A Graph-based Software Tool for the CAD Modeling of Mechanical Assemblies

Stanislao Patalano¹, Ferdinando Vitolo¹ and Antonio Lanzotti²

¹ DiME, University of Naples Federico II, P.le Tecchio 80, Naples, Italy
² DIAS, University of Naples Federico II, P.le Tecchio 80, Naples, Italy

Keywords: Mechanical Design, Graph Theory, Digital Pattern, CAD Modeling, Graphical User Interfaces.

Abstract: The challenge of reducing designing time for new mechanical assemblies, especially in the context of large companies, encourages the use of methods and tools aimed to support designing activities and to re-use the company know-how. Furthermore, the design choices must be rapidly check to avoid errors that could cause delay or expensive re-designing. In such a context, the graph theory and related algorithms could be used to define a transfer function, easily to implement, that governs a software tool able to support the designing activities. Therefore, the paper presents a designing approach, based on the graph theory, aimed to generate the geometric modeling of mechanical assemblies. The approach and the software tool are useful both for designer and companies that want to customize and improve such activities. Finally, the paper shows the case study related to the design of a transversal manual gearbox and the generation of a GUI, developed in MatLAB® environment, to validate the approach.

1 INTRODUCTION

In engineering practice, designers often define mathematical models that describe the behavior of part or system (mechanical, electrical, naval) under development. Then, designers provide the manipulation of a series of equations by using both the knowledge related to each system and the relations between mathematical models and physical reality (Shai and Preiss, 1999). In such context, it is difficult to individuate each parameter and related dependencies with the other ones. This is often due to the large number of equations and parameters that govern the mechanical or electrical system.

A strategic activity is the reuse of past experiences during the design phase. The reuse is often related to the specific designer who has acquired experiences.

Especially large companies adopt structured work teams to manage and distribute engineering knowledge (Sharmin et al., 2009). In such a context, we believe that software tools aimed to support the design tasks must adopt Knowledge Based Engineering (KBE) to capture and reuse knowledge, in automatic way, during design activities, acquiring it from each expert designer. In fact, Sandeberg (2003) define KBE as “the use of advanced software techniques to capture and re-use product and process knowledge in an integrated way”. Several researchers worked on the re-use of product and process knowledge. Chen et al. (2012) developed a multi-level assembly model that is useful to capture the information in different levels of the design phases. Tang et all (2010) use a design structure matrix (DSM) method for capture and reuse the past experience knowledge. In particular, they use additional tables, linked to DSM, to record information as type and level of interaction, milestones and criterions on parameters. This method requires, however, designer defines interactions in a subjective way. Therefore, designer must already have past experiences.

A different approach used especially in the context of small companies is that the designer must work and manage all information (as dependencies, or best cases) in opportune way. This means for the designer to pour on his skill the success of the design. Therefore, a concrete contribute to the growth of small companies is to provide methodological approach and software tools to adopt KBE in the above-mentioned way.

During the designing of mechanical parts as wheels, levers or shafts, after the preliminary sizing
of the part, designers need to verify the correctness of the hypotheses in use, especially in relation with multi-objective tasks. In particular, the use of Multi-Objective Optimization methods induces further decisions for designers. In fact, designers have to choose the a priori articulation of preferences i.e. the specification of parameters, and coefficients when they are sure about decisions; otherwise, they have to adopt methods with a posteriori articulation of preferences when it is difficult to express a preference before the acquisition of results (Marler and Arora, 2004). Then, designers build a digital model by using CAD-CAE environment to deepen form, function and geometrical characteristics of the product or to simulate product performances, as strength or product compliance (Franciosa et al., 2011).

Therefore, there is a huge need for supporting all these phases by simplifying and optimizing the product development process. To meet this need of integrated knowledge, within CAD-CAE environment, it is possible to choose additional knowledge-based modules that make easier the design tasks for classic and common product (Sham, 2010), (VV. AA., 2008), (VV. AA., 2012). The main problem of such modules is that they are difficult to adapt to specific requirements if you are not an experienced programmer.

If designer do not adapt existing knowledge-based modules, they have to find different way to take into account and to represent dependencies among design parameters.

A very useful tool to rapidly represent the dependencies among parameters or variables is provided by the graph theory. Graphs, in fact, provide the representation of binary relations and can be used for schematic modeling of many problems in mathematics, engineering, computer science, economics, sociology, linguistics, and wherever it is necessary to represent a binary correspondence (Deo, 2004), (Franciosa et al., 2010), (Franciosa et al., 2012).

The way to associate a design methodology to graph theory could be expressed as follows. In the axiomatic design, Suh (1990) and (1997) claimed that is necessary to start from a poor-level of details for each domain (requirement domain or physical domain) and to progress using zigzagging technique between the domains. Using the hierarchical decomposition, it’s impossible to represent the dependencies or interactions between the parameters or elements of product (Tang et al., 2009). With the use of graph theory it’s possible to create the directed graph (digraph) that clearly represents the dependencies between the parameters for any level of the product hierarchy structure (Deo, 2004). Therefore, once defined the level of detail it is possible to associate a digraph for the “transversal” dependencies in the level. Some authors reserve the term arc for an oriented or directed edge, while edge is for undirected edge. In the following we deal with digraphs, and therefore the term edge will be used to indicate the oriented edge. For deepening on graph theory, see (Deo, 2004); (Bondy and Murty, 2008) and (Wilson, 1978).

The use of graphs has several advantages in product design:

• to represent all dependencies in a simple and clear way;
• to increase the level of detail by expanding the nodes of the graph.

Therefore, the graph representation fits a common need in designing activities. In fact, designers first define high-level requirements and then increase the level of detail by introducing further parameters and related dependencies among them.

Some works in literature are aimed to integrate tools based on graph within CAD systems. In Franciosa et al. (2009) a tool for a quick checking of tolerance specifications, directly assigned to a CAD model, were proposed. Lockett and Guenov (2005) presented a new approach based on the average area for a module integrated in CAD feature recognition and aimed to molded parts. Also during the generation of CAD models within high-level CAD environments, as CATIA® or UGS-NX - the last one added with Wave Engineering module (Sham, 2012), it is possible to explore links between the assembled parts through graphs.

The present paper proposes an approach and a software tool to accomplish the geometric modeling of mechanical assembly by supporting the preliminary phase of dimensioning as well as the updating during the re-use of consolidated CAD models. In particular, the approach allows the management of company know-how according a KBE point of view and provides an easy-to use graphical interface based on directed graphs.

The paper is arranged as follows. Section 2 summarizes the approach to CAD modeling of assemblies based on graphs. Section 3 and Section 4 provides the application of the approach to the case of an automotive gearbox and the implemented GUI, respectively. Finally Section 5 draws conclusions.
2 THE APPROACH TO CAD MODELING OF ASSEMBLIES BASED ON GRAPHS

2.1 General View and Fields of Intervention

Especially in the context of small or medium companies, designers are often the main actor in decision processes, during designing or CAD modeling activities. The authors here provide an approach and a software tool first to make automatic the calculus chains due for the dimensioning and the locating of assembling parts and then to improve the re-use of company knowledge related to the designing activities of different designers (Figure 1). Therefore, also repetitive activities or manual optimization loops are significantly improved.

The proposed approach to CAD modelling deals with three different levels within the geometric modelling of assemblies. The three levels are (Figure 2):

- **Functional level** (Level 1) where functional groups are defined; at this level it is possible to operate with a series of parameters to accomplish the consistency of functional groups (as for example costs, number of instances, types of unified components);
- **Logical level** (Level 2) where datum, parameters and control rules for dimensions are defined; at this level it is possible to manage datum or equations to accomplish geometrical consistency of assembling parts (curves and surfaces for mating conditions, axes of rotation, distances between planes as functional gaps);
- **Geometrical level** (Level 3) where the geometries of parts, that compose the assembly, are modeled; at this level it is possible to operate local changes to dimensions as well as to further parameters of parts (thickness and length of parts, curvature’s radii, Young’s modulus).

The present approach and the developed software tool allow operating at each of the three levels. In particular, it is possible to proceed from level 1 to level 3 for the generation of a new CAD model or, alternatively, to operate only in one of the three levels.

The preliminary sizing of an assembly often imposes to work with a high number of variables and related equations. Therefore, during this phase it could be strategic to have a tool that simplify the management of such equations and makes easily detectable the dependency of a variable from others.

![Figure 1: a) typical design scheme performed in small-medium company; b) proposed design scheme.](image)

![Figure 2: Three levels of intervention for the proposed approach.](image)
In particular, Shai (2003) claims that it is possible to transform the mathematical problem towards the graph field and then return with the solution to the mathematical field. The approach presented in this paper first represents the mathematical and geometrical relationships by means of graphs; then, it provides the manipulation of the edges in order to change the dependencies and, finally, to return to the mathematical field. This general flow is shown in Figure 3.

2.2 Approach

The approach to geometric modelling deals with an editable transfer function based on graphs as depicted in Figure 4. In particular, the general transfer function \( Y \) is able to correspond \((x_1, x_2, \ldots, x_n)\) input parameters to a set of assembly variables \((y_1, y_2, \ldots, y_n)\) by using parameters, relations and related computation tasks. At this stage, the set of relations, representing the dependencies between parameters, is associated to digraphs. In particular, the nodes are associated to parameters, while directed edges represent existing relations. On the other side, a matrix representation could be used for software implementation.

Furthermore, the relations represented through digraphs can be handled generating new transfer functions. The handling consists of three phases:
- extraction of graphs;
- editing of graphs;
- updating of relations.

2.2.1 Extraction of Graph

The initial set of nodes and relations, assigned by designers, are represented through digraphs. In particular, the nodes are arranged in the space by using the Fruchterman-Reingold’s algorithm as in (Franciosa, 2009). This algorithm generates the layout of the graphs according the degree of attraction or repulsion of nodes. In particular, the degree of repulsion is a function of the number of links between the node under consideration and the surrounding ones linked to the first.

Fully connected graph is the assumption. If the graph is disconnected and it has several components the algorithms can be applied to each component.

Before extracting the graph designer is asked to isolate the input parameters. In fact, these parameters will be the starting nodes from which to generate the digraph until to arrive to ending nodes that represent the outputs i.e. assembly variables.

2.2.2 Editing of Graphs

The phase deals with the editing of digraphs and the running of algorithms to check validity of the digraphs. The designer can reverse or delete a link between two nodes. The possibility to reverse the direction of the edge, and then of a dependency, allows to change relationships without reprogramming the data structure.

When an edge is reversed the algorithm for loop detection, working on edge list, checks digraphs to highlight loops to be avoided. In particular, a series of integers \(1, 2, \ldots, n\) is assigned to the vertices. Starting from the first vertex, the algorithm builds a directed path \(P=(p_1, p_2, \ldots, p_k)\) until there are no further vertices linked. Then the algorithm checks if there is a directed edge from the \(p_k\) vertex to \(p_1\). If there is a directed edge a circuit is found; if there is no directed edge, the algorithm deletes \(p_k\) and checks for \(p_{k-1}\). Then it repeats the previous steps until \(P=(p_1)\). The algorithm checks for isolated vertices. Similarly, when the edge is removed the algorithm for isolated node detection (Figure 5), working on adjacency matrix, verifies if the vertices adjacent to the removed edge have become isolated.

In particular an edge is defined through two ordered vertices. Therefore the algorithm detects the adjacent vertices after the edge removal.

The algorithm proposed in Figure 5 is provided to analyse throughout the adjacency matrix. In particular, the vertex under consideration is indicated with letter “I”, while “J” indicates the node of interaction. If the cell of adjacency matrix, identified by them, shows a zero value, it means that there is no a linked edge between them. If the entire row “I” has all cells with zero value, i.e. \(\sum_{j=1}^n a_{ij} = 0\), then the vertex is isolated.
2.2.3 Updating of Relations

During this phase the relationships between the parameters are updated. In this way, the editing of graphs moves to transfer function by means of the updated relations. The presented approach based on the above-mentioned three phases (extraction, editing and updating) implies a series of advantages for designers. During the generation of initial CAD models designer is supported when he focuses on large set of equations and builds related digraph. In fact, he could easily check, by using the loop detection algorithm, if there are redundant equations and he could provide to delete them. Also the extracting phase supports designer when he locates the parameters, i.e. the initial nodes of digraphs, as first member of only one equation to avoid redundancies. Then, different designers, who have to operate changes and consequent regenerations of CAD models, could operate in an easy and intuitive way on the digraphs without changing the equations set out by other colleagues. In other words, it is not necessary to rewrite one or more equations to operate changes to transfer function, but it is sufficient to change the digraph with simple and intuitive actions.

3 CASE STUDY: GEARBOX “CMT-6M”

The approach to CAD modelling was applied to the design and modelling activities due to develop an automotive manual transverse gearbox with two shafts and six maximum gears, in the following called “CMT-6M”. To accomplish the complete geometrical modelling of the gearbox, the three levels (functional, logical and geometrical levels) were carried out.

3.1 Functional Level

To accomplish the designing of a family of gearboxes a set of functional parameters was taken into account (Table 1), according to know-how of a European automotive group.

<table>
<thead>
<tr>
<th>Materials for gears</th>
<th>MatG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials for shafts</td>
<td>MatS</td>
</tr>
<tr>
<td>Costs</td>
<td>Cs</td>
</tr>
<tr>
<td>Number of shafts</td>
<td>Ns</td>
</tr>
<tr>
<td>Number of gears</td>
<td>Nge</td>
</tr>
<tr>
<td>Teeth Modulus</td>
<td>m</td>
</tr>
</tbody>
</table>

The bounding volume of the assembly is a significant requirement in gearbox design as it deals with the characteristics of layout engine compartment. Often, similar gearboxes have to be assembled within different engine compartments and this induces several changes to the whole architecture of gearbox as for the dimensions of gears, the transmission ratios, the distances between shafts. As same 3D bounding volumes could be accomplished through different combination of
functional or datum parameters, it was not settled as functional parameters but it was used as inspection parameter during re-design activities.

### 3.2 Logical Level

For the sizing of modular toothing the classic theory for spur gears (Charchut and Thomas 1972) and (Juvinall and Marshek, 1994) was used. As the present analysis was not focused to fatigue design, Lewis’ theory together appropriate correction factors, contained in UNI 8862 standards, for both nominal and allowable stresses, was used.

Table 2 summarizes the general equations for gearbox sizing. Table 3 and Table 4 show the additional equations for straight and helical toothed sizing, respectively (Appendix A shows notation used in Table 2, Table 3 and Table 4).

#### Table 2: General equations for external gears.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum number of teeth</td>
<td>$z_{1\text{min}} = \frac{1}{2} \sqrt{1 - \tau (2 - \tau) \sin^2 \theta}$</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>$b = \lambda \cdot m$</td>
</tr>
<tr>
<td>Tooth width</td>
<td>$D_p = m \cdot z$</td>
</tr>
<tr>
<td>Pitch diameter</td>
<td>$D_e = D_p + 2m$</td>
</tr>
<tr>
<td>Outside diameter</td>
<td>$l_{ag} = D_e_1 + D_e_2$</td>
</tr>
<tr>
<td>Max size</td>
<td></td>
</tr>
</tbody>
</table>

#### Table 3: Additional equations to characterize straight-toothed gears.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending</td>
<td>$m_f = C \cdot \sqrt{\frac{M}{\lambda \cdot z_1 \cdot K_v \cdot \sigma_{wmm}}}$</td>
</tr>
<tr>
<td>Wear</td>
<td>$m_w = C \cdot \sqrt{\frac{M}{\lambda \cdot p_2 \cdot \sigma_{wmm}}}$</td>
</tr>
</tbody>
</table>

#### Table 4: Additional equations to characterize helical-toothed gears.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending</td>
<td>$m_f = C \cdot \sqrt{\frac{M}{\lambda \cdot z_1 \cdot K_v \cdot \sigma_{wmm}}}$</td>
</tr>
<tr>
<td>Wear</td>
<td>$m_w = C_{ht} \cdot \sqrt{\frac{M}{\lambda \cdot p_2 \cdot \sigma_{wmm}}}$</td>
</tr>
<tr>
<td>Number of teeth dummy</td>
<td>$z_{id} = \frac{Z_2}{\cos^3 \beta}$</td>
</tr>
<tr>
<td>Gear width</td>
<td>$l = b \cdot \cos \beta$</td>
</tr>
</tbody>
</table>

Furthermore, a set of geometrical datum for the proper layout of the gearbox elements was introduced. In particular, to accomplish the assembling of the manual transverse gearbox with two shafts and six gears, called “CMT-6M”, according to a top-down approach, the datum set depicted in Figure 6 was used. Four axes referred, respectively, to the primary shaft, the secondary shaft, the differential gear and the reverse gear compose the datum set.

The datum set is located by means of four parameters (Figure 6):
- **Axis base primary shaft-idler shaft** ($I_{1F}$) = distance between the axis of the primary shaft and the axis of the reverse idler gear.
- **PS-SD angle** ($\alpha_{1-2}$) = angle between the lines that link the traces of primary-secondary axes and secondary-differential axes, contained in the plane perpendicular to such axes;
- **Axis base primary-secondary** ($l_{12}$) = distance between the drive axis and the driven shaft;
- **Axle base secondary-differential** ($I_{2D}$) = distance between the axis of the drive shaft and the axis of the differential.

#### 3.2.1 Digraph Representation

The parameters and relations defined in functional and logical levels for each spur gear are translated into the digraph depicted in Figure 7.

![Figure 6: Datum set of the CMT-6M gearbox.](image)

The designer defines all dependencies among parameters and sets the input in terms of vector of parameters and list of edges.

The digraph of gearbox (Figure 8) is composed by a number of subgraphs equal to the number of spur gear belonging to gearbox. When the gearbox is
characterized by a different number of gear ratios, it is possible to add or delete the sub-graph depicted in Figure 7.

Figure 7: Sub-graph for helical toothed spur gear.

3.3 Geometrical Level

Figure 9 depicts the 3D model of CMT-6M gearbox corresponding to geometrical level. At this level the independent parameters i.e. the ones that are not included in the digraph could be updated.

Figure 9: 3D CAD model of CMT-6M gearbox at geometrical level.

4 CMT-6M GUI IMPLEMENTATION

A Graphical User Interface (GUI), developed in MatLAB environment, was accomplished in order to support the design of automotive manual transverse gearboxes (Figure 10). The main window is divided in two fields: the upper field where the inputs values of parameters are typed (the field of independent parameters); the lower field where the output values are evaluated (the field of dependent parameters). Furthermore, in the lower field are also located the keys to provide the editing of generated digraphs and the evaluation of relations.

The designer can set the number of gears, belonging to the gearbox, up to a maximum of six.

Figure 11 depicts the digraph of CMT-6M (with six gears). Using such interface, the designer could interact within the digraph, by exploring the whole set of parameters, by modifying the existing links between nodes and by using the algorithms for loop detection and for isolated node detection, respectively. To reduce the computational time, when designer selects an edge to remove it, (Figure 12) the algorithm for isolated node detection works only on the two adjacent nodes. The possibility of reversing the edge was implemented to convert an influenced node in a node that influences.

The inversion of a link has to be carefully used by expert designers. In particular, it should not be used for a large set of parameters as it causes the inversion of corresponding relations with unacceptable consequences.

Otherwise, the deleting of an edge from digraph determines the elimination of dependence in the corresponding equation. So, the father-child relation that link two adjacent nodes is deleted. Designer can interactively deletes a link between two nodes when the corresponding relation is negligible.

When all links associated to a node are deleted, the corresponding parameter becomes “independent”. Therefore, the deleting of links is useful for nodes located at the end of digraph chains i.e. for nodes that have no outgoing edges. In fact, in such case the deleted edge does not cause unpredictable effects to the design process but it allows setting a value to a parameter that is independent from the calculated one. An example is the gear width parameter (Lruota) in the sub-graph for helical toothed spur gear (Figure 7). As the node has not outgoing edges, the designer could disconnect this node (see Figure 12) and provide a new input value, keeping the geometrical correctness of the toothing.
The computational results are currently displayed in a dedicated environment. By clicking on "Plot" command, the characteristic surfaces of toothing i.e. head, sides and foot of helical-toothed gears are displayed on screen (Figure 13). In this environment is also possible to change the mesh degree (Figure 14) to accomplish the desired accuracy of geometries. Then, it is possible to export the points of meshes, related to the geometry of whole assembly, by using a text format to generate the 3D model in any CAD environment.

The designer could also evaluate the bounding volumes of CAD models. In a separate window (Figure 15), in fact, it is possible to evaluate the distances between two points along different directions and, then, to easily assign different input values to functional or datum parameters to fit, after the automatic re-generation of geometries the desired bounding volume.

Figure 10: GUI for CMT-6M gearbox.

Figure 11: Automatic generation of gearbox digraph (left). Legend of digraph (right).
5 CONCLUSIONS

The paper presents an approach, based on the graph theory, aimed to generate the geometric modeling of mechanical assemblies.

The approach focuses on the management and re-using of company know-how, according to a KBE point of view.

The developed GUI enables designers to interactively and easily generate several instances of geometrical models by acting on a digraph representation instead of operating on a large set of equations and parameters.

The introduction of such a GUI implies both significant times for its setting up and costs due to the training of designers. Nevertheless, the use of GUI contributes to the reduction of time to market of new gearboxes, enriches the company know-how, improves the sharing of knowledge between designers and facilitates the division of tasks between designers with different designing experiences. A possible scenario in using such GUI is that “expert designers” could use digraph representation to share knowledge, to add relations among parameters and, more generally, to change in an interactive way, complex transfer functions that control dependent design parameters. Otherwise, “younger designers” could accomplish the requirements related to the overall volume of gearbox by modelling different configurations of gears.

The GUI is actually not embedded in a specific CAD environment to give the possibility to export results towards any CAD environment. In fact, the GUI exports data as a cloud of points assuring an effective integration within the company CAD systems in use.

Further developments, concerning with the GUI definition, will deal with the integration of the GUI in a CAD environment to optimise the generation of CAD models; moreover, the setting up of dependencies among graph nodes will be accomplished in a fully automatic way starting from the existing equations. Finally, different case studies dealing with partially known transfer functions will be taken into account.
ACKNOWLEDGEMENTS

The present paper was developed with the economic support of MIUR (Italian Ministry of University and Research) performing the activities of the project PON01_01268 “Digital pattern product development: a pattern driven approach for industrial product design”.

REFERENCES


Deo, N., 2004. Graph theory with applications to engineering and computer science, PHI Learning. New Delphi.


Sham Tickoo, 2010. CATIA V3R19 for engineers and designers, Dreamtech Press.

Sham Tickoo, 2012. NX8 for designers, CADCIM Technologies.


APPENDIX A - NOTATION

Below are listed the parameters shown in Table 2, Table 3 and Table 4:

- $\vartheta$ = transverse pressure angle;
- $Z_1$ = number of teeth on driven gear;
- $Z_2$ = number of teeth on drive gear;
- $\beta$ = helix angle;
- $m$ = module;
- $\lambda$ = face factor;
- $C$ = Lewis’s geometric factor;
- $C_0 = 0.83 + 0.88 \cdot C$ for $\beta = 15^\circ \div 30^\circ$;
- $M$ = torque;
- $\sigma_{amm}$ = allowable stress;
- $p_{amm}$ = allowable pressure;
- $K_d$ = dynamic factor;
- $\varphi_h$ = helix angle factor.