A Two-step Empirical-analytical Optimization Scheme
A Simulation Metamodeling Approach

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Abstract: This paper presents a two-step optimization scheme developed to find the optimal operational settings of operational systems seeking to optimize their operations using multiple performance measures. The study focuses on two conflicting performance measures, the Throughput Rate (TR) and the Mean Flow Time (MFT). First an empirical approach is used to uncover the near optimal values of the performance measures using an experimental design procedure. Second, an analytical procedure is deployed to find the exact optima using values the near optima found in the first step as target. The analytical procedure uses a non-linear regression meta-model derived from simulation outputs and compromises the two conflicting targets while minimizing the loss incurred to the overall system. This loss is expressed in the form of a multivariate version of the Taguchi quadratic loss function. Although the framework as presented in this paper is derived by analyzing a manufacturing system through discrete-event simulation, the procedure however, can successfully be applied to any processing system in various industries including food production, financial institutions, warehouse industry, and healthcare.

1 INTRODUCTION

The choice of performance measures in a processing system depends highly on management policy and decision-making. Multiple objective measures are needed to describe the dynamic nature of a production system. A single performance measure is not enough to capture and characterize the overall performance of a system. Also, optimizing a system with respect to one single objective only may lead to sacrificing other objective(s) of interest. For example the objective of minimizing in-process inventory might be in conflict with that of maximizing a production rate. Literature on the design and operation of flexible manufacturing systems has shown that most of the past research studies have used only a single performance measure in their objective functions (Blogun et al., 1999). From this point of view, the multi-objectives approach has recently been of interest in a wide range of design and control problems for manufacturing systems, such as machine selection, choice of the manufacturing or processing system configuration architecture, control of automated storage and retrieval systems, and overall scheduling scheme.

The selection of the most appropriate setting of input factors in order to attain the required process objective/target (mean) is of major interest in a variety of production environments. The problem is referred to as the “optimal setting parameters” because it is concerned with selecting the best setting of parameters for an optimal operation of the system. It worth it to mention that the generic term of system is used in this study to designate a process-oriented infrastructure including a warehouse, a manufacturing system, or a operating theater in a hospital. Selecting the optimal setting is critically important since it affects not only performance measures, operations and/or production costs but also the loss incurred to the system in the event of a performance deviation from the company-identified target values. On the other hand, these operational targets need to be frequently reviewed as a result of the unpredictable variations in the shop floor conditions and the fluctuating nature of the market place.

Clearly, there is a true need and a real opportunity to apply a combined scheduling methodology to dynamic and stochastic scheduling problems with the objective of reducing the overall
production cost. This paper analyzes a hypothetical flexible manufacturing system using simulation and proposes a unique and robust scheme in designing, modeling and optimizing systems in a very effective way. The reader is referred to other author’s publications (Bardhan and Tshibangu, 2003), (Tshibangu, 2005), (Tshibangu, 2006) for a detailed description of the hypothetical manufacturing system considered in this study. The system is modeled with a total number of 9 workstations including a receiving and a shipping stations. These 9 stations process are served by a fleet of AGVs while processing fifteen part types, each with a different processing time. The optimization procedure as developed in this paper is carried out at two levels. First an empirical approach is used to uncover near-to-optimal values of the individual performance criteria of interest. These values are subsequently used as targets in the second and more analytical level of the optimization procedure during which a multi-criteria optimization technique eventually uncovers the true optimal setting of the system parameters. Specifically, the analytical optimization is applied to a regression model equation (meta-model) derived from simulation output results. The approach used in this study takes advantage of a robust experimental design methodology to render the system immune to noise. The purpose is to present a pragmatic approach that may enhance the overall performance of process-oriented systems including manufacturing systems, warehouse, airport traffic and hospitals.

2 RESEARCH METHODOLOGY

The various phases of the robust design methodology as applied in this paper is the same as proposed in the literature (Montgomery 2012), (Taguchi, 1987) except that in this study, after completing the simulation experiments and collecting all pertinent data the following additional steps are taken in order to accommodate the subsequent optimization procedures as proposed in this research:

1. Calculate the mean and the variance with respect to noise factors $\sigma^2_{\text{wrnfi}}(i)$ for each treatment $i$ (row of the inner array) and for each performance measure of interest; this variance measures the variation in the performance criterion when there is a change in noise factors.

2. Compute and use $\log \sigma^2_{\text{wrnfi}}(i)$ of each performance measure to improve statistical properties of analysis.

3. Apply the normal probability plotting technique to the calculated mean and the log $\sigma^2_{\text{wrnfi}}$ of each control factor setting to determine the significance of the main factors and their interaction effects on each performance measure of interest.

4. Develop and implement the four-step optimization procedure to predict the factors and their associated settings that will simultaneously minimize $\sigma^2_{\text{wrnfi}}$ and optimize the mean of the performance measures. Adjust and fine-tune the settings to the most appropriate economical levels.

5. Perform a second analytical optimization procedure using a Bi-variate Quadratic Loss Function (BQLF) inspired from Taguchi Methodology.

6. Run confirmatory simulation experiments.

7. Make the conclusions on the multi-criteria optimization procedure.

2.1 The Robust Design Formulation

Implementing the robust design formulation requires the following steps:

- Define the response or dependent variables (performance measures of interest), the independent variables (including the controllable factors and the uncontrollable factors or source of noise).

- Plan the experiment by specifying how the control parameter settings will be varied and how the effect of noise will be measured.

- Carry out the experiment and use the results to predict improved control parameter settings (e.g., by using the optimization procedure developed in this study).

- Run a confirmation experiment to check the validity of the prediction.

This study takes advantage of a robust design configuration inspired by the Taguchi robust design methodology. However, because of the high amount of criticism against Taguchi’s experimental design tools such as orthogonal arrays, linear graphs, and signal-to-noise ratios, this study avoids the use of Taguchi’s statistical methods but rather uses an empirical technique developed by the author.

The paper develops and proposes an optimization scheme by studying an AGV-served FMS and evaluating its overall performance using the mean flow time ($MFT$) and the throughput rate ($TR$). The study considers as controllable variables 5 design parameters, designated by $X_i$ ($i=1\ldots5$), namely: i) the number of AGVs ($X_1$), ii) the speed of AGV ($X_2$), iii)
the queue discipline \((X_3)\), iv) the AGV dispatching rule \((X_4)\), v) and the buffer size \((X_5)\). These variables have a direct impact on the performance of machines and material handling (AGVs) as they are considered in most literature not only as the most expensive (some even as the most sensitive) components of the overall system and also as potential sources of operational disturbances. The natural values assigned to these design variables are displayed in Table 1. In this study, the controllable parameters \(X_i\) through \(X_f\) to set and tested at two setting levels (min and max).

The principal sources of noise tested in this study (and also considered as the most commonly investigated and documented in the reported literature (Montgomery, 2013) are: i) the arrival rate between parts (or orders), \((X_6)\), the mean time between failures of the machines \((X_7)\) and the associated mean time to repair \((X_8)\). These factors are also tested at two levels in combination with each control factor \((X_i)\) through \(X_f\) at each setting level.

Table 1: Natural values and setting of control factors.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Control Factor</th>
<th>Low Level (-1)</th>
<th>High Level (+1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_1)</td>
<td>Number of AGVs</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>(X_2)</td>
<td>Speed of AGV</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>(X_3)</td>
<td>Queue Discipline</td>
<td>FIFO</td>
<td>SPT</td>
</tr>
<tr>
<td>(X_4)</td>
<td>AGV Dispatching Rule</td>
<td>FCFS</td>
<td>SDT</td>
</tr>
<tr>
<td>(X_5)</td>
<td>Buffer Size</td>
<td>8</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 2 depicts settings and natural values for noise factors as assigned and simulated in the experiments. For both controllable and noise factors, the coded levels are \((-1)\) and \((+1)\) for the low and high level, respectively.

### 2.2 Planning the Experiment

Planning the experiment is a two-part step that involves deciding on how to vary the parameter settings and how to measure the effect of noise (Kacker and Shoemaker, 1986). Using a full factorial experimental design with the 5 controllable factors \(X_1, X_2, X_3, X_4, X_5, X_6\) and \(X_7\) set at two levels in combination with three noise factors \(X_6, X_7, X_8\), varied at two settings would require \(2^5 \times 2^3 = 256\) simulation runs.

Two-level, full factorial or fractional factorial designs are the most common structures used in constructing experimental design plans for system design variables. Montgomery (2013) recommends appropriate fractional factorial designs of resolution \(IV\) or \(V\) in the design of robust manufacturing systems. In this study a two-level fractional factorial design of resolution \(V\), denoted \(2^{5-1}\) has been used. This design requires only 16 runs. Across the full set of noise factors, the implemented robust design leads to a total of 16 x 8 = 128 simulation runs (instead of 256 as required by a full factorial design). The study also decides to use a robust design of resolution \(V\) in order to allow an estimation of both main factors and two-way interactions effects, as they are necessary and very crucial for the first step of the proposed optimization scheme, and referred to as the empirical step.

A standard statistical experimental design, also known as a data collection plan is normally advocated and recommended when conducting simulation experiments. The data collection plan used in this study was inspired from Genichi Taguchi’s strategy for improving product and process quality in manufacturing (Taguchi, 1986). It has been first used and proposed by Wild and Pignatiello (Wild et al., 1991). Their proposed design strategy includes simultaneous changing of input parameter values. Therefore, the uncertainty (noise) associated with not knowing the effect of shifts in actual parameter values such as shifts in mean inter-arrival times, mean service times, or the effect of not knowing the accuracy of the estimates of the input parameter values, is introduced into the experimental design itself. Tshibangu, 2003, 2005 provides detailed information about this specific data collection plan. This plan has been also used in this study to run the simulation experiments and effectively collect the statistics thereof.

Table 2: Natural values and setting of noise factors.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Noise Factor</th>
<th>Low Level ((-1))</th>
<th>High Level ((+1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_6)</td>
<td>Inter-arrival</td>
<td>EXPO(15)</td>
<td>EXPO(5)</td>
</tr>
<tr>
<td>(X_7)</td>
<td>MTBF</td>
<td>EXPO(300)</td>
<td>EXPO(800)</td>
</tr>
<tr>
<td>(X_8)</td>
<td>MTTR</td>
<td>EXPO(50)</td>
<td>EXPO(90)</td>
</tr>
</tbody>
</table>

### 3 EMPIRICAL OPTIMIZATION

Because flexible manufacturing systems and any other process-oriented systems are subject to various uncontrollable factors that may adversely affect their performance, a robust design of such systems is crucial and unavoidable. The author has developed a four-step optimization procedure to be used simultaneously with the robust design as first step of the optimization scheme as proposed in this study:

Let \(\bar{y}_j\) represent the average performance measure across all the set of noise factors.
combination, averaged across all the simulation replications for each treatment combination (or design configuration) \(i\). Let \(\log \sigma_{\text{wrtnf}(i)}^2\) be the associated logarithm of the variance with respect to noise for that particular treatment \(i\). Kacker and Shoemaker, 1986 recommend to use the logarithm of the variance in order to improve statistical properties of the analysis, and to employ the “effects” values and/or graphs in association with normal probability plots and or ANOVA procedures to identify and partition the following three categories of control factor vectors:

Assuming that we have partitioned three categories of control vectors as non-empty sets \(X^T\), containing the factors that have a significant effect on the variances, \(X^T\), containing factors significant on the means (and their interactions), and \(X^T\) as the set of the factors that affect neither the mean nor the variance, respectively, then a four-step empirical optimization procedure may be implemented as follows:

1. **Step 1**
   Identify the vector \(X^T\) and adjust the controllable factors members of this set to their values that minimize \(\sigma_{\text{wrtnf}}^2\) of the performance measure \(y\).

2. **Step 2**
   Identify vector \((X_{\text{tr}}^T)\) of factors having a significant effect on the mean \(\overline{y}\) and set the controllable factors members of this set to their level values that optimize the mean \(\overline{y}\) of the objective performance \(y\). Also, identify \((X_{\text{tr}}^T)\) vector of factors having a significant effect on mean \(\overline{y}\) and on the variance \(\sigma_{\text{wrtnf}}^2\). Simultaneously set the factor members of this set to their level values that optimize the mean \(\overline{y}\) if this setting does not act in opposition with the minimization of the variance. Otherwise, find a compromise between minimizing the variance and optimizing the mean as suggested in **Step 4** where the final setting is to be decided.

3. **Step 3**
   Identify the vector \(X^T\) and set the control factors members of this set to the values of their interaction with members of vector \(X^T\) that minimize the variance or \(\log \sigma_{\text{wrtnf}}^2\) or the values of their interaction with members of \(X^T\) that optimize the mean \(\overline{y}\). Otherwise, set the factors at their economical settings.

4. **Step 4**
   Conduct a small follow-up experiment to find trade-off between members of \((X_{\text{tr}}^T)\) containing factors with effects on variance and mean acting in opposition and or the overall economical settings. A suggestion from this study is that in finding the overall economical setting, the step involves only those factors that have the greatest effect on either the variance \(\sigma_{\text{wrtnf}}^2\) or the mean \(\overline{y}\).

Using the above-developed procedure with related plots and tables, and applying it to the data as derived from the experiments for the two performance measures, i.e., Mean Flow Time (MFT) and Throughput Rate (TR) performance measures, the following coded results are obtained: \(MFT = 0.3666\) units time/part and \(TR = 3000\) parts/month (100 parts/day). These values will be considered as the optimal target values to be achieved in the second level of the optimization procedure (multi-criteria optimization).

### 3.1 Simulation Meta-modeling

Tshibangu (2005) redefines the purpose of metamodeling as the method by which to measure the sensitivity of the simulation response to various factors that may be either decision (controllable) variables or environmental (non-controllable) variables (Kleijnen, 1977).

After completing the robust design process, the 128 simulation experiments were carried out as initially recommended in the experiment plan and the main statistics describing the system were collected following the proposed data collection plan. These values were subsequently fed into a non-linear regression meta-model to derive the estimate-equations \(\hat{y}_{\text{TR}}\) and \(\hat{y}_{\text{MFT}}\) for the throughput rate (TR) and the mean flow time (MFT), respectively. Meta-models are usually constructed by running a special RSM (Response Surface Methodology) experiment and fitting a regression equation that relates the responses to the independent variables or factors.

### 3.2 Determination of Variances, Main and Interaction Effects

A well-planned experiment makes simple the analysis subsequently needed to predict the improved (optimal) parameter settings. In this study, for each of the simulated design configurations \(i\), eight measurements (over the set of noise factor combinations) were taken for each performance measure of interest and averaged across the replications to obtain \(\overline{y}\) for each \(i^{th}\) row of the inner array. Sixteen design configurations and five center-points (for a total of 21) designs were simulated over a set of eight noise factor combinations, leading to
21x8 =128 runs. The results of these various simulation experiments, too large to be displayed in this paper, but available upon request, were subsequently averaged up across the three replications.

This research intends to minimize the variances of the performance measures with respect to the noise factors for each run. The reported variances across the text, denoted \( \sigma^2_{\text{wrtnf}} \) is calculated as follows:

\[
\sigma^2_{\text{wrtnf}} = \frac{1}{f-1} \sum_{f=1}^{f} (y_{ij} - \bar{y})^2, \quad f = 1, 2, ... , f, \quad (1)
\]

where \( y_{ij} \) is the observed value of a given performance measure for a particular design configuration \( i \) and a noise factor configuration \( j \); \( \bar{y} \) is the average value of a given performance criterion considering that particular design configuration \( i \). In this study, \( f = 8 \) (eight noise combinations).

Table 3: Effects of Control Factors MFT Variance.

<table>
<thead>
<tr>
<th>Control Factors</th>
<th>Effect on MFT log ( \sigma^2_{\text{wrtnf}} ) at Level (1)</th>
<th>Effect on MFT log ( \sigma^2_{\text{wrtnf}} ) at Level (-1)</th>
<th>Absolute Value Difference between High and Low levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
<td>1.6159</td>
<td>1.657502</td>
<td>0.04155</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>1.6081</td>
<td>1.558286</td>
<td>0.04982</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>1.4921</td>
<td>1.781325</td>
<td>0.28920</td>
</tr>
<tr>
<td>( X_4 )</td>
<td>1.6338</td>
<td>1.639566</td>
<td>0.00568</td>
</tr>
<tr>
<td>( X_5 )</td>
<td>1.6032</td>
<td>1.670230</td>
<td>0.06701</td>
</tr>
</tbody>
</table>

Table 4: Effects of Control Factors on MFT Mean.

<table>
<thead>
<tr>
<th>Control Factors</th>
<th>Effect on MFT Average at Level (1)</th>
<th>Effect on MFT Average at Level (-1)</th>
<th>Absolute Value Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
<td>8.238767</td>
<td>25.97234</td>
<td>17.73358</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>12.62954</td>
<td>20.89605</td>
<td>8.26650</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>13.86047</td>
<td>20.55636</td>
<td>6.69016</td>
</tr>
<tr>
<td>( X_4 )</td>
<td>16.97108</td>
<td>17.24002</td>
<td>0.26893</td>
</tr>
<tr>
<td>( X_5 )</td>
<td>17.61093</td>
<td>16.60017</td>
<td>1.01076</td>
</tr>
</tbody>
</table>

The objective is to make the variances of the responses (performance measures) as small as possible while the means are brought to their optimum settings, which would consist of a minimum for the MFT and a maximum for the TR. The study then computes the values of \( \bar{y} \) and \( \log \sigma_{\text{wrtnf}} \) at each design configuration. Subsequently, the effects of each control factor on the overall mean and the variance (or \( \log \sigma_{\text{wrtnf}} \)) are calculated by using the normal probability data plotting technique (Box et al., 1978). Tables 3 and 4 display the effects on the MFT variance and mean, respectively.

As it can be seen, these effects on the mean and the variance are also partitioned into high level and low level effects. The same procedure is applied to the throughput rate TR and the results, not displayed in this paper, are available upon request. The process is conducted for all the control factors. Then each controllable factor is tested at two levels, the magnitude of its effect on variability is measured by the difference between the average values of \( \log \sigma_{\text{wrtnf}} \) at those settings. The computed effects at high and low levels will be used in identifying those controllable factor levels (settings) that have the largest effect on \( \log \sigma_{\text{wrtnf}} \).

The same procedure is also applied to the mean values to determine the effects of the control parameters. Note that a visual summary of the magnitude of each control factor’s effect can also be used for analysis of various effects. From analysis of the results in Table 3 for example, it can be seen on one hand for instance (in bold) that the parameter \( X_2 \) (queue discipline) has the most significant effect on the MFT variability. These results agree with previous findings (Egbelu and Tanchoco 1984); (Sabuncuoglu, 1989); (Bardhan and Tshibangu 2003). On the other hand, the effect at high level is compared to the effect at low level, and the better setting of each control parameter is the one that gives the smaller average value of \( \log \sigma_{\text{wrtnf}} \) Table 4 results indicate that factor \( X_1 \) (the number of AGVs), when set at its high level, has the most significant effect on the mean value of the MFT (see results in bold). Once identified, these factors will be set at the settings (levels) that minimize \( \log \sigma_{\text{wrtnf}}, \) i.e., \( X_1 \) and \( X_2 \) at high settings. Proceeding the same way for the TR similar results are obtained and the settings implemented.

Now that the first empirical optimization step has revealed the near optimal settings of the system, it becomes appropriate to move to the second step of the optimization procedure, referred to in this study as the analytical phase of the proposed optimization scheme. For one to perform the analytical optimization step, a mathematical model of the system is required. This paper proposes to feed the simulation results into a non-linear regression meta-model to derive the mathematical model. Applying the meta-modeling technique to the flexible manufacturing system under study in this research yield the following equations for the estimates of two performance measures of interest, \( \hat{y}_{TR} \) and \( \hat{y}_{MTF} \).
\[
\hat{\gamma}_T = 90.7617 + 20.6726x_1 + 2.5357x_2 \\
+2.6977x_3 + 0.5617x_4 - 4.712x_1^2 - 9.042x_2^2 \\
-8.732x_1^2 - 7.923x_1x_2 - 7.458x_1x_4 \\
+3.5513x_1x_2 - 0.5315x_2x_4 - 0.3304x_4^2 
\] (2)

\[
\hat{\gamma}_{MF_T} = 4.6503 - 8.8668x_1 - 4.4760x_1^2 \\
-3.2451x_1 - 0.1345x_1 + 0.5054x_1 \\
+14.1731x_1^2 + 1.5309x_2^2 - 1.3399x_2^2 \\
-0.4519x_1^2 - 1.4569x_2^2 + 5.3816x_1x_2 \\
-0.7952x_1x_1 - 0.0335x_1x_4 - 0.504x_1x_5 \\
-0.1457x_1x_1 - 0.3251x_1x_4 + 0.4863x_1x_5 \\
-0.7655x_1x_5 
\] (3)

where \(x_1, x_2, x_3, x_4, x_5\) are the coded units for the operating variables \(X_1, X_2, X_3, X_4,\) and \(X_5,\) respectively.

4 ANALYTICAL APPROACH

The Taguchi’s loss function discussed in the literature (Montgomery, 2013) for a single objective criterion can be extended to the case of multiple quality characteristics or objective performances, and then referred to as a “multivariate quality loss function”. The author (Tshibangu, 2006) shows how the traditional and simple QLF can be extended to a multivariate QLF.

Let \(y_j\) and \(T_j\) be the performance measures of interest \((j=1, 2, ..., Q)\), where \(Q\) is the total number of performance measures, and the target for objective performance \(y_j\), respectively, and be denoted by \(y = (y_1, y_2, ..., y_Q)^T\) and \(T = (T_1, T_2, ..., T_Q)^T\) under the assumption that \(L(y)\) is a twice-differentiable function in the neighborhood of \(T\).

Assuming that each objective performance has a mean \(\mu(y_j)\) and a variance \(\sigma^2(y_j)\), then, after some mathematical developments and manipulations (Tshibangu, 2005), (Ribeiro and Elsayed, 1995) the expected value of the quadratic loss function for a bivariate QLF can be derived and written as follows:

\[
E[L(y_1, y_2)] = \sum_{i=1}^{O} \theta_i \left[ (\mu_i - T_i)^2 + \sigma_i^2 \right] \\
+ \theta_2 \left[ (y_1 - T_1)(y_2 - T_2) \right] 
\] (4)

The first term of the second hand side of Eqn. (4) is known as a weighted sum of mean squares, while the second term is called the gradient term. It is important to note that three aspects are of interest in formulating robust design systems:

(i) deviation from targets; (ii) robustness to noise; (iii) robustness to process parameters fluctuations. A weighted sum of mean squares is appropriate to capture (i) and (ii), while gradient information is necessary to capture (iii). This research is particularly interested in deviation from target and robustness to noise. Therefore, only the first term of Equation (9) is needed.

The next step consists of applying the derived QLF to the FMS meta-models Eqns. (2) and (3) obtained from simulation outputs. In order to determine the optimal input parameters, an objective function is developed from Eqn. (4), following a framework adopted by Ribeiro and Elsayed (1995).

Because of the robust design configuration adopted during the experiments, it can be assumed that the variability of the system due to fluctuations of the operating parameters is negligible, then, for a given treatment, the loss incurred to a system as the result of a departure of the system performance \(\hat{y}_j\) from the target \(T_j\) can be estimated as:

\[
L(i) = \sum_{j=1}^{Q} w_j \left[ (\hat{y}_j - T_j)^2 + \hat{\sigma}_{yj}^2 \right] 
\] (5)

where \(L(i)\) is the loss at treatment \(i; w_j\) is a weight to take into account in order to consider the relative importance of a individual performance measure \(y_j\) \((j=1, 2, ..., Q)\), \(\hat{y}_j, \hat{\sigma}_{yj}\) are respectively the predicted (estimate) mean and standard deviation of the performance measures of interest \(y_j\), and \(T_j\) is the target for the system performance measure \(y_j\). \(L(i)\) is the objective function to be minimized. In this particular form, the objective function has two terms. The first term of the objective function, \((\hat{y}_j - T_j)^2\), accounts for deviations from target values. The second term, \(\hat{\sigma}_{yj}^2\) accounts for the source of variability due to non-controllable factors (noise).

5 RESULTS

For the twenty-one treatment combinations simulated in this study, the resulting normalized values are displayed in Table 5 showing the values...
of \( y_j \) and \( \log \sigma_y^2 \) at each design configuration for each of the two primary performance criteria of interest in this study. Note that only the throughput rate (TR) seeks a maximization. The mean flow time (MFT) and the variances of both TR and MFT need be minimized. Therefore, the normalization procedure of these values will consist of maximizing the inverse. Further analysis of output results indicates that design configurations labeled #3, 7, 16, 20, and 21 are the most cost effective as they yield the least cost. This finding suggests that operating the studied system under any of these design settings would be far more economically attractive than operating the same system under other design settings even when they are also identified as the most robust designs. For example, the difference between the most cost-effective design (configuration design #3) and the most expensive one (design #13) represents approximately 56 monetary units in normalized values. This may represent a significant amount of money if the value of the monetary loss coefficient factor is important. Using for example $10.00 value for the loss coefficient will lead to a difference of $123.00 in expected losses between design #3 and design #13 representing \((1- \frac{38}{161})\)*100 = 76% of savings when operating under design #3 setting parameters. Design #13 has been used for the comparison because it is among the strongest design candidates in terms of robustness of the system (i.e., insensitivity to noise factors). This example shows that significant savings (e.g. 76%) can be generated when switching ample from design #13 to design #3.

6 CONCLUSIONS

This study first uses an empirical optimization procedure to avoid the controversial Taguchi statistical tools. Then a metamodel is derived from the simulation outputs. The study also derives a multivariate quadratic loss function (QLF) from the traditional Taguchi loss function in order to capture the loss incurred to the overall system when attempting to optimize a set of two objective performances (throughput rate TR and mean flow time MFT). Hence, the QLF is referred in this study to as a bivariate quadratic loss function (BQLF).

Next (second level of the optimization scheme), the BQLF is analytically applied to the metamodel derived from the simulation outputs to fine-tune the optimization process em with respect to the two objective performances. From the results obtained in step 1 of the optimization scheme as developed in this paper, optimum/target values of 100 parts/day and 3,666 units time/part (in coded data) have been fixed for the TR, and MFT, respectively. This two-level optimization procedure lead to a solution that yield a minimum cost to be incurred to the overall system as a penalty for missing the objective targets. The values of 98 parts/day (-2% from target) and 0.3459 units time/part (+5.6% from target) are obtained as optima, for TR and MFT, respectively. These maximum outputs will be obtained under a overal system configuration that is considered to be the most robust and economical, leading to the following settings in natural values: Number of AGVs \((X_1)\): 6; Speed of AGV \((X_1)\): 150 feet/min; Queue discipline (machine rule) \((X_2)\): SPT; AGV dispatching rule \((X_2)\): STD; Buffer size: \((X_3)\): 4.

Although, conceptually validated on a flexible manufacturing system, the above-developed and proposed optimization scheme can be easily extended to other process-oriented industries including banks, warehouse, ticketing lines at airports, restaurants, healthcare facilities, pharmaceutical industries, and others.

<table>
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<tr>
<th>Design Configuration</th>
<th>TR</th>
<th>TR log (Var)</th>
<th>MFT</th>
<th>MFT log (Var)</th>
<th>Pred. Loss</th>
<th>e.g. K= $10.00</th>
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<td>0.075</td>
<td>3.792</td>
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REFERENCES


