Evaluation of Efficiency of Brazilian Airlines using the MCDEA-TRIMAP Model

Eliane Ribeiro Pereira1, Maria Cecília de Carvalho Chaves1 and João Carlos C. B. Soares de Mello2

1Accounting Department, Universidade Federal do Rio de Janeiro, Av. Pasteur, 250, Urca, RJ, Brazil
2Production Engineering Department, Universidade Federal Fluminense, R. Passo da Pátria, 156, Niterói, RJ, Brazil

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Abstract: The deregulation of the domestic airline industry brought a significant increase in competition between airlines, pushing them to continuously update their strategies. This article reviews the operating performance of these companies, from the contrast in the results of the DEA classic model and MCDEA-TRIMAP efficiency indicators. The use of such methodology allows the increase of discriminatory power of the productive units evaluated and a better evaluation of them. The application of the DEA model to the data from the Brazilian Airline industry for the year 2008 indicated three companies as efficient, while the use of MCDEA TRIMAP index allowed the identification of the most efficient company in 2008. The index provided better discrimination of the production units in the study.

1 INTRODUCTION

The continued development of commercial aviation has led to it being one of the main means of transportation for both passengers and cargo, when it comes to medium and long distances. Governments and analysts often highlight the airline industry as playing an essential and strategic role for the country (Pasin and Lacerda, 2003). In Brazil, which has large territorial dimensions, the use of commercial aviation is highlighted as an element of integration between its farthest points. Its unquestionable importance can be measured by it being the largest market in Latin America, accounting for 3% of national GDP (Araújo et al., 2006).

The Brazilian air transportation sector has undergone profound changes over time, mainly due to the deregulation of the sector, (Soares de Mello et al., 2003), which brought increased competition into the air transportation industry. The entry into the market of GOL Airlines, the first LCC (low cost carrier) airline in Latin America in 2001, further intensified competition (Evangelho et al., 2005). To ensure competitiveness, the airlines have been forced to aim for better use of their resources. New transformations occurred due to both physical infrastructure problems and operational issues, brought to light by the occurrence of serious accidents in the years 2006 and 2007. The domestic airline industry was then placed in check and has slowly been recovering from this period, marked by insecurity and mistrust.

This article aims to investigate the performance of Brazilian airlines within the scenario described. The analysis carried out took into account the staff involved in the operation of the means of transportation, and the use of the fleet to transport cargo and passengers within national territory and internationally, as done by Silveira et al. (2008). Other studies regarding operational efficiency can be found in Soares de Mello et al. (2003) and Araújo et al. (2006). Fernandes and Capobianco (2001) and Fernandes and Capobianco (2004) used the DEA tool to study the capital structure of companies. Lopes et al. (2006) investigated the pricing of airlines in domestic long-distance flights. The DEA methodology is widely used in the study of air transportation and the efficiency of airports. (Fernandes and Pacheco, 2002); (Pacheco and Fernandes, 2003); (Soares de Mello and Gomes, 2004); (Pacheco et al., 2006).

To measure the effectiveness of airlines, the model MCDEA created by Li and Reeves (1999) was applied, which allows the ordination of efficient units, with the use of two objective functions in addition to the classic DEA. Next, the TRIMAP tool (developed by Climaco and Antunes 1987) was
used, which allows investigation of the decomposing of the area of the weights space of the objective functions. Finally, by calculating the values of objective functions for each region of weight space, the efficiency index MCDEA-TRIMAP was determined proposed by Soares de Mello et al. (2006) in order to assess each airline.

This article is organized as follows: section 2 is a brief summary of the evolution of the Brazilian airline industry. Section 3 presents a review of the tool and the DEA model created by Li and Reeves. Section 4 discusses the use of the TRIMAP tool in the solving of the MCDEA model, and Section 5 presents the characterization and modeling of the problem. Section 6 discusses the results of applying the MCDEA model, together with the calculation of the efficiency index, as well as conducting a comparison of them with the results obtained from the classic DEA. Finally, Section 7 describes the conclusions of the study.

2 AIR TRANSPORT SECTOR

The first big boost for the airline industry took place in World War I when aircrafts were used in military strategies. However, it was only after World War II that air transportation began to be used on a larger scale.

The large extension of Brazilian territory and the precarious nature of other means of transportation favored the expansion of commercial aviation in the country, which began in 1927 with the founding of the Airline Rio-Grandense (VRG). In 1933 another major company was founded: VASP Airlines, which three years later inaugurated a regular flight between Rio de Janeiro and São Paulo, which remains until today the largest line of domestic aviation traffic, (Oliveira, 2005).

In the 40s, the market gained tremendous momentum with the purchase of American aircrafts from the World War II surplus (Novais and Silva, 2006) sold at low prices and financing, which favored the emerge of airlines. The poor administrative and financial structure of companies at that time resulted in the realization of many mergers and bankruptcies in the 50s, and few Brazilian companies were maintained, operating mainly regionally.

The 60s were marked by the search for solutions to the problems the sector faced. The government created the National Aviation Conference (CONAC), with the aim of defining strategies to reverse the crisis which the airline industry was engulfed in. Defined policies and guidelines, which remained in place until the late 80s, encouraged the merger of companies, improving their financial situation. The industry began to operate under a model of "managed competition" in which the government controlled both the entry of airlines into the domestic market, as well as the defining of fees to be charged (Guimarães and Salgado, 2003).

The early 90s brought the introduction of a new policy for the Brazilian air transportation industry, resulting in the loosening of regulations in the sector. Companies were able to start working with different rates, due to tariff bands, controlled by the government. In 1992, the abolition of regional monopolies contributed to increased competition in the sector. At the end of the decade, the competition received a new impetus - the tariff bands were extinguished and exclusivity in the operation of some airlines to regional companies was removed, (Lima and Soares de Mello, 2009). All companies became domestic, ending the classification of regional and national companies.

With the absorption of the company Cruzeiro, VARIG became, at the beginning of the XXI Century, the largest carrier in Latin America. During this same period, the DAC (Civil Aviation Department), currently the ANAC (National Civil Aviation Agency) promoted the freedom to set rates, allowing each company to define their marketing strategy. TAM which had been a regional airline, became the second largest company in the South American continent, while Transbrasil ceased operations in 2001.

The process of creating new companies and airlines, and the frequency of flights and aircrafts was made more flexible, which enabled the creation of GOL Airlines in 2001, the first Brazilian company to apply the model LCC (Low Cost Carriers). In this model, lower rates are charged, with high use of the aircrafts, increased use of the internet for sales, service on board reduced and better use of the maximum takeoff weight (Evangelho et al., 2005).

The year 2005 was marked by change. VASP operations ceased, while new companies started regular operations. Some companies that were traditionally part of the irregular segment became concessionary companies and started to operate regular airlines. TAF Airlines, for example, returned to the market and to this field of work from which it had retired in 2002.

In September 2006, the positive momentum gained by a sector seen as having excellence in safety, quality and punctuality was shaken by the
early indications of serious problems. Air disasters revealed the serious lacks in the physical infrastructure of the Brazilian airline industry. Flight controllers were removed, others went on strike; flights were continually canceled and airlines who could not meet the scheduled times for their flights aggrivated the crisis by overselling tickets, outnumbering the seats available. In July 2007, yet another accident resulted in further uncertainty for air travelers.

After the heavy crisis the airline industry had faced in 2006 and 2007, 2008 came as a year of recovery for the sector, which sought to adopt new measures to restore the confidence and credibility they had previously enjoyed. Even so, Visagiao (2008) cautions that although the symptoms of the crisis have been reduced, consequences generated by the global financial crisis can bring difficulties for the sector, due to decreased availability of credit.

Few companies dominated the domestic market between 2003 and 2008. The Statistical Yearbook of Air Transport for the year 2008 highlights the presence of four major companies that year: Tam, Gol, VRG and Oceanair, the first 3 being considered the biggest of the year. In October of that year, Gol merged with VRG, resulting in the VRG Linhas Aéreas S.A. , (VRG Airlines) leaving the domestic market basically divided between two companies.

3 DATA ENVELOPMENT ANALYSIS AND LI AND REEVES’ MODEL

The Data Envelopment Analysis (DEA), developed by Charnes, Cooper and Rhodes (1978), uses mathematical optimization in order to estimate the technical efficiency of production units (Decision Making Units - DMUs) without have to arbitrate the weights for each input variables (available resources) or output (results obtained) or consider financial matters to make comparative analyses (Estellita Angulo-Meza and Lins, 2000).

Multiobjective Linear Programming (PLMO) and DEA have in common the concept of Pareto efficiency, since both approaches consider that production units are efficient, if, and only if, it is not possible to improve one of its features without worsening any of the others.

The DEA methodology constructs an efficient frontier, which vertices are formed by DMUs deemed efficient (Pareto efficient) because they have a better input/ output ratio, while the others are located in a region below the boundary (Gomes et al., 2003). Since each DMU chooses its own set of multipliers, so that the effectiveness is the best possible in relation to others, it is possible that a large number of DMU's are located on the efficient frontier, revealing the benevolent structure of the method and its low discriminatory power. According to Leta et al. (2005) for empirical determination, the tie of production units happens when the number of DMU's is not very large compared to the total number of inputs and outputs.

The MCDEA model, by Li and Reeves (1999), uses Multiobjective Linear Programming in order to solve the problems of discrimination of the DMU's and promote a better distribution of the multipliers for the variables. It proposes a multicriteria approach to DEA, including two other objective functions, in addition to the traditional objective function of the DEA model, each representing a new criterion to measure the efficiency (maximum deviation and sum of deviations). Since the criteria provides less flexibility for optimization, it tends to restrict the freedom of DMU's in the quest for efficiency.

It should also be noted that the MCDEA model can be considered a method of joint evaluation (Angulo-Meza et al., 2003) because it is a multicriteria method which presents non-dominated solutions, taking into account all the objective functions without being limited by the solutions obtained from the individual optimization of each function.

The model originally defined as CCR, considers the maximizing of the efficiency of the production unit, which is calculated according to the model (1) where $v_i$ and $u_r$ are the multipliers of inputs $i$, $i = 1,...,m$, and outputs $r$, $r = 1,...,s$, respectively; $x_{ij}$ and $y_{ro}$ are the inputs $i$ and outputs $r$ of DMU $j$, $j = 1,..., n$; $x_{io}$ and $y_{ro}$ are the inputs $i$ and outputs $r$ of DMU 0.

$$\text{Max } h_o = \sum_{r=1}^{s} u_r y_{ro}$$

Subject to

$$\sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 , j=1,...,n$$

$$u_r, v_i \geq 0 , \forall r, i$$

A DMU is efficient when $h_o = 1$, which means that the restriction relating to the DMU is active and therefore has to be equal to zero. The MCDEA
The model proposes the use of the variable \( d \), as the measurement rather than \( h \). Thus, in this model the DMU is efficient if, and only if, \( d = 0 \). Therefore, it could be assumed that this formulation minimizes the inefficiency of the DMU with the restriction that the weighted sum of outputs be less than or equal to the weighted sum of inputs of each DMU. Thus, the CCR model is reformulated as shown in (2):

\[
\begin{align*}
\text{Min } d_o \\
\text{Subject to} \\
\text{Max } h_o = u_r y_{ro} \\
\sum_{i=1}^{m} v_i x_{io} = 1 \\
\sum_{r=1}^{s} u_r y_{ij} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \ldots, n \\
u_r, v_i, d_j \geq 0, \quad \forall \ r, i
\end{align*}
\]

From the model presented in (2), and in order to restrict the freedom of choice of the multipliers, the MCDEA model adds two objective functions: minimization of the sum of deviations ("general benevolence") and minimizing the maximum deviation ("equity"). According to Li and Reeves (1999), each of the three objective functions is independent of the others, since there is no order of priority among the criteria of efficiency. The MCDEA formulation is shown in (3):

\[
\begin{align*}
\text{Min } d_o \\
\text{Min Max } d_j \\
\text{Min } \sum_{j=1}^{n} d_j \\
\text{Subject to} \\
\sum_{i=1}^{m} v_i x_{io} = 1 \\
\sum_{r=1}^{s} u_r y_{ij} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \ldots, n \\
u_r, v_i, d_j \geq 0, \quad \forall \ r, i
\end{align*}
\]

In evaluating the results, a DMU is minimax if, and only if, the value of \( d_o \) corresponding to the solution that minimizes the second objective function of the MCDEA is zero. Likewise, a DMU minisum is efficient if, and only if the value of \( d_o \) corresponding to the solution which minimizes the third objective function of the model is zero.

When a DMU is minimax or minisum efficient, it must also be DEA efficient because, by definition, the efficiencies minisum and minimax require \( d_o = 0 \). It can be concluded, as a result, that the minimax and minisum objectives do not, as a general rule, favor the classical efficiency of DMU under evaluation.

To solve the model being studied, a decision support tool for TRIMAP was used, which has an excellent graphic interface for analysis and is suitable for linear programming problems which have three objective functions.

4 TRIMAP AND MCDEA

The TRIMAP method (Climaco and Antunes, 1989), combines three fundamental procedures: decomposition of the weights space, introduction of restrictions in objective space, and introduction of restrictions in the weights space. The interactive environment enables the decision-maker to research efficient solutions, based on past progressive learning findings of efficient solutions. The combination of reduced permissible space, along with the reduction in the weights space of objective functions, allows the agent to specify lower constraints for the values of the objective function, and/or restrictions in the weights space (Climaco et al., 2003).

The TRIMAP calculates efficient solutions that optimize each objective function and the efficient solution that minimizes a weighted Tchebycheff distance to the ideal solution. During the interactions, as unwanted solutions are eliminated, the preferences of the decision maker are perceived. The development of the method enables the reduction of permissible space, saving computational effort and progressively increasing the focus on the sub-region of greatest interest to the efficient decision maker, facilitating the decision making.

The use of this tool allows for key graphs to be obtained for the study of the MCDEA model. The graph obtained shows the weights space decomposed in indifferent regions - regions where the weights of the objective functions may vary without changing the value of the solution obtained - that correspond to the basic non-dominated solutions obtained. In addition, the chart can show direct restrictions on weights and permissible values of objective functions. The TRIMAP also offers a summary of the numerical results obtained, providing for each nondominated basic solution, the value of the basic variables, the objective functions, the percentage of area occupied by the region of indifference, among other data. In this study
TRIMAP was used to generate solutions and served as an analysis tool for studying the weighting space, as in Soares de Mello et al. (2006).

The TRIMAP is a tool that is well suited for the study of the MCDEA model, once it calculates all the solutions of the optimal objective function of the classic DEA, identifying the nondominated solutions. Soares de Mello et al. (2006) highlights the importance of this result, because the knowledge of the existence of alternative multipliers and the identification which corresponds to basic solutions enables the realization of further analysis. In addition, knowledge of the decomposition of weighting space allows the investigation of the stability of the efficient solutions, as well as the identifying of combinations of weights which, while not giving maximum efficiency to a DMU, enable them to be considered a good solution, as they don’t excessively reduce the efficiency, and by providing better values to other objective functions, are more in line with the preferences of the decision-maker.

In Climaco et al. (2008) one can find discussions on how to do qualitative analysis of the MCDEA model with TRIMAP. Soares de Mello et al. (2006) proposed an MCDEA-TRIMAP evaluation index, which considers the properties derived from the use of the TRIMAP in the Li and Reeves model. Considering that the calculation of efficiency in this model requires the consideration of all the possible combinations of the weights of objective functions and that the values assumed by the classical objective function undergo continuous variation, this objective function is integrated when the weighted sum of three objective functions is optimized. This integration must be done in the entire space of possible weights and, the result divided by the size of this space, provides the average value of the classic objective function in this space. The complement of this average value represents the efficiency ratio, as detailed in (4):

$$I(\text{Eff}_{MCDEA-TRIMAP}) = 1 - \left( \frac{\int \int \int FO1(\lambda_1, \lambda_2, \lambda_3) dS/\text{area } \Delta}{} \right)$$  \hspace{1cm} (4)

As the integration is continued by parts in the weights space, assuming a constant value in each region of continuity, the calculation of the index can be simplified by making the weighted sum of the first objective function, using as a measure the percentage area in each valid solution. It is important to note that all these values are easily obtained through TRIMAP. To avoid distortions in the integration of the weights space the expression on the objective function minisum must be divided by the total number of DMU’s under analysis (Soares de Mello et al, 2006). In accordance with the model properties of Li and Reeves, MCDEA index is less than or equal to the classic DEA efficiency. In this study the evaluation of the efficiency is performed by comparing the index with the results obtained by classic DEA.

5 CHARACTERIZATION AND MODELING OF THE PROBLEM

Annually, the ANAC (National Civil Aviation Agency) publishes the Statistical Yearbook of Air Transport, which contains statistical data on the national air transportation sector. Silveira et al. (2008) investigated the efficiency of the national airlines, considering the data for the year 2005, using a methodology similar to that used in this study. The big changes that occurred in the scenario of the air transport industry in recent years motivated the realization of this study, which considers the data for the year 2008, available at www.anac.gov.br. The recent scenario in the airline industry shows significant changes from one year to another with mergers, creation, and closure of companies. Therefore, it was not possible to perform a comparative analysis of the results obtained from the previous years.

Data provided by ANAC is made available at the end of the year, and therefore not considered for purposes of analyzing any possible variations throughout the year. In October 2008, Varig was bought by Gol and the national market was basically divided between two companies: Gol/VRG and TAM. In order to simplify things, this study considered only one company (GOL/VRG), taking into account the sum of each company’s information during the period.

The companies AZUL, MASTER TOP, SKYMASTER and VARG LOG did not present their data in the time frame for disclosure and are not included in this study. The company ABSA - Aerolineas Brasileiras, used only for cargo transportation, was also not considered.

In 2008, the serious challenges the airline industry faced in 2006 and 2007 were, if not solved, at least softened. The government adopted a series of measures throughout this time, with the objective being to solve the problems of the infrastructure and other particular problems which unfolded during the crisis.
The evaluation of Brazilian airlines was done from a comparative analysis of the results obtained by applying the DEA methodology and the indexes obtained by applying the MCDEA model.

Air transport is characterized by the transport of passengers and cargo over long distances. Therefore, for this study two outputs related to these variables were chosen: number of paid seats used, and the metric tons used per kilometer. The choice of inputs considered the main capital asset of these companies: airplanes. Since the aircraft from each company has distinct characteristics, the maximum takeoff weight input was used, being a variable that is linked to the ability to transport both passengers and cargo. On the other hand, staff is needed to operate the aircraft, perform services, and manage the company. For that reason, the other input used was the total staff amount of each company.

Each individual company was considered a DMU. As mentioned, Gol and Varig were considered as a single DMU, due to the purchase of the latter in October 2008. As the study aims to evaluate the performance of airlines in terms of their operational management, an input- oriented model was used to evaluate companies that have the ability to reduce their fleet and manage their workforce without causing damage to the total transported.

Despite the difference in size between airlines, there is no guarantee of disproportionality between the inputs and outputs. Silveira Soares de Mello (2009) formulated the MCDEA-BCC model and applied it to the airline industry, with the data pertaining to the year 2005. However, due to the formulation of classic BCC, the MCDEA-BCC can generate slacks larger than 1, which means negative efficiencies. According to Soares de Mello et al (2002), this phenomenon occurs especially when a DMU has an increasing return to scale. Thus, the CCR model was adopted for comparison with the model used by Reeves and Li (1999), which is based on this model.

### 6 RESULTS

The classic DEA-CCR was applied to the 17 DMU’s that represent regular Brazilian air transport companies, with the movement of cargo and passengers in 2008.

According to ANAC, the year 2008 was marked by the presence of four companies who represented 80% of the total number of stages performed: TAM, GOL, VRG, and OCEANAIR, the first considered the top three of the year. In October 2008 the merger of GOL and VRG companies took place which resulted in the continuation of a single company, VRG Linhas Aereas SA. For simplicity sake, this study considered conjunctively the data for both companies, as GOL/VRG.

The 2008 report of the ANAC featured a retrospective perspective of the information disclosed in the period 2000/2008, noting that between the years 2003 to 2008, the domestic market share had the predominance of few companies in the sector. In 2008, TAM had a 50% stake, Gol 29%, and VRG, 14%.

The first step of the study was to apply the DEA-CCR model to the DMU’s investigated, using the software SIAD of Meza Angulo et al. (2005). The results are summarized in Table 1.

<table>
<thead>
<tr>
<th>DMU</th>
<th>Airline</th>
<th>Classical Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU1</td>
<td>Abaceté Linhas Aéreas</td>
<td>0,098966</td>
</tr>
<tr>
<td>DMU2</td>
<td>Gol Transp. Aéreo Ltda/VRG Linhas Aéreas</td>
<td>1,000000</td>
</tr>
<tr>
<td>DMU3</td>
<td>Meta Mesquita</td>
<td>0,414240</td>
</tr>
<tr>
<td>DMU4</td>
<td>Oceanair</td>
<td>0,619809</td>
</tr>
<tr>
<td>DMU5</td>
<td>Puma Air</td>
<td>0,091950</td>
</tr>
<tr>
<td>DMU6</td>
<td>Passaredo Transp.Ae.S/A</td>
<td>0,384954</td>
</tr>
<tr>
<td>DMU7</td>
<td>Pantanal L.A.Sul-Matogrossense</td>
<td>0,255394</td>
</tr>
<tr>
<td>DMU8</td>
<td>Rico Linhas Aéreas S/A</td>
<td>0,378091</td>
</tr>
<tr>
<td>DMU9</td>
<td>Tam Linhas Aéreas S/A</td>
<td>1,000000</td>
</tr>
<tr>
<td>DMU10</td>
<td>Trip T. A. R. Interior Paulista</td>
<td>0,431652</td>
</tr>
<tr>
<td>DMU11</td>
<td>Taf Linhas Aéreas S/A</td>
<td>1,000000</td>
</tr>
<tr>
<td>DMU12</td>
<td>Total Linhas Aéreas S/A</td>
<td>0,713920</td>
</tr>
<tr>
<td>DMU13</td>
<td>Webjet</td>
<td>0,694577</td>
</tr>
<tr>
<td>DMU14</td>
<td>Air Minas</td>
<td>0,192657</td>
</tr>
<tr>
<td>DMU15</td>
<td>Nht</td>
<td>0,129612</td>
</tr>
<tr>
<td>DMU16</td>
<td>Sete</td>
<td>0,150975</td>
</tr>
<tr>
<td>DMU17</td>
<td>Team</td>
<td>0,078830</td>
</tr>
</tbody>
</table>

The classic DEA-CCR model provided 3 efficient DMU’s - Gol Transportes Aereos Ltda/VRG Linhas Aereas, TAM Airlines S/A, and TAF Linhas Aéreas S/A, without it being possible to make any distinctions between them. At this point, the model MCDEA was applied in order to increase the power of discrimination between the units studied and the TRIMAP was used in order to assess the weights space.
Figures 1, 2 and 3 show the decomposition of the weight space for the DMU's effective in the classical model. The analysis (details in Climaco et al., 2008) shows that the DMU TAM was the only efficient minisum and minimax. Since the solutions that optimize the objective function related to the DEA CCR model cover the entire weights space, the latter is better assessed by the MCDEA model.

The DMU GOL/VRG is minimax efficient but not minisum efficient. However, the DMU TAM is only minisum inefficient in a small area. The DMU TAF was only efficient achieving this efficiency with very specific multipliers.

The calculation of the evaluation index for MCDEA-TRIMAP DMU's considers the weighted sum of the solutions of the first objective function which weighting space is the percentage of the areas where each solution is valid, using the data provided by TRIMAP.

From these results, it was discovered that the only DMU which remained efficient was the company TAM, followed by GOL/VRG, confirming the graphical analysis carried out and the fact that the model restricts the optimization of MCDEA DMU's.

In addition, the analysis of the results show the benevolence of the Classic DEA model, which identified the company TAF Airlines as efficient, while the MCDEA model showed the same company as having very low efficiency ratios, 0.2568.

7 CONCLUSIONS

The application of the DEA model to the data from the Brazilian Airline industry for the year 2008, provided by ANAC, indicated three companies as efficient - Gol Transportes Aereos Ltda/VRG Linhas Aereas, TAM Airlines S/A, and TAF Linhas Aéreas S/A, without it being possible to make any distinctions between them. The use of MCDEA TRIMAP index allowed the identification of the most efficient company in 2008 - TAM airlines. Therefore, the index provided, once again, better discrimination of the production units in the study.

It is important to note that GOL, shown as efficient in the application of the DEA model, obtained a very similar index in the application of MCDEA. However, TAF reached a much lower index. This difference can be seen in the qualitative analysis of Figure 3, where TAF barely achieved being efficient, and even then, in a very small area.

The decomposition of the weight space for the DMU's effective in the classical DEA model is illustrated in Figures 1, 2 and 3. It shows that the DMU TAM was the only efficient minisum and minimax, being considered the company more efficient. This result could be expected, since the solutions which optimize the objective function for the DEA CCR model overlying the entire space of weights. The DMUs GOL / VRG is only minimax efficient, while TAF is only efficient DMU, which is not getting thus able to overcome the result of the first DMU.

It should be noted that since the departure from the market of VARIG, in 2006, TAM has been responsible for the largest share of the Brazilian market. The result obtained in this study is quite consistent, since efficiency is crucial for a company that wishes to remain an industry leader. The merge of GOL/VRG, which occurred at the end of 2008, will undoubtedly bring dispute in the near future and generate strong reactions in the market.

The MCDEA model (Li and Reeves, 1999) was developed for DEA-CCR models. Future studies may investigate the use of MCDEA with DEA-BCC models (Banker et al., 1984). Future studies may also explore the potential interactivity of TRIMAP and DEA models, with constrains to the values of the objective functions, and constraints with values of the multipliers.

REFERENCES

Angulo-Meza, L., Biondi Neto, L., Soares de Mello, J. C.


Anuário Estatístico do Transporte Aéreo – disponível em http://www.anac.gov.br/


envelope of data in the study of efficiency and benchmarks for Brazilian airlines, *Pesquisa Operacional*, 23(2), 325-345.


