Keywords: Neural Networks, Large Training Sets, SOM-RBF Mixed Model, Ensemble of Neural Networks, Environmental Applications.

Abstract: The paper presents a technique to partition and sort data in a large training set for building models of environmental function approximation using RBF networks. This process allows us to make very accurate approximations of the functions in a time fraction related to the RBF networks classic training process. Furthermore, this technique avoids problems of buffer overflow in the training algorithm execution. The results obtained proved similar accuracy to those obtained with a classical model in a time substantially less, opening, on the other hand, the way to the parallelization process using GPUs technology.

1 INTRODUCTION

The concept of subdividing a complex problem into simpler ones whose solutions are the basis for constructing an overall solution is a much-studied field in computing. The concept of modular networks was first formalized by (Jacobs, 1991) as neuronal architecture in which different processing modules, with different inputs can be distinguished, which use different mechanisms than the final output, which, in turn is a function of the outputs of the various modules. The concept of modularity is inherent to the nervous system of living beings, where specialized neuronal structures make interconnections in order to realize superior capabilities (de Felipe, 1997). If we restrict ourselves to the field of connectionist computing, many authors have applied calculus models based on the composition of neuronal classifiers as a means of problem-solving. Thus, in a study by (Arriaza et al., 2003), a system for classification and detection of clouds in satellite images was developed in which the classification is processed by means of a cascade of interconnected networks. Perhaps, the most accepted model in this context would be the utilization of specialized classifiers in one domain of the problem, whose outputs are studied and treated by a subsequent function that delivers the final output.

These structures, known as Neural Network Ensembles have been described in the literature as a finite set of neural networks trained to perform a common task, together with a process that combines their outputs to obtain a general solution. This technique, combined with various techniques for recombining outputs, generates systems with a greater capacity for generalization than the simple neural network models (Rao, 2005).

Radial basis function (RBF) networks are very useful for solving problems where knowledge is scarce, fitting non-adjustable functions using statistical procedures and, in some cases, of conflicting knowledge. RBFs were introduced in the literature by (Broomhead, 1988) but it was (Poggio, 1990) who later offered the technique that allows a RBF the possibility of generalizing the solution of a problem.

2 RADIAL BASIS FUNCTION NETWORKS

RBF networks make approximations of functions that are scarcely known, that cannot be fitted using statistical procedures and, in some cases, are contradictory. RBFs were first described by (Broomhead, 1988), whilst (Poggio, 1990) offered the technique by which an RBF could be applied to provide a generalized solution to a problem. A RBF is a net-
work consisting of two levels: one level is constructed from a set of kernel functions, called the radial base functions, while the second, called the integration layer integrates the output of these functions in a linear fashion.

RBFs networks have certain functional features that make them attractive for solving problems, such as:

- Pattern recognition
- Generalization of future events based on past actions
- Non-linear regressions.

### 2.1 Training of a RBF Net

The scientific literature shows a number of RBF networks training techniques.

We consider, however, these techniques are divided into two blocks:

- Algorithms for exact alignment of the training set. In this case the number of kernel functions of the network is equal to the cardinal of the training set.
- No exact training algorithms. In these algorithms, the number of kernel functions is below the cardinal of the training set.

For the present case, the process of training a network using RBFs accurate approximation of all the elements of the training set is the following:

In a set of $m$ elements that belongs to a training set $E_{mn} = \{e_1, e_2, ..., e_m\}$ (with $n$ characteristics per sample), and where $W = \{w_1, w_2, ..., w_m\}$ are the weights of the neurons that comprise the network, the expression on which training of the RBF is based is:

$$ rbf(\vec{x}) = \sum_{i=0}^{m} w_i \cdot \varphi_i(||\vec{e}_i - \vec{x}||) $$

where $\varphi$ is the radial base function (Gaussian).

If we express this as a matrix we have (Haykin, 1994):

$$ \begin{pmatrix} \varphi_1(\vec{e}_1) & \cdots & \varphi_1(\vec{e}_m) \\ \vdots & \ddots & \vdots \\ \varphi_m(\vec{e}_1) & \cdots & \varphi_m(\vec{e}_m) \end{pmatrix} \times \begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} $$

Thus, to obtain the unknown $\hat{W}$ (which is what the training stage consists of), the expression would be:

$$ \begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix} = \begin{pmatrix} \varphi_1(\vec{e}_1) & \cdots & \varphi_1(\vec{e}_m) \\ \vdots & \ddots & \vdots \\ \varphi_m(\vec{e}_1) & \cdots & \varphi_m(\vec{e}_m) \end{pmatrix}^{-1} \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} $$

### 2.2 Layout, Typeface, Font Sizes and Numbering

The time required for the training process means that it is very costly to train a RBF. If we analyse the final expression of the training process, we observe that to train a RBF we have two large operations (Brassard, 1995):

- Finding the inverse of a $m \times m$ matrix: order of complexity $O(n^3)$
- Multiplying the two matrices: order of complexity $O(n^3)$

Although the order of complexity is polynomial in some applications where the data set is very large, the general algorithm is inapplicable, because there are problems of buffer overflow and excessive CPU time.

The use of non exact algorithms to solve the problem at the expense of CPU time increase and/or decrease the precision of approximation.

### 3 MIXED OPERATING MODEL (SOM-RBF)

The proposed operational model, once trained, makes use, on one hand, of a SOM reference vectors to determine the "position" within the approximate curve of the input vector, and furthermore, a set RBFs networks formed, trained, each in the context of the curve to approximate.

The proposed architecture is defined by a pair $(\{RBF_i\}, \{C_i\})$, where $\{RBF_i\}$ is a set of RBFs applied, one by one, to a subset of the input space, and $\{C_i\}$ are the reference vectors of a Self-Organizing Map (SOM). The vectors $C_i$ act as activators of the RBFs that have to be used for a given input. Schematically, the model has the structure given in Figure 1.

X is an input vector belonging to the input space, SOM is a Self-Organizing Map that generates an output $S_i$ for a given input, and which operates as an activation signal for the RBF, which is responsible for generating the output. As the scheme shows, the input X is also applied to each of the RBFs in this structure, since it is they that really generate the output.

### 3.1 Training the RBF-SOM Mixed Operating Model

The new algorithm proposed in this paper consists of dividing the training set into $N$ subgroups and, in
Figure 1: Structure of the RBF-SOM mixed Operating Model. The signal $s_i$ enables or disables for each input vector, a unique network of RBFs.

this way, creating $N$ RBFs that approximate the curve of the problem. To illustrate the process, we start with a real situation: the process of approximation of an environmental variable, of which there are 16,000 samples obtained with 20 variables that define them.

Nevertheless, this division of elements from the training set must be ordered rather than randomized. The ordering process uses the SOMs described in (Kohonen, 1990) as tools for separating and ordering the training vectors.

Algebraically, the ordering process is as follows:

1. Make a preliminary random partition of the training set into $N$ groups:

   $\begin{bmatrix}
e_{i,1} & \cdots & e_{i,20} \\
   e_{x,1} & \cdots & e_{x,20} \\
   \vdots & \ddots & \vdots \\
   e_{16000,1} & \cdots & e_{16000,20}
\end{bmatrix}$

   (4a)

   The reason for this (using a first partition at random and not based on any statistical procedure) is due to our efforts to reduce the computational requirements.

2. Training a SOM for each of the training subsets. (In our case, we used a hexagonal topology of $p \times k$ rows and columns)

3. From step 2, we are interested in the $p \times k$ centroids of each of the SOMs.

   $\begin{bmatrix}
e_{i,1} & \cdots & e_{i,20} \\
   e_{x,1} & \cdots & e_{x,20} \\
   \vdots & \ddots & \vdots \\
   e_{16000,1} & \cdots & e_{16000,20}
\end{bmatrix}$

   (4b)

   After this step, we will have $N \times p \times k$ centroids: $C = \{c_1, c_2, \ldots, c_{N \times p \times k}\}$

4. Apply a new SOM to the set of centroids $C$, in order to generate $z$ new centroids:

   $\begin{bmatrix}
c_{i,1} & \cdots & c_{i,20} \\
   \vdots & \ddots & \vdots \\
   c_{N \times p \times k,1} & \cdots & c_{N \times p \times k,20}
\end{bmatrix} \rightarrow z$ centroids

   (4c)

5. Use the $z$ centroids to divide the elements of the training set into $z$ training subsets.
   a. Measure the Euclidean distance of the training element to each centroid.
   b. Calculate which is the minimum distance.
   c. Assign the training element to the closest training subset (as measured to its reference centroid).

6. Train the $z$ RBFs using the exact algorithm procedure.

In this way, the product of the training is not a single trained RBF, but $z$ RBFs. Faced with this change, when we undertake the classification, we have to:

1. Check which RBF it is most suitable to use for classification (using the Euclidean distances to the centroids mentioned).
2. Classify using the RBF obtained.

3.2 Theoretical Study of the Improvement

The improvement arises mainly from the fact that the most costly operations of training the RBF, namely the inverse of the matrix and the multiplication of the matrices together, do not have to be executed so many times, since we are working with $z$ RBFs and not just a single RBF.

In this way, we can express the order of complexity of both algorithms, after the proposed improvement, as:

$$O\left(z \cdot \frac{m^3}{z}ight).$$

(5)

where $z$ is the number of centroids and $n$ the number of elements in the matrices that must be operated on.

Simply put, it can be demonstrated that a cubic algorithm takes substantially less time to solve after the improved method is applied. Let us see the theoretical demonstration.

$$n^3 \geq z \left(\frac{m}{z}\right)^3 \Rightarrow n^3 \geq z \left(\frac{m}{z}\right)^3 \Rightarrow n^3 \geq \frac{m^3}{z} \Rightarrow 1 \geq \frac{m}{z} \Rightarrow z \geq 1$$

(6)

Then, since $z \geq 1$, we have $z^2 \geq 1$, and, therefore, we conclude that the efficiency of the algorithm of
order $x^3$ is greater than that of an algorithm of order $n^3$.

This variation significantly delays the process of polynomial growth, reducing the cost of memory and CPU time.

4 CASE STUDY. MODELING ECOCOLOGICAL VARIABLES USING REMOTE SENSING DATA

Within the framework of a study of environmental variables, the objective of the present study has been to substitute a RBF network used as an approximation function for the ecological variables, by a RBF structure that approximates these variables with comparable precision, but with shorter training times.

The study of ecological variables is a fundamental component of the environmental sciences, for environmental impact analysis and for determining ecological partitions in a geographical area. This work is essentially carried out by means of costly field surveys, which require heavy investment and slow down the rate at which results can be obtained. The use of satellite information to obtain the same data comprises an effective tool that is quick and of modest cost.

Nevertheless, there is a need to understand the relationship between the satellite information and the ecological variables. Studies undertaken until now have determined the relationship between information collected by LANDSAT sensors using RBFs, and have identified the geographical zones that share the same values of the ecological variables. There are a good number of studies which correlate LANDSAT information with vegetation data; however, the correlation of this information with ecological variables is more complex (Cruz et al., 2010).

Two restrictions were imposed on the new method of approximation: It should yield a precision similar to that obtained using a single RBF and there should be an exact fit for all cases in the input set.

One of the operating characteristics of these problems is that training sets are very large (order of tens of thousands of vectors). In this case, training sets containing 22000 vectors were used, each with 20 elements, obtained from LANDSAT information, with 16000 used for training of the RBFs and 6000 for calibrating the results.

We experimented using the 25 test sets corresponding to the 25 different ecological variables (out of a total of 45 that were used in the environmental study (Cruz et al., 2010)).

Applying the algorithm our experiment took the following values:

1. The random partition contained 100 groups.
   a) To each group, we applied the SOM, with the following features: Topology: hexagonal, Number of rows: 32, Number of columns: 1.

2. We obtained 32 centroids for each of the 100 SOM, yielding a total of 3,200 centroids.
   a) In the fourth step, we applied the SOM to the 3200 earlier centroids, with the following features: Topology: hexagonal, Number of rows: 5, Number of columns: 5.

3. In this way, we obtained 25 centroids and, therefore, 25 RBFs.

When we analyze the results of the experiment, we can see that, while the average time of execution of the classic training of an RBF is approximately 1 hour 30 minutes for each training, the proposed new method gives better results, Table 1 show the result of these experiments.

Of course, we checked that the improvement in the training phase did not cause a deterioration of the goodness of fit in the subsequent classification.

Table 1: Results of experiments using as application field, the approximation of environmental variables 25 and the training process described above.

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<tr>
<td>Training time</td>
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process, by measuring the classification error against several calibration files (one for each training set). In this way, we measured the percentage accuracy, by comparing the output value against a threshold error of either ±0.01 and ±0.001.

In both cases, the accuracy was similar to results obtained using a single RBF.

Therefore, since the percentage effectiveness using the actual algorithm is basically the same, we have demonstrated the efficiency of the proposed method on reducing training time (Sedgewick, 2002).

\[
\text{Time efficiency} = \frac{\text{New process time}}{\text{Usual process time}} \times 100
\]

The results give a mean efficiency of 0.836182336%; in other words the execution time, on average, is reduced by 99.1638177%, and thus we can conclude that the proposed technique is quite acceptable.

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