect sizes at voxels where it is 99% sure that the effect size is greater than 2% of the global mean. And it compares the similar covariance among group in Table 1.

Table 1 is arranged columns which are from right to left as: (i) region of interest; (ii) voxel-level t-value; (iii) Z-value; (iv) means; and (v) standard deviate. The maximum intensity projection (MIP) of the statistical map is displayed (Friston, 2002).Throughout the Figure 4, it is shown the fitted response through even-relative response results among some subjects. With the activation on voxels for individual subjects, we can compare different
subjects in the group with similar variances and then we can choose the some subjects as priors for next group computation.

5 CONCLUSIONS

Any approach to variance estimation can easily be combined with the multilevel GLM to provide a practical multilevel method (Beckmann et al., 2003). Indeed, Bayesian approaches present the significant effects by combination hierarchical model with posterior probability. And we can set prior as multiple levels by comparing subjects as prior in group analysis to increase computational speed and more precise effects.

All the above ideas would be the objectives for next research hot points. Furthermore, Bayesian would be served for brain science.

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