FREQUENCY SPECTRUM OF THE WAVE BACKSCATTERED TO TRANSCEIVER MOVING TOWARDS ROUGH SURFACE

Alexander B. Shmelev
Radiotechnical Institute by Academician A.L.Mints, 8 Marta Str.,Bld.10-1, Moscow, 127083, Russia
abshmelev@yahoo.com

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Abstract: Frequency spectrum of the wave scattered by randomly rough surface back to transceiver located on aerospace vehicle moving towards the surface is evaluated and investigated in explicit form. Kirchhoff's (physical optics) method is applied for scattered field evaluation. It is assumed that transceiver irradiates directive spherical wave illuminating circled area on the surface. Distribution of the rough surface height is assumed to be normal with isotropic Gaussian correlation function. Application the effective approximation formula for characteristic functions difference in integrand gives rise to spectrum evaluation for arbitrary height of surface irregularities. Frequency spectrum is shown to exist in two forms. The first one is represented by monotonic curve, depending on correlation distance of the rough surface. The second form includes one maximum, which position and amplitude are related with the roughness' mean square slope. On the parameter plane the curve is plotted which separates regions with abovementioned spectrum forms.

1 INTRODUCTION

The purpose of this paper is theoretical evaluation frequency spectrum of radiowave backscattered to transceiver moving towards randomly rough surface. This situation occurs, for example, before spacecraft landing on the Moon or planet surface. Irregularities of such surfaces are formed by natural factors and may be described as random fields. Frequency spectrum provides information about statistical characteristics of rough surface, such as correlation distance, mean square height and slope of its irregularities.

The scattering problem on randomly rough surface may be formulated as follows. Let scalar (sound) or vector (electromagnetic) wave fall on the surface *S* separating two media. The surface is described by equation $z = \zeta(x, y, t)$, where ζ is random function of coordinates *x*, *y* and time *t*. It is required to establish relation between statistical parameters of rough surface and characteristics of scattered field. Approaches to this problem as well as results obtained were described in literature at various times (Beckman and Spizzichino, 1963), (Bass and Fuks, 1972), (Shmelev, 1972). Nevertheless this problem is actual up to now because of application peculiarity variety.

We use in this paper Kirchhoff's (physical optics) method – the most developed and effective in wave scattering problems. It is based on assumption that reflection of incident wave at every point of rough surface locally obeys geometric optics laws. This means that our consideration is restricted to rather smooth and gentle irregularities, which curvature radius is large in comparison with wave length. We don't take into account shadowing effects, so surface slopes are assumed to be not too sharp.

The problem solution by Kirchhoff's method is used to include two stages. At the first stage dynamical part of the problem is considered. General expression for wave field diffracted on the surface *S* is composed in Kirchhoff's approximation. The surface height is described herein by arbitrary function $z = \zeta(x, y, t)$. At the second stage this function is declared to be random and various statistical characteristics of scattered field, such as middle value, average intensity, correlation function etc., are evaluated by averaging over rough surfaces ensemble. In this paper we are interested in frequency spectrum of backscattered field when transceiver is moving towards rough surface (in

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vertical direction). Analogous problem in the case when transceiver is moving along rough surface (in horizontal direction) was studied earlier (Shmelev, 1973). More accurate explicit results may be obtained in the case under consideration.

2 DYNAMICAL PART

For the sake of simplicity we consider scalar (sound) waves taking in mind that vector character of electromagnetic wave acts on polarization but not on spectrum shape. Let the directional spherical wave

$$\Phi(\mathbf{r}) = \frac{F(\mathbf{n}_r)}{|\mathbf{r}-\mathbf{R}_0|} \mathrm{e}^{ik|\mathbf{r}-\mathbf{R}_0|} \tag{1}$$

fall on the rough surface *S* which height $z = \zeta(x, y)$ diverges from the mean plane $z = \langle \zeta(x, y) \rangle = 0$ denoted by S_0. The wave number in upper medium is $k = \omega/c$, directivity pattern of transmitter is $F(\mathbf{n}_r)$, where $\mathbf{n}_r = \dfrac{\mathbf{r}-\mathbf{R}_0}{|\mathbf{r}-\mathbf{R}_0|}$, $\mathbf{R}_0$ - transmitter position vector.

At first we consider motionless surface *S*. Its movement towards transceiver will be taken into account in quasi-static approximation by time dependence restoration in final expression for diffracted field. This approximation is valid if transmitter velocity is small in comparison with light velocity $v \ll c$. Geometric scheme of the wave scattering problem is shown on Figure 1 for general case of spaced transmitter *Q* and receiver *P*.

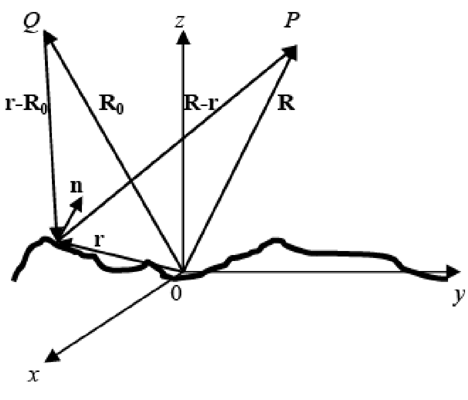

Figure 1: The scattering problem geometry for spaced transmitter and receiver.

Here $\mathbf{R_0}$ and $\mathbf{R}$ are transmitter *Q* and receiver *P* position vectors, $\mathbf{r} = (x, y, \zeta(x, y)) = (\mathbf{r}_\perp, \zeta)$ is radius-vector of rough surface point, $\mathbf{n}$ - surface normal at this point.

Diffracted field at observation point *P* is related with values of the field φ and its normal derivative $\partial\varphi/\partial n$ on the rough surface *S* by Green's formula

$$\varphi(\mathbf{R}) = \frac{1}{4\pi} \int_S \left(\varphi \frac{\partial}{\partial n} \frac{\mathrm{e}^{ik|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}|} - \frac{\partial \varphi}{\partial n} \frac{\mathrm{e}^{ik|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}|} \right) dS\,. \tag{2}$$

In Kirchhoff's approximation following relations are valid at the every point of surface *S*

$$\varphi = V\Phi, \quad \frac{\partial \varphi}{\partial n} = -V \frac{\partial \Phi}{\partial n}, \tag{3}$$

where $V = V(\mathbf{r}, \mathbf{n})$ is local Fresnel reflection coefficient. Substitution (1) and (3) into (2) gives

$$\varphi(\mathbf{R}) = \frac{1}{4\pi} \int_S V \frac{\partial}{\partial n} \left[\frac{F(\mathbf{n}_r) \mathrm{e}^{ik|\mathbf{r}-\mathbf{R}_0| + ik|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}| \cdot |\mathbf{r}-\mathbf{R}_0|} \right] dS\,. \tag{4}$$

Let us assume now that transmitter and receiver are situated far enough from the rough surface, so that $R, R_0 \gg \lambda, \sigma, k\sigma^2$, where σ is mean square height of surface irregularities and $\lambda = 2\pi/k$ - wave length. Then we separate two types of multipliers in (4) – rapidly oscillating exponent and slowly varying functions weakly dependent on rough surface height $\zeta(\mathbf{r}_\perp)$. Setting $\zeta = 0$ in this functions, keeping linear with respect to ζ terms in exponent power expansion and changing integration over surface *S* by integration over middle plane S_0, we come to known expression for diffracted field

$$\varphi(\mathbf{R}) = \frac{i}{4\pi} \int_{S_0} VF(\mathbf{n}_{01}) \frac{\mathrm{e}^{ikR_{01} + ikR_1}}{R_{01} R_1} \frac{q^2}{q_z} \mathrm{e}^{iq_z \zeta} d^2 \mathbf{r}_\perp, \tag{5}$$

where $\mathbf{r}_\perp = (x, y, 0)$, $\mathbf{R}_{01} = \mathbf{r}_\perp - \mathbf{R}_0$, $\mathbf{R}_1 = \mathbf{R} - \mathbf{r}_\perp$, $\mathbf{n}_{01} = \mathbf{R}_{01}/R_{01}$, $\mathbf{n}_1 = \mathbf{R}_1/R_1$ and $\mathbf{q} = k(\mathbf{n}_{01} - \mathbf{n}_1)$ is the scattering vector.

Let us now take into consideration motion of the rough surface along *z*-axis with the constant velocity *v*. In quasi-static approximation we have to set in (5) $\zeta = \zeta(\mathbf{r}_\perp, t) = \zeta(\mathbf{r}_\perp) + vt$. In addition we assume that surface *S* is absolutely reflecting (*V*=1) – absolutely rigid in acoustics or perfectly conductive in electrodynamics. This removes influence of surface material properties on spectrum under investigation. Solution of dynamical part of the problem takes on final form

$$\varphi = \frac{i\,\mathrm{e}^{i\omega_0 t}}{4\pi}\int_{S_0} F\left(\mathbf{n}_{01}\right)\frac{\mathrm{e}^{ik\left(R_{01}+R_1\right)}}{R_{01}R_1}\frac{q^2}{q_z}\mathrm{e}^{iq_z\zeta(\mathbf{r}_\perp)+iq_z vt}\,d^2\mathbf{r}_\perp\,. \quad (6)$$

This expression provides basis for further evaluation statistical characteristics of diffracted field.

3 AVERAGE FIELD

Averaging expression (6) over ensemble of random field $\zeta(\mathbf{r}_\perp)$ realizations leads to average value of diffracted field

$$\langle\varphi\rangle = \frac{i\,\mathrm{e}^{i\omega_0 t}}{4\pi}\int_{S_0} F\,\frac{\mathrm{e}^{ik\left(R_{01}+R_1\right)}}{R_{01}R_1}\frac{q^2}{q_z}\,f_{1\zeta}\left(q_z\right)\mathrm{e}^{iq_z vt}\,d^2\mathbf{r}_\perp\,, \quad (7)$$

where corner brackets denote averaging operation, $f_{1\zeta}\left(q_z\right)=\left\langle\exp\left(iq_z\zeta\right)\right\rangle$ is characteristic function of the rough surface *S*. We suppose that random field $\zeta(\mathbf{r}_\perp)$ is statistically homogeneous.

Evaluation this integral by means of stationary phase technique gives physically transparent result

$$\left\langle\varphi(\mathbf{R},t)\right\rangle = f_{1\zeta}(q_{zs})\varphi^{(0)}(\mathbf{R})\exp\left(iq_{zs}vt\right), \quad (8)$$

where $q_{zs}=2k\cos\theta_s$ is *z*-component of scattering vector at stationary point coinciding with the point of mirror reflection from the mean plane S_0. This point is chosen to be origin of coordinates. Incidence angle at stationary point is denoted by θ_s. The field mirrored from the plane S_0 is designated as $\varphi^{(0)}(\mathbf{R})$.

Average field is interpreted like coherent part of diffracted field. Multiplier $f_{1\zeta}\left(q_{zs}\right)$ is effective reflection coefficient of average field. If surface height is distributed under Gaussian law, it has the form

$$f_{1\zeta}(q_{zs}) = \exp\left(-2k^2\sigma^2\cos^2\theta_s\right) = \mathrm{e}^{-\Delta^2/2}, \ldots\ldots(9)$$

where $\Delta = 2k\sigma\cos\theta_s$ is the Rayleigh parameter characterizing degree of surface roughness.

Doppler shift of average field is equal to

$$\Delta\omega = q_{zs}v = 2kv\cos\theta_s\,. \quad (10)$$

Backscattering case follows by setting $\theta_s = 0$ in formulas obtained.

4 SCATTERED FIELD

Scattered field is meant to be diffracted field minus its average value $\Delta\varphi = \varphi - \langle\varphi\rangle$. Let us evaluate temporal correlation function of the scattered field $\psi\left(t-t'\right)=\left\langle\Delta\varphi\left(\mathbf{R},t\right)\cdot\Delta\varphi^*\left(\mathbf{R},t'\right)\right\rangle$. Using (6) and (7) we obtain

$$\psi\left(\tau\right) = \frac{\mathrm{e}^{i\omega_0\tau}}{16\pi^2}\int_{S_0}\frac{\left|F\left(\mathbf{n}_{01}\right)\right|^2 q^4}{R_{01}^2R_1^2q_z^2}J\left(\mathbf{q}\right)\mathrm{e}^{iq_z v\tau}\,d^2\mathbf{r}_\perp\,, \quad (11)$$

where

$$J\left(\mathbf{q}\right) = \int_{-\infty}^{\infty}\left[f_{2\zeta}\left(q_z,-q_z,\boldsymbol{\rho}\right)-\left|f_{1\zeta}\left(q_z\right)\right|^2\right]\mathrm{e}^{i\mathbf{q}_\perp\boldsymbol{\rho}}\,d^2\boldsymbol{\rho}\,, (12)$$

$f_{2\zeta}\left(u,w,\boldsymbol{\rho}\right)=\left\langle\exp\left[iu\zeta\left(\mathbf{r}_\perp\right)+iw\zeta\left(\mathbf{r}_\perp+\boldsymbol{\rho}\right)\right]\right\rangle$ is two-dimensional characteristic function of the rough surface.

Integral $J\left(\mathbf{q}\right)$ is well studied in cited literature. There are known its explicit expressions for irregularities distributed under Gaussian law. In the case of small irregularities in comparison with the wave length $\delta=\left(k\sigma\right)^2\ll 1$ and Gaussian spatial correlation coefficient $K\left(\rho\right)=\exp\left(-\rho^2/l^2\right)$ with the single correlation distance *l* this expression has the form

$$J\left(\mathbf{q}\right)=\pi\sigma^2l^2q_z^2\exp\left(-q_\perp^2l^2/4\right),\quad \delta\ll 1\,. \quad (13)$$

In the opposite case of high irregularities $\delta\gg 1$ and arbitrary spatial single-scale correlation function integral *J* equals to

$$J\left(\mathbf{q}\right)=\frac{4\pi}{\beta q_z^2}\exp\left(-\frac{q_\perp^2}{\beta q_z^2}\right),\quad \delta\gg 1\,, \quad (14)$$

where parameter $\beta=\sigma_s^2=-2\sigma^2K''\left(0\right)=4\sigma^2/l^2$ characterizes mean-square slope of the surface roughness.

In this paper we use the integral J evaluation technique valid for arbitrary values of parameter δ, i.e. for arbitrary height of surface irregularities. It is based on approximation formula

$$\exp\left[-\gamma\left(1-\mathrm{e}^{-x^2}\right)\right]-\mathrm{e}^{-\gamma}\simeq\left(1-\mathrm{e}^{-\gamma}\right)\exp\left(-\frac{\gamma x^2}{1-\mathrm{e}^{-\gamma}}\right), \quad (15)$$

where γ is positive parameter. This approximation was studied in detail in (Vinogradov and Shmelev, 2008).

Let us assume that the surface height is distributed under Gaussian law with Gaussian spatial correlation coefficient $K(\boldsymbol{\rho})=\exp\left(-\rho_x^2/l_x^2-\rho_y^2/l_y^2\right)$ taking into consideration possible non-isotropy of surface irregularities. Difference of characteristic functions in integrand (12) may be represented in accordance with (15) by following expression

$$\begin{aligned} &f_{2\zeta}\left(q_z,-q_z,\boldsymbol{\rho}\right)-\left|f_{1\zeta}\left(q_z\right)\right|^2= \\ &=\exp\left[-q_z^2\sigma^2\left(1-\mathrm{e}^{-\rho_x^2/l_x^2-\rho_y^2/l_y^2}\right)\right]-\exp\left(-q_z^2\sigma^2\right)\simeq \\ &\simeq\left(1-\mathrm{e}^{-q_z^2\sigma^2}\right)\exp\left(-\rho_x^2/l_{\Phi x}^2-\rho_y^2/l_{\Phi y}^2\right), \end{aligned} \quad (16)$$

where effective correlation distances have the values

$$l_{\Phi x}^2=\frac{l_x^2}{q_z^2\sigma^2}\left(1-\mathrm{e}^{-q_z^2\sigma^2}\right),\quad l_{\Phi y}^2=\frac{l_y^2}{q_z^2\sigma^2}\left(1-\mathrm{e}^{-q_z^2\sigma^2}\right), \quad (17)$$

dependent on wave length.

Substitution (16)-(17) into (12) and immediate evaluation of the integral lead to result

$$J(\mathbf{q})\simeq\pi l_{\Phi x}l_{\Phi y}\left(1-\mathrm{e}^{-q_z^2\sigma^2}\right)\exp\left(-\frac{q_x^2 l_{\Phi x}^2+q_y^2 l_{\Phi y}^2}{4}\right). \quad (18)$$

In the case of statistically isotropic surface we have to set $l_x=l_y=l$. This gives

$$\begin{aligned} &J(\mathbf{q})=\pi l_{\Phi}^2\left(1-\mathrm{e}^{-q_z^2\sigma^2}\right)\exp\left(-\frac{q_\perp^2 l_\Phi^2}{4}\right), \\ &l_\Phi^2=\frac{l^2}{q_z^2\sigma^2}\left(1-\mathrm{e}^{-q_z^2\sigma^2}\right). \end{aligned} \quad (19)$$

In limiting cases of small and high irregularities these expressions lead to (13) and (14).

Let us consider now backscattering case, when transmitter and receiver positions coincide, i.e. $\mathbf{R}=\mathbf{R}_0=(0,0,Z)$. Let rough surface be statistically isotropic, and multiplier describing antenna pattern be in the form

$$\left|F\left(\mathbf{n}_{01}\right)\right|^2=\begin{cases}1, & \text{if } r_\perp\le a,\\ 0, & \text{if } r_\perp>a,\end{cases} \quad (20)$$

where a is radius of illuminated area on the mean plane S_0. Transformation to polar coordinates in integrand (11) gives then required expression for temporal correlation function of backscattered field

$$\psi(\tau)=\frac{\mathrm{e}^{i\omega_0\tau}}{16\pi^2}\int_0^{2\pi}d\varphi\int_0^a r_\perp dr_\perp\frac{q^4}{R_{01}^2R_1^2q_z^2}J(\mathbf{q})\mathrm{e}^{iq_z v\tau}, \quad (21)$$

where following relations are valid:

$$\begin{aligned} &\mathbf{R}_{01}=-\mathbf{R}_1=\left(\mathbf{r}_\perp,-Z\right),\quad R_1^2=R_{01}^2=r_\perp^2+Z^2, \\ &\mathbf{q}=2k\mathbf{n}_{01}=2k\frac{\left(\mathbf{r}_\perp,-Z\right)}{\sqrt{r_\perp^2+Z^2}},\quad q^2=4k^2, \\ &l_\Phi^2=\frac{l^2\left(r_\perp^2+Z^2\right)}{4k^2\sigma^2Z^2}\left[1-\exp\left(-\frac{4k^2\sigma^2Z^2}{r_\perp^2+Z^2}\right)\right]. \end{aligned} \quad (22)$$

5 FREQUENCY SPECTRUM

As is known, frequency spectrum may be evaluated by Fourier transformation of temporal correlation function

$$G(\omega)=\int_{-\infty}^{\infty}\psi(\tau)\mathrm{e}^{i\omega\tau}d\tau. \quad (23)$$

Insertion (21), (22) into (23) and application of δ-function integral representation $\delta(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{e}^{i\omega\tau}d\tau$ gives following expression for frequency spectrum of backscattered field

$$G(\omega)=\frac{1}{4}\int_0^a\frac{q^4J(\mathbf{q})}{R_{01}^2R_1^2q_z^2}\delta\left(\omega-\omega_0-q_z v\right)r_\perp dr_\perp. \quad (24)$$

Introducing new integration variable $x=q_z v=-\frac{2kvZ}{\sqrt{Z^2+r_\perp^2}}$ and performing integration of δ-function, we obtain frequency spectrum of backscattered field in explicit form

$$G(\Omega) = \frac{\pi\left(1-e^{-\delta\Omega^2}\right)^2}{2kvZ^2\beta\Omega^3}\exp\left[\frac{\left(\Omega^2-1\right)}{\beta\Omega^2}\left(1-e^{-\delta\Omega^2}\right)\right]. \quad (25)$$

Dimensionless frequency $\Omega = \frac{\omega-\omega_0}{2kv} = \frac{\Delta\omega}{2kv}$ varies within $\cos\theta \le \Omega \le 1$, where angle 2θ is the beam width of transmitter, so that $\tan\theta = a/Z$. Beyond this interval $G(\Omega) \equiv 0$. Physically this means that frequency spectrum includes all possible Doppler shifts of scattered field – from maximum $\Delta\omega_{max} = 2kv$ in vertical direction till minimum $\Delta\omega_{min} = 2kv\cos\theta$ in direction of illuminated area border.

To avoid dependence on inessential parameters let us consider spectrum normalized to its value at Ω=1, i.e. $G_0(\Omega) = G(\Omega)/G(1)$:

$$G_0(\Omega) = \frac{\left(1-e^{-\delta\Omega^2}\right)^2}{\left(1-e^{-\delta}\right)^2\Omega^3}\exp\left[\frac{\left(\Omega^2-1\right)}{\beta\Omega^2}\left(1-e^{-\delta\Omega^2}\right)\right]. \quad (26)$$

Analysis of this expression shows that there exist two forms of frequency spectrum. The first one is represented by monotonic curve, depending on correlation distance of the rough surface. The second form includes one maximum, which position and amplitude are related with irregularities mean square slope. Typical examples of these spectrum forms are shown on Figures 2 and 3.

Regions on the plane (δ,β) corresponding to one or another form of spectrum differ in sign of derivative $G_0'(1)$. Region, where $G_0'(1) > 0$, corresponds to the first form and region, where $G_0'(1) < 0$, – to the second form. The curve corresponding to $G_0'(1) = 0$ separates these two regions. Simple calculations using (26) lead to equation of this curve plotted on Figure 4:

$$\beta = \frac{2\left(1-e^{-\delta}\right)^2}{3-\left(3+4\delta\right)e^{-\delta}}. \quad (27)$$

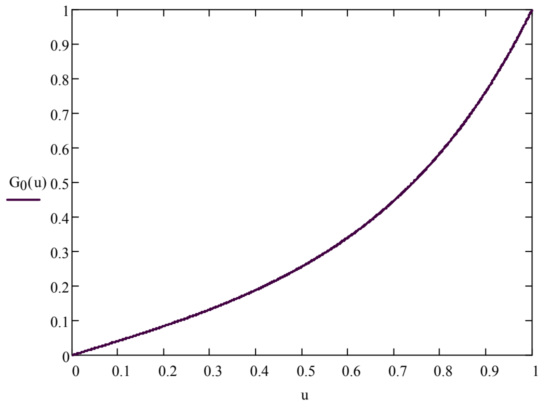

Figure 2: The first form of frequency spectrum for parameter values δ=0.1, β=0.1.

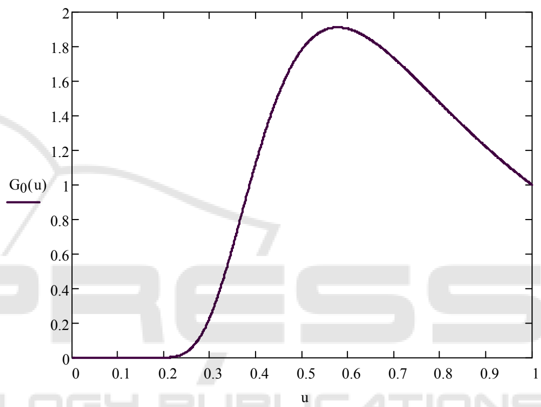

Figure 3: The second form of frequency spectrum for parameter values δ=100, β=2.

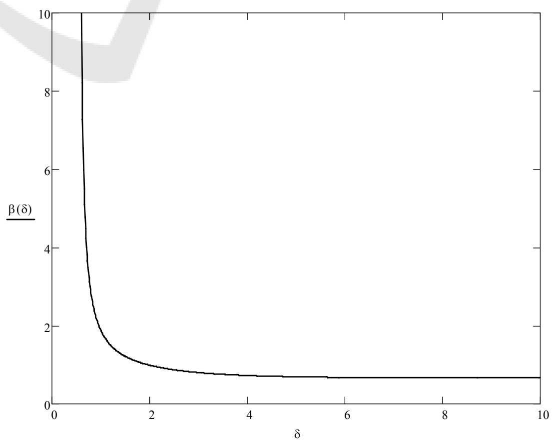

Figure 4: The curve separating regions with the first (under the curve) and the second (above the curve) forms of spectrum.

The first spectrum form results in the case when irregularities are rather small $(\delta < 1)$ or gentle $(\beta < 0.7)$. High and sharp irregularities lead to the second form of spectrum.

In the case of extreme low roughness $(\delta \ll 1)$ expression (26) is simplified to

$$G_0(\Omega) \simeq \Omega \cdot \exp\left[-\alpha\left(1-\Omega^2\right)\right], \quad \delta \ll 1, \qquad (28)$$

where $\alpha = \delta/\beta = (kl)^2$ is mean square correlation distance in the scale of wave length. Differentiation (28) gives relation

$$\alpha = \frac{G_0'(1)-1}{2}, \qquad (29)$$

which may be used for experimental estimation of parameter α.

In the opposite case of very high roughness $(\delta \gg 1)$ expression (26) takes on the form

$$G_0(\Omega) = \frac{1}{\Omega^3} \exp\left[-\frac{\left(1-\Omega^2\right)}{\beta\Omega^2}\right], \quad \delta \gg 1. \qquad (30)$$

For sharp irregularities $(\beta > 0.67)$ it describes the second spectrum form having maximum at $\Omega_m = \sqrt{2/(3\beta)}$. Thus position of maximum carries information on mean square slope of the rough surface β. Additional information on this parameter contains height of this maximum

$$G_0(\Omega_m) = \left(\frac{3\beta}{2}\right)^{3/2} \cdot \exp\left(-\frac{3\beta-2}{2\beta}\right). \qquad (31)$$

This function is plotted on Figure 5.

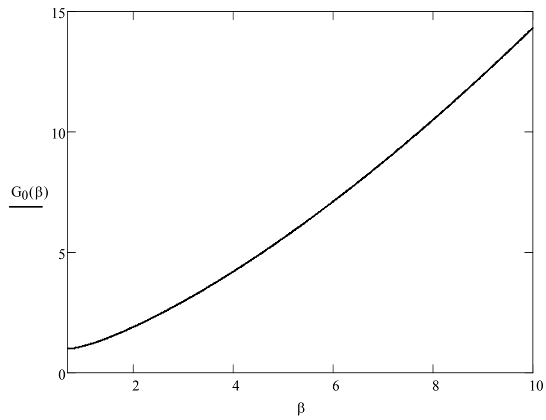

Figure 5: View of the function (31).

6 CONCLUSIONS

Proposed evaluation the frequency spectrum of the wave backscattered from rough surface in explicit form and for arbitrary roughness height lets us establish detailed relations between spectrum parameters and statistical characteristics of the surface. Results obtained may be useful for further development of rough surfaces remote sensing technique.

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