Using Domain Knowledge in Association Rules Mining

Case Study

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Abstract: A case study concerning an approach to application of domain knowledge in association rule mining is presented. Association rules are understood as general relations of two general Boolean attributes derived from columns of an analysed data matrix. Interesting items of domain knowledge are expressed in an intuitive form distinct from association rules. Each particular pattern of domain knowledge is mapped onto a set of all association rules which can be considered as its consequences. These sets are used when interpreting results of data mining procedure. Deduction rules concerning association rules are applied.

1 INTRODUCTION

An approach to dealing with domain knowledge in association rules data mining is sketched in (Rauch and Šimůnek, 2011). Association rules of the form \( \varphi \approx \psi \) are used. Here \( \varphi \) and \( \psi \) are general Boolean attributes derived from columns of an analysed data matrix. Symbol \( \approx \) is a 4ft-quantifier, it corresponds to a condition concerning a contingency table of \( \varphi \) and \( \psi \) (Rauch, 2013).

Boolean attributes \( \varphi \) and \( \psi \) are built from basic Boolean attributes of the form \( A(\alpha) \) where \( A \) is an attribute i.e. a column of a data matrix with possible values \( a_1, \ldots, a_t \) and \( \alpha \subseteq \{a_1, \ldots, a_t\} \). Basic Boolean attribute \( A(\alpha) \) is true in row \( o \) of an analysed data matrix if it holds \( A(o) \in \alpha \) i.e. if the value \( A(o) \) of the attribute \( A \) in row \( o \) belongs to the set \( \alpha \).

We use the 4ft-Miner procedure (Rauch, 2013; Rauch and Šimůnek, 2005) which mines for such association rules. The 4ft-Miner procedure is an enhanced implementation of the ASSOC procedure introduced in (Hálek and Havránek, 1978). Its implementation is based on dealing with suitable strings of bits making possible to easy deal with basic Boolean attributes \( A(\alpha) \). We do not use the a-priori algorithm (Agrawal et al., 1996).

We deal with SI-formulas expressing mutual influence of attributes. The expression \( BMI \uparrow \uparrow \text{Diastolic} \) is an example of SI-formula. It concerns attributes \( BMI \) i.e. body mass index and \( \text{Diastolic} \) i.e. diastolic blood pressure. Its meaning is: if \( BMI \) increases then diastolic blood pressure increases as well.

For each SI-formula \( \Omega \) and 4ft-quantifier \( \approx \) a set \( \text{Cons}(\Omega, \approx) \) of association rules \( \varphi \approx \psi \) which can be considered as consequences of \( \Omega \) is defined. This set is then used when interpreting results of the 4ft-Miner procedure. Deduction rules rules of the form \( \varphi \approx \psi \) where both \( \varphi \approx \psi \) and \( \varphi' \approx \psi' \) are association rules play an important role. If the deduction rule \( \varphi \approx \psi \) is correct and the association rule \( \varphi \approx \psi \) is true in a given data matrix \( \mathcal{M} \), then the association rule \( \varphi' \approx \psi' \) is also true in \( \mathcal{M} \). These deduction rules are studied in (Rauch, 2013) in details together with additional features of special logical calculi of association rules.

An application of this approach for SI-formula \( BMI \uparrow \uparrow \text{Diastolic} \) is outlined in (Rauch and Šimůnek, 2011). The goal of the paper is to present this approach for additional SI-formulas in details. The goal of this paper is not to get new medical knowledge, the goal is to present new possibilities of dealing with domain knowledge in association rules data mining. Well known items of domain knowledge together with freely downloadable medical data set are used to achieve this goal.

No similar approach based on domain knowledge and logical calculi of association rules is known to the authors. However, various alternative approaches are published e.g. in (Delgado et al., 2001; Delgado et al., 2011; Brossette et al., 1998; Ordonez et al., 2006; Roddick et al., 2003). Their detailed comparison with the approach presented here is beyond of the scope of this paper and it is left as a further work.

The STULONG medical data set is introduced in section 2 together with related items of domain knowledge. An analytical question concerning this
data set and related items of domain knowledge is presented in section 3. Applications of the 4ft-Miner procedure relevant to this analytical question are described in section 4. Sets of association rules which can be considered as consequences of items of domain knowledge in question are introduced in section 5. These sets are used to interpret results of the 4ft-Miner procedure, see section 6. Results related to an additional analytical question are shortly presented in section 7. Concluding remarks are in section 8.

### 2 STULONG DATA SET

#### 2.1 Data Matrix Entry

We use data set STULONG concerning Longitudinal Study of Atherosclerosis Risk Factors, see http://euromise.vse.cz/challenge2004/. Data set consists of four data matrices, we deal with data matrix Entry only. It concerns 1,417 patients – men that have been examined at the beginning of the study. Each row of data matrix describes one patient. Data matrix has 64 columns corresponding to particular attributes – characteristics of patients. The attributes can be divided into various groups, see e.g. http://euromise.vse.cz/challenge2004/data/entry/.

We use four groups defined for this paper – Social + BMI, Vices, Problems, and Examinations. The groups are introduced in Tab. 1 together with attributes belonging to particular groups. Let us note: names of categories are followed by the frequencies of particular categories. married/1207 means that there are 1,207 married patients in the data matrix Entry.

Frequencies of categories of the attribute BMI are in Fig. 1. Frequencies of categories of the attribute Cholesterol are distributed similarly. There are missing values in the data matrix Entry, thus the sum of frequencies of all particular categories of some attributes can be less 1,417. Names of some categories are abbreviated, we use manager instead of managerial worker, etc., see http://euromise.vse.cz/challenge2004/data/entry/social.html#zodpov. Tric means Skinfold above musculus triceps (mm) and Subsc means skinfold above musculus subcapularis (mm).

![Figure 1: Frequencies of categories of BMI.](image-url)

<table>
<thead>
<tr>
<th>Group of attributes</th>
<th>Attribute</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social + BMI</td>
<td>M_Status</td>
<td>married/1207, divorced/104, single/95, widower/10</td>
</tr>
<tr>
<td></td>
<td>Education</td>
<td>basic/151, apprentice/405, secondary/444, university/397</td>
</tr>
<tr>
<td></td>
<td>Responsibility</td>
<td>manager/286, independent/435, others/636, pensioner/25</td>
</tr>
<tr>
<td>BMI</td>
<td></td>
<td>&lt; 22, 22,…, 32, &gt; 32</td>
</tr>
</tbody>
</table>

#### 2.2 Mutual Influence of Attributes

There are various items of domain knowledge concerning mutual influence of attributes and related to the STULONG data set. We use several of them, they correspond to the following SI-formulas.

SI-formula BMI $\uparrow+$ Hypertension(yes) means: if BMI increases, then the relative frequency of Hypertension(yes) increases as well. Here BMI is a general ordinal attribute, the attribute BMI i.e. a column of data matrix Entry is an example of an instance of the general ordinal attribute BMI. Similarly, Hypertension(yes) is a general Boolean attribute, the attribute Hypertension(yes) is its instance. This approach can be described formally, SI-formulas can be understood as an enhancement of a logical calculus of association rules (Rauch, 2011). We use a less formal approach here.

We assume that BMI $\uparrow+$ Hypertension(yes) is an approved and generally accepted medical knowledge. There are additional and similar SI-formulas BMI $\uparrow+$ Infarction(yes), BMI $\uparrow+$ Diabetes(yes), BMI $\uparrow+$ Hyperlipidemia(yes), and a simi-
lar set of SI-formulas can be created for each of ordinal attributes Education, Beer, Vine, Liquors, Smoking, Tríc, Subsc, and Cholesterol. However, to show possibilities of formulation and answering analytical questions based on SI-formulas, we assume that BMI ↑ Hyper tension(yes) is the only approved relevant medical knowledge here.

3 ANALYTICAL QUESTIONS

We have groups of attributes Social + BMI, Vices, Problems, and Examinations, see Tab. 1. In addition, we have SI-formulas concerning ordinal attributes from the groups Social + BMI and Vices and Boolean attributes from the group Problems. In addition, we assume that SI-formula BMI ↑ Hyper tension(yes) is the only approved and generally accepted medical knowledge concerning data matrix Entry. Thus, it is natural to ask the following question Q1:

Q1: In the data matrix Entry, are there any interesting relations between attributes of the groups Social + BMI and Vices on the one side and the attributes of the group Problems on the other side which cannot be considered as consequences of BMI ↑ Hyper tension(yes)?

We dealt with association rules and thus the question Q1 can be formulated as the question QAR1:

QAR1: In the Entry data matrix, are there any interesting true association rules ϕ ≈ ψ such that ϕ is a Boolean characteristics of the groups Social + BMI and Vices, ψ is a Boolean characteristics of the group Problems, ≈ is a suitable 4ft-quantifier, and these rules ϕ ≈ ψ cannot be considered as consequences of BMI ↑ Hyper tension(yes)?

We use the procedure 4ft-Miner to solve QAR1 in the following four steps. Below, we write Hpt(yes) instead of Hypertension(yes) (also BMI ↑ Hpt(yes) etc.):

1. We define a set Φ of interesting Boolean characteristics of the groups Social + BMI and Vices and a set Ψ of interesting Boolean characteristics of the group Problems. An example is in section 4.2.

2. We find a set True(Entry, Φ, ≈, Ψ) of all rules ϕ ≈ ψ which are true in Entry, ϕ ∈ Φ, ψ ∈ Ψ, and ≈ is a suitable 4ft-quantifier. Several variants of definitions of Φ, Ψ, and ≈ can be used. Examples are in section 4.3.

3. We define a set Cons(BMI ↑ Hpt(yes), ≈) of all rules ϕ ≈ ψ which can be considered as consequences of BMI ↑ Hpt(yes), see section 5.

4. We investigate the set TC defined as

   True(Entry, Φ, ≈, Ψ) ∩ Cons(BMI ↑ Hpt(yes), ≈).

   Depending on results of investigation we can get the following conclusions.

   If TC contains only rules from the set Cons(BMI ↑ Hpt(yes), ≈) then we conclude: All rules from True(Entry, Φ, ≈, Ψ) can be considered as consequences of BMI ↑ Hpt(yes); there is no interesting rule ϕ ≈ ψ indicating an additional item of knowledge.

   If TC contains no (or only several) rules from the set Cons(BMI ↑ Hpt(yes), ≈) then we conclude: There are no (or only too few) rules in True(Entry, Φ, ≈, Ψ) which can be considered as consequences of BMI ↑ Hpt(yes). Assuming that the definitions of sets Φ and Ψ are reasonable we can further conclude that this is suspicious and start investigation of circumstances of acquisition of the Entry data matrix.

   If TC contains rules which are not from the set Cons(BMI ↑ Hpt(yes), ≈) then we start interpretation of these rules. One of ways how to do this is to look if there are rules which can be considered as consequences of additional SI-formulas which correspond to items of knowledge which are not approved and generally accepted. This way is outlined below.

   We use additional SI-formulas BMI ↑ ATR(yes) where ATR is one of the attributes Infarction(yes), Diabetes(yes), and Hyperlidaemia(yes). For each such SI-formula Ω we continue this way: We define a set Cons(Ω, ≈) of rules ϕ ≈ ψ which can be considered as consequences of Ω. Then we investigate the set TC defined as True(Entry, Φ, ≈, Ψ) ∩ Cons(Ω, ≈) and further investigate the set TC defined as True(Entry, Φ, ≈, Ψ) ∩ Cons(Ω, ≈). Remember that we assume that Ω is not and approved and generally accepted item of medical knowledge.

   Depending on results of investigation we can get the following conclusions.

   If TC does not contain rules from the set Cons(Ω, ≈) then we conclude: There is no indication of Ω in the Entry data matrix.

   If TC contains some rules from the set Cons(Ω, ≈) then we conclude: There are indications of Ω in the Entry data matrix. Then we can start suitable activity (e.g. confirmation analysis starting with getting additional observations) to decide if Ω is a generally acceptable item of knowledge.

   Examples of such conclusions related to the question QAR1 are given in section 6. Examples of conclusions related to an additional task are in section 7.
4 APPLYING 4ft-Miner

4.1 Association Rules

Association rule is an expression \( \varphi \approx \psi \) where \( \varphi \) and \( \psi \) are Boolean attributes. It means that \( \varphi \) and \( \psi \) are associated in a way given by the symbol \( \approx \). \( \varphi \) is called antecedent, \( \psi \) is called succedent. Symbol \( \approx \) is a 4ft-quantifier. It corresponds to a condition concerning a four-fold contingency table of \( \varphi \) and \( \psi \). The association rule \( \varphi \approx \psi \) concerns analysed data matrix. Data matrix \( M \) is an example of such data matrix.

The rule \( \varphi \approx \psi \) is true in data matrix \( M \) if the condition corresponding to 4ft-quantifier \( \approx \) is satisfied in a four-fold contingency table of \( \varphi \) and \( \psi \) in \( M \). The four-fold contingency table \( 4ft(\varphi, \psi, M) \) of \( \varphi \) and \( \psi \) in data matrix \( M \) is a quadruple \( (a, b, c, d) \) where \( a \) is the number of rows of \( M \) satisfying both \( \varphi \) and \( \psi \), \( b \) is the number of rows of \( M \) satisfying \( \varphi \) and not satisfying \( \psi \), \( c \) is the number of rows satisfying \( \psi \) but not \( \varphi \), and \( d \) is the number of rows satisfying neither \( \varphi \) nor \( \psi \), see Table 2. There are various 4ft-quantifiers, see e.g. (Rauch, 2013).

Table 2: 4ft table \( 4ft(\varphi, \psi, M) \) of \( \varphi \) and \( \psi \) in \( M \).

<table>
<thead>
<tr>
<th>( M )</th>
<th>( \varphi )</th>
<th>( \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi \approx \psi )</td>
<td>( a )</td>
<td>( b )</td>
</tr>
<tr>
<td>( \sim \varphi \approx \psi )</td>
<td>( c )</td>
<td>( d )</td>
</tr>
</tbody>
</table>

We use here the 4ft-quantifier \( \Rightarrow_{p,B} \) of founded implication. It is defined for \( 0 < p < 1 \) and \( B > 0 \) by the condition \( \frac{a}{p} \geq p \frac{a + B}{B} \). The association rule \( \varphi \Rightarrow_{p,B} \psi \) means that at least 100\( p \) per cent of rows of \( M \) satisfying \( \varphi \) satisfy also \( \psi \) and that there are at least \( B \) rows of \( M \) satisfying both \( \varphi \) and \( \psi \).

We also use 4ft-quantifier \( \sim_{p,B} \) of above average dependence defined for \( 0 < q, B > 0 \) by the condition \( \frac{a}{p} \geq (1 + q) \frac{a + B}{B} \). This says that among rows satisfying \( \varphi \) at least 100\( q \) per cent more rows satisfying \( \psi \) than among all rows and that there are at least \( B \) rows satisfying both \( \varphi \) and \( \psi \).

4.2 Set of Relevant Rules

We solve analytical question QAR1: In the Entry data matrix, are there any interesting true association rules \( \varphi \approx \psi \) such that \( \varphi \) is a Boolean characteristics of the groups Social + BMI and Vices, \( \psi \) is a Boolean characteristic of the group Problems, \( \approx \) is a suitable 4ft-quantifier, and \( \varphi \approx \psi \) cannot be considered as a consequence of BMI \( \uparrow \) Hypertension(\( \text{yes} \))?

We use the 4ft-Miner procedure in four steps introduced in section 3. In the first step we define a set \( \Phi \) of relevant antecedents i.e. Boolean characteristics of the groups Social + BMI and Vices and a set \( \Psi \) of relevant succedents i.e. Boolean characteristics of the group Problems. The set \( \Phi \) is defined as a set of all conjunctions \( \varphi_B \wedge \psi_V \) where \( \varphi_B \in B(\text{Social} + \text{BMI}) \) and \( \psi_V \in B(\text{Vices}) \). Here \( B(\text{Social} + \text{BMI}) \) means a set of all Boolean attributes derived from the attributes of the group Social + BMI we consider relevant to our analytical question, similarly for \( B(\text{Vices}) \). The set \( \Psi \) can be similarly denoted as \( B(\text{Problems}) \).

The set \( B(\text{Social} + \text{BMI}) \) is defined in the frame ANTECEDENT in Fig 2 in row Social + BMI Conj, 1-4 and in four consecutive rows. Each \( \varphi_B \) is a conjunction of 1-4 basic Boolean attributes derived from particular attributes of the group Social + BMI.

Set of basic Boolean attributes derived from attribute \( \text{M Status} \) is defined by the row \( \text{M Status}(\text{subset}) \), \( 1-4 \), \( B \), pos. It means that all basic Boolean attributes \( \text{M Status}(\alpha) \) where \( \alpha \) is a subset of all categories of attribute \( \text{M Status} \) containing just one category are generated: \( \text{M Status}(\text{married}), \text{M Status}(\text{divorced}), \text{M Status}(\text{single}), \text{M Status}(\text{widower}) \). Set of basic Boolean attributes derived from attribute Responsibility is defined similarly.

Set of basic Boolean attributes derived from attribute Education is defined by the row Education(basic school), 1-2, \( B \), pos. Thus, all 7 basic Boolean attributes Education(\( \alpha \)) where \( \alpha \) is a set of 1 or 2 consecutive categories (i.e. interval of categories) are generated. Education(basic school), Education(basic school, apprentice school) are examples.

Set of all Boolean attributes derived from the attribute BMI is defined by the row BMI(int), 1-4 \( B \), pos. It means that all Boolean attributes BMI(\( \alpha \)) where \( \alpha \) is a set of 1-4 consecutive categories (i.e.
31
¬ 70

Various optimization techniques are implemented. For example, the set 
\( B(Problems) \) of relevant succedents is defined in second step in section 3. We used 
\( \Phi \) and \( \Psi \) defined in the previous section.

The 4ft-quantifier \( \Rightarrow_{0.9,30} \) defined by the condition \( \frac{\phi}{\Psi} = \alpha \geq 30 \) (see section 4.1) was used first. The task was solved in 4 minutes (PC with 2GB RAM and Intel T7200 processor at 2 GHz) and 3.35 x 10^6 association rules were generated and tested. Various optimization techniques are implemented in the 4ft-Miner procedure, (Rauch and Šimůnek, 2005). Thus, not all 10^6 rules are truly generated and tested. However, no true rules was found, i.e. \( True(Entry, \Phi, \Rightarrow_{0.9,30}, \Psi) = \Theta \).

Thus we used 4ft-quantifier \( \Rightarrow_{0.9,30} \) instead of \( \Rightarrow_{0.9,30} \). This setting led to 24 true rules. In other words, the set \( True(Entry, \Phi, \Rightarrow_{0.9,30}, \Psi) \) contains 24 rules. The strongest rule is the rule (we denote this rule \( R_1 \)) \( BMI(\geq 31) \land \Gamma \Rightarrow_{0.9,30} Hpt(\text{yes}) \) with 4ft-table 4 ft (\( BMI(\geq 31) \land \Gamma \), Hpt(\text{yes})). See Figure 4. We write Hpt(\text{yes}) instead of Hpt(\text{yes}).

<table>
<thead>
<tr>
<th>Entry</th>
<th>Hpt(\text{yes})</th>
<th>Hpt(\text{no})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( BMI(\geq 31) \land \Gamma )</td>
<td>32</td>
<td>70</td>
</tr>
<tr>
<td>( -(BMI(\geq 30) \land \Gamma) )</td>
<td>187</td>
<td>1118</td>
</tr>
</tbody>
</table>

Figure 3: 4 ft (\( BMI(\geq 30) \land \Gamma \), Hpt(\text{yes}), Entry).

5 CONSEQUENCES OF ITEMS OF DOMAIN KNOWLEDGE

Runs of the 4ft-Miner procedure resulted into two non-empty sets – \( True(Entry, \Phi, \Rightarrow_{0.9,30}, \Psi) \) containing 24 rules and \( True(Entry, \Phi, \Rightarrow_{0.9,30}, \Psi) \) containing 3 754 rules. We define a set \( Cons(BMI \uparrow \uparrow Hpt(\text{yes})) \) of all rules \( \phi \approx \Psi \) which can be considered as consequences of \( BMI \uparrow \uparrow Hpt(\text{yes}) \) for both \( \Rightarrow_{0.9,30} \) and \( \Rightarrow_{0.9,30} \). See section 3.

The set \( Cons(BMI \uparrow \uparrow Hpt(\text{yes}), \Rightarrow_{0.9,30}) \) of all rules \( \phi \Rightarrow_{\Psi,B} \Psi \) which can be considered as consequences of \( BMI \uparrow \uparrow Hpt(\text{yes}) \) is defined in four steps:

1. A set \( AC(BMI \uparrow \uparrow Hpt(\text{yes}), \Rightarrow_{0.9,30}) \) of atomic consequences of \( BMI \uparrow \uparrow Hpt(\text{yes}) \) for \( \Rightarrow_{0.9,30} \) is defined as a set of simple rules \( BMI(\delta) \Rightarrow_{\Psi,B} Hpt(\text{yes}) \).
such that $p' \geq 0.3$, $B' \geq 30$ and $BMI(\delta)$ is a basic Boolean attribute expressing (informally speaking) that $BMI$ is high enough.

2) A set $AgC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30})$ of agreed consequences of $BMI \quad \uparrow \quad Hpt(\text{yes})$ for $\Rightarrow_{0.3,30}$ is defined. A rule $\rho \Rightarrow_{p,B} \sigma$ belongs to the set $AgC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30})$ if the following conditions are satisfied:

- $\rho \Rightarrow_{p,B} \sigma \not\subset AC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30})$
- there is no $\kappa \Rightarrow_{p',B'} \lambda$ belonging to $AC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30})$ such that $\rho \Rightarrow_{p,B} \sigma$ logically follows from $\kappa \Rightarrow_{p',B'} \lambda$.

3) A set $LtC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30})$ of logical consequences of $BMI \quad \uparrow \quad Hpt(\text{yes})$ for $\Rightarrow_{0.3,30}$ is defined. A rule $\varphi \Rightarrow_{p,B} \psi$ belongs to the set $LtC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30})$ if the following conditions are satisfied:

- $\varphi \Rightarrow_{p,B} \psi \not\subset (AC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30}) \cup AgC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30}))$
- there is $\tau \Rightarrow_{p',B'} \omega$ belonging to the set $AC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30}) \cup AgC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30})$ such that $\varphi \Rightarrow_{p,B} \psi$ logically follows from $\tau \Rightarrow_{p',B'} \omega$.

4) We define $Cons(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30}) = AC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30}) \cup AgC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30}) \cup LtC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30})$.

We give more details to particular steps 1) – 3).

1) The set $AC(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.3,30})$ is defined as a set of all rules $BMI(\delta) \Rightarrow_{p,B} Hpt(\text{yes})$ where $p' \geq 0.3$, $B' \geq 30$ and $BMI(\delta)$ is a basic Boolean attribute expressing that $BMI$ is high enough.

Procedure 4ft-Miner has a simple tool facilitating a definition of a set of basic Boolean attributes $BMI(\delta)$ expressing that $BMI$ is high enough, see Fig. 5. This is a contingency table of the attributes $BMI$ and Hypertension. Based on this table we can decide in cooperation with a domain expert that we will consider all basic Boolean attributes $BMI(\delta)$ such that $\delta \in \{28, 29, 30, 31, 32, > 32\}$. It is crucial that this definition is stored and further used.

2) An association rule $\varphi' \Rightarrow_{p,B} \psi'$ logically follows from a rule $\varphi \Rightarrow_{p,B} \psi$ if the following is true: If $\varphi \Rightarrow_{p,B} \psi$ is true in a given data matrix $M$ then $\varphi' \Rightarrow_{p,B} \psi'$ is also true in $M$. It is easy to prove that the association rule $BMI(\delta) \land \chi \Rightarrow_{0.3,30} Hpt(\text{yes})$ does not logically follow from $BMI(\delta) \Rightarrow_{0.3,30} Hpt(\text{yes})$.

The core of the proof is the fact that if there are at least $B$ rows of a data matrix $M$ satisfying $BMI(\delta) \land Hpt(\text{yes})$ then there still can be no row of $M$ satisfying $BMI(\delta) \land \chi \land Hpt(\text{yes})$.

However, in some cases it can be reasonable from the point of view of a domain expert to agree that $BMI(\delta) \land \chi \Rightarrow_{0.3,30} Hpt(\text{yes})$ is a consequence of $BMI(\delta) \Rightarrow_{0.3,30} Hpt(\text{yes})$. Then we call the rule $BMI(\delta) \land \chi \Rightarrow_{0.3,30} Hpt(\text{yes})$ an agreed consequence of the rule $BMI(\delta) \Rightarrow_{0.3,30} Hpt(\text{yes})$.

The rule $BMI(\delta) \land \chi \Rightarrow_{0.3,30} Hpt(\text{yes})$ is an example of an agreed consequence of $BMI \quad \uparrow \quad Hpt(\text{yes})$ because the truthfulness of Boolean attribute $Beam$ (not) has no influence on the relation of $BMI$ and $Hpt(\text{yes})$. The same is true for all basic Boolean attributes we can derive from the attributes of the groups Social + BMI and Vicos (except $BMI$).

3) A criterion making possible to decide if an association rule $\varphi' \Rightarrow_{p,B} \psi'$ logically follows from a rule $\varphi \Rightarrow_{p,B} \psi$ is proved in (Rauch, 2013). It is, e.g., easy to prove that $BMI(\delta) \Rightarrow_{0.3,30} Hpt(\text{yes}) \lor \text{Infarction}(\text{yes})$ logically follows from $BMI(\delta) \Rightarrow_{0.3,30} Hpt(\text{yes})$.

Let us emphasize that the same approach can be used to get the set $Cons(BMI \quad \uparrow \quad Hpt(\text{yes}), \Rightarrow_{0.1,30})$. In addition, it is important that the 4ft-Miner procedure has tools making possible to apply this approach to get a set $Cons(\Omega, \approx)$ of all rules $\varphi \approx \psi$ which can be considered as consequences of an item $\Omega$ of domain knowledge for a 4ft-quantifier $\approx$. This is possible for various types of items $\Omega$ of domain knowledge and many important 4ft-quantifiers $\approx$. Results on deduction rules of the form $\varphi \Rightarrow_{p,B} \psi$ (Rauch, 2013) are used.

6 INTERPRETING RESULTS

We have two non-empty sets of rules – True(Entry, $\Phi, \Rightarrow_{0.3,30}, \Psi$) containing 24 rules...
and $\text{True}(\text{Entry}, \Phi, \sim_{0.130} \Psi)$ containing 3754 rules, see section 4.3. We interpret these sets according the point 4 introduced in section 3. The 4ft-Miner procedure makes possible to easy compare these sets with sets $\text{Cons}(\text{BMI} \uparrow Hpt(\text{yes}), \sim_{0.330})$ and $\text{Cons}(\text{BMI} \uparrow Hpt(\text{yes}), \sim_{0.130})$ respectively. Results of comparison are summarized in Table 3. Here $\mathcal{C}(\Omega, \approx)$ abbreviates $\text{Cons}(\text{BMI} \uparrow Hpt(\text{yes}), \approx)$.

<table>
<thead>
<tr>
<th>$\Rightarrow_{0.330}$</th>
<th>$\Rightarrow_{0.130}$</th>
<th>$\sim_{0.130}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1407</td>
<td>3754</td>
</tr>
<tr>
<td>0</td>
<td>1 347</td>
<td></td>
</tr>
<tr>
<td>Σ</td>
<td>1 347</td>
<td></td>
</tr>
</tbody>
</table>

The first row in the body of Table 3 means that all rules in $\text{True}(\text{Entry}, \Phi, \Rightarrow_{0.330} \Psi)$ can be considered as consequences of $\text{BMI} \uparrow Hpt$. Thus we can conclude that there are no interesting rules $\Phi \Rightarrow_{0.330} \Psi$ that are true in data matrix $\text{Entry}$ such that $\Phi$ is a Boolean characteristics of the groups $\text{Social} \leftrightarrow \text{BMI}$ and $\text{Vices}, \Psi$ is a Boolean characteristics of the group $\text{Problems}$, and $\Phi \Rightarrow_{0.330} \Psi$ cannot be considered as a consequence of $\text{BMI} \uparrow Hpt(\text{yes})$.

The second row in the body of Table 3 means that there are 1 347 interesting rules $\Phi \sim_{0.130} \Psi$ that are true in data matrix $\text{Entry}$ such that $\Phi$ is a Boolean characteristics of the groups $\text{Social} \leftrightarrow \text{BMI}$ and $\text{Vices}, \Psi$ is a Boolean characteristics of the group $\text{Problems}$, and $\Phi \sim_{0.130} \Psi$ cannot be considered as a consequence of $\text{BMI} \uparrow Hpt(\text{yes})$.

The 4ft-Miner procedure has additional tools enabling the following conclusions:

(i) There are only 4 rule concerning Infarction(\text{yes}) and 1 rule concerning Hyperlipidemia(\text{yes}) among the mentioned 1 347 rules, all remaining 1 342 rules concern $Hpt(\text{yes})$. This is because frequencies of attributes Infarction(\text{yes}), Hyperlipidemia(\text{yes}), and Diabetes(\text{yes}) are very low, see Table 1.

(ii) Among remaining 1 342 rules concerning $Hpt(\text{yes})$, there are 411 rules which can be written as $\text{BMI}(\delta) \land \tau \sim_{0.130} Hpt(\text{yes})$ where $\delta \subset \{24, 25, 26\}$. This means that these rules are not consequences of $\text{BMI} \uparrow Hpt(\text{yes})$ in the sense of the definition in section 5. They can be seen as candidates of exceptions from the item $\text{BMI} \uparrow Hpt(\text{yes})$. However, a deeper discussion on this topic is out of the scope of this paper.

These conclusions are made under the assumptions that the sets of interesting rules are defined in the way described in section 4.2 and that the sets $\text{Cons}(\text{BMI} \uparrow Hpt(\text{yes}), \Rightarrow_{0.330})$ and $\text{Cons}(\text{BMI} \uparrow Hpt(\text{yes}), \sim_{0.130})$ defined in section 5 are used. Let us emphasize that all these definitions can be modified in various ways and thus various variants of these conclusions can be formulated.

Let us also emphasize that rules with minimal confidence 0.3 used above are not too much suitable to express interesting relations. We use them here only to show principles of the presented approach. In addition, there are lot of rules with stronger quantifiers than $\sim_{0.130}$ in the set $\text{True}(\text{Entry}, \Phi, \sim_{0.130} \Psi)$ containing $3754$ rules. The strongest one is a rule with quantifier $\sim_{0.0231}$ (i.e. lift 2.02).

### 7 ADDITIONAL RESULTS

The analytical questions $\text{QAR}_1$ solved above can be modified to the question $\text{QAR}_2$ (note that attribute $\text{BMI}$ is not involved):

$\text{QAR}_2$: In the $\text{Entry}$ data matrix, are there any interesting true association rules $\Phi \Rightarrow \Psi$ such that $\Phi$ is a Boolean characteristics of the groups $\text{Social} \leftrightarrow \text{BMI}$ and $\text{Vices}, \Psi$ is a Boolean characteristics of the group $\text{Problems}$, $\Phi \Rightarrow \Psi$ is a suitable 4ft-quantifier?

To solve this question, we used a run of the 4ft-Miner procedure with the definition of relevant antecedents according to Fig. 6, the definition of relevant succedents according to Fig. 2, and the quantifier $\sim_{0.330}$ (i.e. lift 1.5). This resulted to 71 true rules, all of them concern $Hpt(\text{yes})$.

![Figure 6: Definition of relevant antecedents for $\text{QAR}_2$.](image)

If we define a set of atomic consequences of $\text{Subsc} \uparrow Hpt$ as a set of all rules $\text{Subsc}(<\delta) \sim_{0.530} Hpt$ where $\delta \subset \{(20; 25), (25; 30), (30; 55)\}$ (see “$>$ 20” in the row $\text{Subsc}$ of Table 4) and if we use an analogous approach as described in Section 5, we can conclude that there are 28 rules which can be considered as consequences of $\text{Subsc} \uparrow Hpt$ and 1 rule which cannot be considered as a consequence of $\text{Subsc} \uparrow Hpt$. In addition there are 29 rules concerning the attribute $\text{Subsc}$, see row $\text{Subsc}$ of Table 4. There are analogous information for attributes $\text{Tric}$ and $\text{Cholesterol}$ which is abbreviated as $\text{Chol}$, see Table 4.

We can conclude that there are lot of rules which can be considered as consequences of the SI-formula $\text{Subsc} \uparrow Hpt$ and only one rule which cannot be con-
considered as a consequence of this SI-formula. Thus it seems reasonable to try to confirm the hypothesis Subsc ↑+ Hpt. In addition, we can conclude that there are no strong indications of Tric ↑+ Hpt and Chol ↑+ Hpt.

However, there are various possibilities of modifications of parameters of the set of relevant antecedents, modifications of the quantifier $\sim_0^{30}$ and modifications of the definitions of sets of consequences of Tric ↑+ Hpt and Chol ↑+ Hpt. This can lead to revision of the introduced conclusions.

### 8 CONCLUSIONS

We have presented a new way of dealing with domain knowledge in association rules data mining. This is based on mapping items of domain knowledge to sets of association rules which can be considered as their consequences. It was shown that there is both necessary theory based on logic of association rules and the 4ft-Miner procedure realizing relevant operations with data, items of knowledge and rules. This makes possible to formulate interesting analytical questions and answer them in an efficient way. There is a very fine way to define sets of relevant association rules. These association rules, when true in data, can be considered as the smallest possible indications of more complex dependences among related attributes.

However, there is still a challenge concerning sensitivity of the presented approach to various parameters. There is also a challenge of combining of the 4ft-Miner procedure for mining the presented syntactically rich association rules with additional data mining procedures, namely with procedures of the LISp-Miner system dealing with various contingency tables (Hájek et al., 2010). These topics are subjects of further work.

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### REFERENCES


