Analysis of MIMO Systems with Transmitter-side Antennas Correlation

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Abstract: Due to its potential performance multiple input multiple output (MIMO) systems are being included in the current standard developments. Nevertheless issues like antennas proximity at the transmitter and receiver arrays can limit the achievable performance. Antennas proximity produces a phenomenon called correlation which affects the channel performance by reducing the capacity and increasing the BER. Hence, the aim of this paper is investigating the transmitter-side antennas correlation modelling and effects. Together with the appropriate signal processing (e. g. singular values decomposition), the effect of transmitter-side antennas correlation is studied. Our results show that under the effect of antennas correlation not necessarily all layers might be used for the data transmission since the weighting of the stronger layer within the MIMO system becomes even stronger respect to non-correlated channels. Simulation results are shown to underline these effects.

1 INTRODUCTION

Multiple Input Multiple Output (MIMO) systems have been studied during the last decades due to their ability to increase the channel capacity and decrease the bit error rate (BER) without increasing the transmit power needed at the transmitter side. In order to obtain the full advantages of the MIMO systems perfect channel state information is required at both the transmitter and receiver sides in order to perform the appropriate signal processing tasks at the transmitter (pre-processing) as well as at the receiver (post-processing) side. A popular technique used for those signal processing operations is the singular values decomposition (SVD). By introducing both operations inter-antenna interferences are avoided and the full MIMO system capabilities can be exploited assuming a highly scattered environment. Due to the antennas physical proximity compared to the wavelength additional effects must be taken into consideration in the analysis and implementation of a MIMO system. Under that condition antennas correlation effect appears affecting the MIMO channel capacity and (bit-error rate) BER. MIMO systems require a highly scattered environment in order to benefit from the use of multiple antennas to select the appropriate transmit and receive conditions. This is synonymous of having multiple paths which largely differ.

Unfortunately, due to the proximity of the antennas separation the theoretically degree of design-freedom of the MIMO system decreases (Lee, 1973; Ertel et al., 1998; Wang et al., 2009). In the presence of antennas correlation the environment is less scattered and hence the MIMO channel capacity decreases and the BER rises. Antennas correlation implies the similarity in the antennas paths and in consequence the environment becomes less scattered. This means that the off-diagonal elements of the channel matrix become similar and it is not possible to exploit the full capabilities of the MIMO system any longer. In order to predict the effects of antenna correlation a proper correlation model is required. The effect of antennas correlation can be separated into two independent effects: the one corresponding to transmitter-side antennas correlation and that due to receiver-side antennas correlation. In order to analyse and predict the behaviour of a correlated MIMO system two key points must be solved: The first one concerns the model of the antennas correlation (i. e. the description of the correlation coefficients) and the second one is related to analysing the correlation effect compared to an uncorrelated MIMO system.

The antennas correlation is usually described by the antennas correlation matrices one for transmitter-side and other for receiver-side (assuming independence between correlations) which collect the cor-

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relation coefficients between the antennas, while the global effect is represented by a called system correlation matrix which is the Kronecker product (Laub, 2005) between the transmitter and receiver side antennas correlation matrices (Taparugssanagorn et al., 2006; Salz and Winters, 1994; Shiu et al., 2000).

In (Yueyu and Lili, 2007) the antennas correlation effect on the channel capacity is studied when using a circular array compared to a linear one showing the decrease in the channel capacity with the antennas correlation effect.

This paper is aimed at the analysis of MIMO systems performance in the presence of transmitter-side antennas correlation. The main contribution of this paper is the definition of the correlation coefficients between transmitter-side antennas as a function of the main parameters showing how they affects the characteristic of the layer-specific weighting factors.

In this paper linear antennas arrays are studied. The authors do not focus on concrete spatial antennas distributions. Instead they use a general propagation model in order to compute the correlation coefficient between antennas.

The remaining part of this contribution is organized as follows: Section 2 describes the physical antennas adjustment as well as the corresponding variables that will impact the computation of the antennas correlation. The corresponding correlated MIMO system model is introduced in Section 3. The associated performance results are presented and interpreted in section 4. Finally, in section 5 the concluding remarks are discussed.

2 BASE-STATION RELATED ANTENNAS CORRELATION

This section describes the physical antennas adjustment as well as the corresponding variables that will impact the computation of the antennas correlation. Starting with the analysis of the correlation between any pair of transmit antennas with respect to a given receive antenna, the result will be extended to any antennas configuration as the correlation is computed for each antennas pair separately. At first, only line of sight (LOS) trajectories are considered as highlighted in Fig. 1.

Fig. 1 represents the physical set-up for a pair of transmit antennas and one receive antenna. The distance between the receive antenna and the reference point (centre of the physical disposition) of the transmit antennas is D. The distance between the transmit antenna #1 and the receive antenna is d_1, while the distance between the transmit antenna #2 and the receive antenna is d_2. The transmit antennas itself are separated by the distance d. Considering the spacings and angles introduced in Fig. 1 some relations can be stated for each transmit antenna. Considering transmit antenna #1, the following relation can be established:

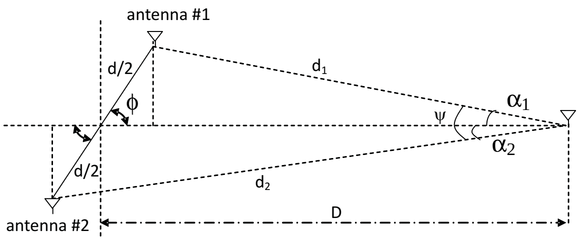

Figure 1: Antennas' physical disposition: two transmit and one receive antennas.

$$d_1 \cdot \cos(\alpha_1) = D - \frac{d}{2} \cdot \cos(\phi) \ . \quad (1)$$

Equation (1) describes the relation among the various physical parameters described in Fig. 1. A similar relation can be obtained for transmit antenna #2 dealing with the following relation:

$$d_2 \cdot \cos(\alpha_2) = D + \frac{d}{2} \cdot \cos(\phi) \ . \quad (2)$$

Now, the focus is set on the computation of the correlation between transmit antennas. Here, the antennas set-up shown in Fig. 1 is considered. Let's assume that the same signal $s(t)$ is simultaneously transmitted through the transmit antennas #1 and #2. Under this conditions the signals arriving at the receive antenna can be described as follows: The signal arriving at the receive antenna from transmit antenna #1 is given by:

$$s_{\mathrm{r}1}(t) = s(t) \cdot G_1 \cdot A(d_1) \cdot \mathrm{e}^{-\mathrm{j}2\pi d_1/\lambda} \ , \quad (3)$$

where G_1 describes the transmit antenna #1 radiation pattern gain in the direction of departure and $A(d_1) \leq 1$ describes the path attenuation effect (given in terms of gain) for the distance d_1. The complex exponential in (3) introduces the phase change suffered by the signal from the transmit antenna #1 to the receive antenna. The signal arriving at the receive antenna from transmit antenna #2 is given by:

$$s_{\mathrm{r}2}(t) = s(t) \cdot G_2 \cdot A(d_2) \cdot \mathrm{e}^{-\mathrm{j}2\pi d_2/\lambda}. \quad (4)$$

where G_2 describes the transmit antenna #2 radiation pattern gain in the direction of departure and $A(d_2) \leq 1$ describes the path attenuation effect for the distance d_2. Given $D >> d$ it can be assumed that $A(d_1) \approx A(d_2)$. The antennas correlation coefficient (path correlation) is given by the correlation between received signal $s_{\mathrm{r}1}(t)$ and $s_{\mathrm{r}2}(t)$ and can be expressed as follows:

$$\rho = \frac{E\{s_{r1}(t)\cdot s_{r2}^*(t)\} - E\{s_{r1}(t)\}\cdot E\{s_{r2}(t)\}}{\sqrt{E\{s_{r1}(t)\cdot s_{r1}^*(t)\}}\cdot\sqrt{E\{s_{r2}(t)\cdot s_{r2}^*(t)\}}} \quad (5)$$

which can be rewritten as:

$$\rho = \frac{E\{s_{r1}(t)\cdot s_{r2}^*(t)\}}{\sqrt{E\{|s_{r1}(t)|^2\}}\cdot\sqrt{E\{|s_{r2}(t)|^2\}}} \quad (6)$$

under the assumption that the transmitted signal $s(t)$ is zero mean and hence $s_{r1}(t)$ and $s_{r2}(t)$ are zero mean valued variables, too. In consequence, the expression $E\{s_{r1}(t)s_{r2}^*(t)\}$ results in

$$\begin{aligned} E\{s_{r1}(t)\cdot s_{r2}^*(t)\} &= E\{|s(t)|^2\}\cdot G_1\cdot A(d_1)\cdot G_2\cdot \\ &\quad A(d_2)\cdot e^{-j2\pi(d_1-d_2)/\lambda} \quad . \end{aligned} \quad (7)$$

In order to simplify the analysis described above, further assumptions should be considered: First, it is assumed that the transmit signal $s(t)$ is unitary, i. e., $E\{|s(t)|^2\} = 1$. Additionally it is assumed that the transmit and receive antennas are isotropic with unitary gain, i. e., $G_1 = G_2 = 1$. Furthermore, given $D >> d$ and $d_1 \approx d_2$ then $A(d_1) \approx A(d_2) \approx A(D)$ can be concluded. Under these conditions equation (7) can be reduced to:

$$E\{s_{r1}(t)\cdot s_{r2}^*(t)\} = A^2(D)\cdot e^{-j2\pi(d_1-d_2)/\lambda} \quad . \quad (8)$$

Concerning the terms in the denominator in (6) the same assumptions are applied obtaining:

$$\begin{aligned} E\{s_{r1}(t)\cdot s_{r1}^*(t)\} &= E\{|s(t)|^2\}\cdot G_1^2\cdot A^2(d_1) \\ &= A^2(d_1) \approx A^2(D) \end{aligned} \quad (9)$$

and

$$\begin{aligned} E\{s_{r2}(t)\cdot s_{r2}^*(t)\} &= E\{|s(t)|^2\}\cdot G_2^2\cdot A^2(d_2) \\ &= A^2(d_2) \approx A^2(D) \quad . \end{aligned} \quad (10)$$

Finally, substituting (8), (9) and (10) in (6) the antennas correlation coefficient can be expressed as:

$$\rho = e^{-\frac{j2\pi(d_1-d_2)}{\lambda}} \quad . \quad (11)$$

In order to rearrange equation (11) as a function of those parameters described in Fig. 1, equations (2) and (3) should be taken into consideration to compute the distance difference $d_1 - d_2$ which should take into account the phase difference between the signals received from each transmit antenna. The difference between (2) and (1) can be expressed as

$$d_2\cdot\cos(\alpha_2) - d_1\cdot\cos(\alpha_1) = d\cdot\cos(\phi) \quad . \quad (12)$$

Considering that the separation between transmit and receive antenna is large enough compared to the separation between the transmit antennas, i. e., $D >> d$ then it can be assumed that $\alpha_1 \approx \alpha_2 \approx \psi/2$, where ψ is called the *spread angle*. In consequence (12) can be expressed as:

$$(d_2 - d_1)\cdot\cos(\psi/2) = d\cdot\cos(\phi) \quad . \quad (13)$$

Substituting the result in (13) into (11) the following expression is obtained:

$$\rho = e^{\frac{-j2\pi d\cos(\phi)}{\lambda\cos(\psi/2)}} = e^{\frac{-j2\pi d_\lambda\cos(\phi)}{\cos(\psi/2)}} \quad , \quad (14)$$

where $d_\lambda = d/\lambda$ is the transmit antennas separation given in wavelengths units. Equation (14) reveals that the transmit antennas path correlation coefficients depends on the antennas separation d_λ, the spread angle ψ and the transmit antennas reference axe rotation angle ϕ (or *signals angle of departure*). Assuming a far field communication the term $\cos(\psi/2)$ in (13) can be approximated to unity (assuming ψ is close to zero). Under this assumption, the correlation coefficient results in:

$$\rho = e^{-j2\pi d_\lambda\cos(\phi)} \quad . \quad (15)$$

Up to now, the antennas correlation coefficient concentrates only on the line of sight (LOS) trajectories. However, wireless channels requires scattered environments to be taken into consideration. In scattered environments the signals transmitted by the transmit antennas are radiated and bounce in multiples obstacles producing multipath signals which arrive at the receive aerial in various arrival directions. In this case the result obtained in (14) must be extended to such scattered environments. Besides, the scatter departs from the transmit antenna with random angles and hence (13) can be extended to the scattered environment case as follows:

$$(d_2 - d_1) \approx d\,\cos(\phi + \xi_i) \quad (16)$$

where ξ_i states for the random variable modelling the angles for the various scatters. Fig. 2 represents the antennas disposition with the scatters representation, where $\xi_{1\nu}$ and $\xi_{2\nu}$ refer to scatter ν for transmit antenna #1 and scatter ν for transmit antenna #2 respectively. The signals arriving at the receive antenna

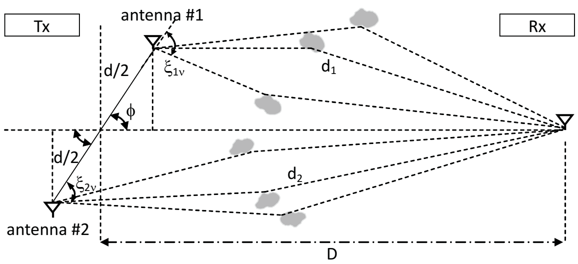

Figure 2: Antennas' physical disposition: two transmit and one receive antennas in a scattered environment.

from each transmit antenna must be appropriately statistically modeled in order to obtain a proper model.

Considering equation (14), assuming far field communication conditions and substituting (16) into (14) the correlation coefficient becomes:

$$\rho = E\{\mathrm{e}^{-\frac{\mathrm{j}2\pi d\cos(\phi+\xi_i)}{\lambda}}\} = E\{\mathrm{e}^{-\mathrm{j}2\pi d_\lambda \cos(\phi+\xi_i)}\} \ , \quad (17)$$

where ξ_i represents the arriving scatter angle deviation from the shortest angle (that corresponds to the LOS when feasible) and $d_\lambda = d/\lambda$ is the antennas separation in wavelengths. The computation of the expectation in (17) is given by:

$$\rho(\phi,\xi) = \int_{-\infty}^{\infty} e^{-\mathrm{j}2\pi d_\lambda \cos(\phi+\xi)}\, p(\xi)\,\mathrm{d}\xi \ , \quad (18)$$

where $p(\xi)$ is the probability distribution function (pdf) of the scatters' angles ξ_i described previously. An appropriate pdf should be defined for the random variable ξ, also called spread angle. It is reasonable to assume that scatters most concentrate around the mean of the scatters angles. In consequence a normal distribution with mean $\mu = 0$ and variance σ_ξ^2 looks appropriate for this purpose. Under this assumption, the pdf can be expressed as

$$p(\xi) = \frac{1}{\sqrt{2\pi}\sigma_\xi}\mathrm{e}^{-\frac{\xi^2}{2\sigma_\xi^2}} \ . \quad (19)$$

This model considers that most of the scatters concentrate around the shorter distance one and the probability of having scatters with large spread angles is low. Analysing (18) and (19), the term $\cos(\phi+\xi)$ in (18) can be developed as:

$$\cos(\phi+\xi) = \cos(\phi)\cos(\xi) - \sin(\phi)\sin(\xi) \ . \quad (20)$$

In the case ξ is small enough, i. e., $\cos(\xi) \approx 1$ and $\sin(\xi) \approx \xi$, (20) can be approximated by:

$$\cos(\phi+\xi) = \cos(\phi) - \xi\sin(\phi) \ . \quad (21)$$

Substituting (21) into (18) leads to:

$$\rho(\phi,\xi) = \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{j}2\pi d_\lambda(\cos(\phi)-\xi\sin(\phi))}\, p(\xi)\,\mathrm{d}\xi \ , \quad (22)$$

where the complex exponential can be separated as the product of two terms, one that doesn't depend on ξ and hence (22) can be rewritten as:

$$\rho(\phi,\xi) = \mathrm{e}^{-\mathrm{j}2\pi d_\lambda \cos(\phi)} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{j}2\pi d_\lambda \xi \sin(\phi)}\, p(\xi)\,\mathrm{d}\xi \ . \quad (23)$$

Taken under consideration that:

$$\int_{-\infty}^{\infty} \mathrm{e}^{-\frac{1}{2}a\cdot x^2 + \mathrm{j}bx} dx = \left(\frac{2\pi}{a}\right)^{(1/2)} \cdot \mathrm{e}^{-\frac{b^2}{2a}} \quad (24)$$

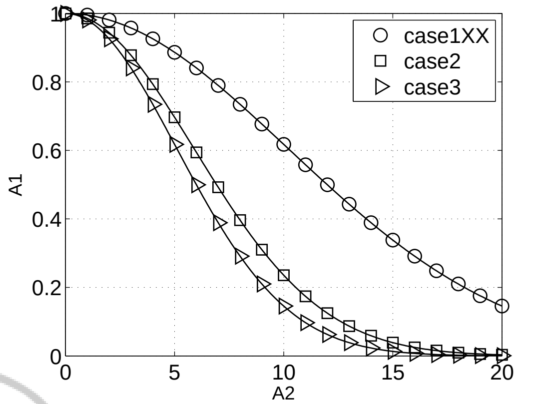

Figure 3: Dependency of $|\rho(\phi,\sigma_\xi)|$ as a function of σ_ξ and ϕ assuming an antennas separation in wavelengths of $d_\lambda = 1/32$.

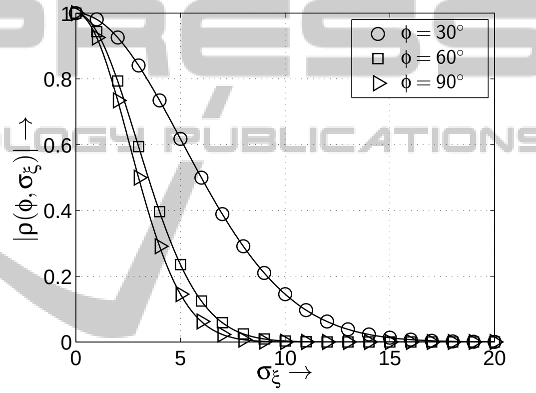

Figure 4: Dependency of $|\rho(\phi,\sigma_\xi)|$ as a function of σ_ξ and ϕ assuming a wavelength specific antenna separation of $d_\lambda = 1/16$.

and identifying $b = 2\pi d_\lambda \sin(\phi)$ and $a = 1/\sigma_\xi^2$, finally (23) becomes

$$\rho(\phi,\sigma_\xi) = \mathrm{e}^{-\mathrm{j}2\pi d_\lambda \cos(\phi)}\, \mathrm{e}^{-\frac{1}{2}(2\pi d_\lambda \sin(\phi)\,\sigma_\xi)^2} \ . \quad (25)$$

Equation (25) allows determining the complex correlation coefficient for a pair of antennas. This result can be extended to multiple antennas and various space antennas distributions.

Fig. 3 depicts the modulus of the correlation coefficient for an antennas separation $d_\lambda = 1/32$ (wavelengths) for various departure angles as a function of the spread angle standard deviation σ_ξ. For a given departure angle, the correlation coefficient modulus increases as the spread angle decreases. A lower value of σ_ξ means that the scatters concentrate in a narrower space and the correlation increases. On the other hand, for a given spread angle, the correlation coefficient modulus decreases with the departure angle. Fig. 4 and 5 represent the correlation coefficient modulus for $d_\lambda = 1/16$ and $d_\lambda = 1/8$ respectively. The

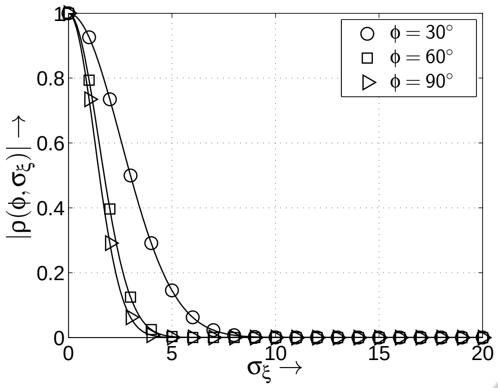

Figure 5: Dependency of $|\rho(\phi, \sigma_\xi)|$ as a function of σ_ξ and ϕ assuming a wavelength specific antenna separation of $d_\lambda = 1/8$.

effects of the departure angle and spread angle on the correlation coefficient are the same as those described in Fig. 3. Now, comparing Fig. 3–5 the effect of antennas separation can be analysed. It can be noticed that for the same spread angle and the same departure angle the correlation coefficient modulus increases as the antennas become closer.

3 MIMO SYSTEM MODEL

It is quite common to assume that the coefficients of the $(n_R \times n_T)$ channel matrix $\mathbf{H}$ are independent and Rayleigh distributed with equal variance. However, in many cases correlations between the transmit antennas as well as between the receive antennas can't be neglected. The way to include the antenna signal correlation into the MIMO channel model with n_T transmit and n_R receive antennas for Rayleigh flat-fading like channels is given by (Oestges, 2006) and results in

$$\text{vec}(\mathbf{H}) = \mathbf{R}_{\text{HH}}^{\frac{1}{2}} \cdot \text{vec}(\mathbf{G}) \quad (26)$$

where $\mathbf{G}$ is a $(n_R \times n_T)$ uncorrelated channel matrix with independent, identically distributed complex Rayleigh distributed elements and $\text{vec}(\cdot)$ being the operator stacking the matrix $\mathbf{G}$ into a vector column-wise. The matrix $\mathbf{R}_{\text{HH}}$ describing the correlation within the channel coefficients $h_{\nu,\mu}$ (with $\nu = 1, \ldots, n_R$ and $\mu = 1, \ldots, n_T$) is defined as

$$\mathbf{R}_{\text{HH}} = \text{E}\left\{\text{vec}(\mathbf{H}) \cdot \text{vec}(\mathbf{H})^{\text{H}}\right\} \quad (27)$$

with $\text{vec}(\mathbf{H})$ resulting exemplarily for the considered (2×2) MIMO system in

$$\text{vec}(\mathbf{H}) = \begin{pmatrix} h_{1,1} \\ h_{2,1} \\ h_{1,2} \\ h_{2,2} \end{pmatrix} . \quad (28)$$

Assuming that the correlation introduced by the antenna elements at the transmitter side is independent from the correlation introduced by the antenna elements at the receiver side, the correlation matrix can be defined over the transmitter side correlation matrix $\mathbf{R}_{\text{TX}}$ as well as the receiver side correlation matrix $\mathbf{R}_{\text{RX}}$. In this case the matrix $\mathbf{R}_{\text{HH}}$ results in

$$\mathbf{R}_{\text{HH}} = \mathbf{R}_{\text{TX}} \otimes \mathbf{R}_{\text{RX}} \quad (29)$$

where $\otimes$ represents the Kronecker product.

For the exemplarily investigated (2×2) MIMO system, the transmitter side correlation matrix $\mathbf{R}_{\text{TX}}$ is given by

$$\mathbf{R}_{\text{TX}}^{(2\times2)} = \begin{pmatrix} \rho_{1,1}^{(\text{TX})} & \rho_{1,2}^{(\text{TX})} \\ \rho_{2,1}^{(\text{TX})} & \rho_{2,2}^{(\text{TX})} \end{pmatrix} = \begin{pmatrix} 1 & \rho^{(\text{TX})} \\ \rho^{*(\text{TX})} & 1 \end{pmatrix} \quad (30)$$

and describes the correlation between the transmit antennas k and ℓ, independent from the receive antenna m. The transmitter side correlation coefficient between the transmit antennas k and ℓ is obtained as

$$\rho_{k,\ell}^{(\text{TX})} = E\{h_{m,k} \cdot h_{m,\ell}^*\} \quad (31)$$

It should be taking under consideration that the value of the correlation coefficient depends on the reference antenna. That is, the correlation coefficient between antenna ℓ and antenna k is given by:

$$\rho_{\ell,k}^{(\text{TX})} = E\{h_{m,\ell} \cdot h_{m,k}^*\} = \rho_{k,\ell}^{*(\text{TX})} . \quad (32)$$

Hence in the correlation matrix the symmetric elements with respect the main diagonal are complex conjugated. This relationship is due to the sign change when computing the distance difference between antennas with different antenna reference.

In this work it is assumed that no correlation between the antennas at the receiver side appears. Under this assumption the receiver side correlation matrix $\mathbf{R}_{\text{RX}}$ results in

$$\mathbf{R}_{\text{RX}}^{(2\times2)} = \begin{pmatrix} \rho_{1,1}^{(\text{RX})} & \rho_{1,2}^{(\text{RX})} \\ \rho_{2,1}^{(\text{RX})} & \rho_{2,2}^{(\text{RX})} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (33)$$

The receiver side correlation matrix $\mathbf{R}_{\text{RX}}$ describes the correlation between the receive antennas m and n, independent from the receive antenna k. The receiver side correlation coefficient between the receive antennas m and n can be calculated as follows

$$\rho_{m,n}^{(\text{RX})} = E\{h_{m,k} \cdot h_{n,k}^*\} . \quad (34)$$

Finally, the overall correlation matrix $\mathbf{R}_{\text{HH}}$ with the elements

$$\mathbf{R}_{\text{HH}}^{(2\times2)} = \begin{pmatrix} \rho_{1,1,1,1} & \rho_{1,1,1,2} & \rho_{1,2,1,1} & \rho_{1,2,1,2} \\ \rho_{1,1,2,1} & \rho_{1,1,2,2} & \rho_{1,2,2,1} & \rho_{1,2,2,2} \\ \rho_{2,1,1,1} & \rho_{2,1,1,2} & \rho_{2,2,1,1} & \rho_{2,2,1,2} \\ \rho_{2,1,2,1} & \rho_{2,1,2,2} & \rho_{2,2,2,1} & \rho_{2,2,2,2} \end{pmatrix} \quad (35)$$

results in

$$\mathbf{R}_{\mathrm{HH}}^{(2\times 2)} = \begin{pmatrix} 1 & 0 & \rho^{(\mathrm{TX})} & 0 \\ 0 & 1 & 0 & \rho^{(\mathrm{TX})} \\ \rho^{*(\mathrm{TX})} & 0 & 1 & 0 \\ 0 & \rho^{*(\mathrm{TX})} & 0 & 1 \end{pmatrix} . \quad (36)$$

Therein, the elements $\rho_{m,n,k,\ell}$ of the overall correlation matrix $\mathbf{R}_{\mathrm{HH}}$ are given by the following equation

$$\rho_{m,n,k,\ell} = E\{h_{m,k} \cdot h_{n,\ell}^{*}\} = \rho_{k,\ell}^{(\mathrm{TX})} \cdot \rho_{m,n}^{(\mathrm{RX})} \ . \quad (37)$$

When considering a non-frequency selective SDM (space division multiplexing) MIMO link composed of n_{T} transmit and n_{R} receive antennas, the system is modelled by

$$\mathbf{u} = \mathbf{H} \cdot \mathbf{c} + \mathbf{w} \ . \quad (38)$$

In (38), $\mathbf{u}$ is the $(n_{\mathrm{R}} \times 1)$ received vector, $\mathbf{c}$ is the $(n_{\mathrm{T}} \times 1)$ transmitted signal vector containing the complex input symbols and $\mathbf{w}$ is the $(n_{\mathrm{R}} \times 1)$ vector of the additive, white Gaussian noise (AWGN). The interference between the different antenna's data streams, which is introduced by the non-diagonal channel matrix $\mathbf{H}$, requires appropriate signal processing strategies. Common strategies for separating the data streams are linear equalization at the receiver side or linear pre-equalization at the transmitter side, if channel state information is available. Unfortunately, linear equalization suffers from noise enhancement and linear pre-equalization of the transmit signal from an increase in the transmit power. Both schemes only offer poor power efficiency. Therefore, other signal processing strategies have attracted a lot of interest. Another popular technique is based on the singular value decomposition (SVD) (Haykin, 2002) of the system matrix $\mathbf{H}$, which can be written as $\mathbf{H} = \mathbf{S} \cdot \mathbf{V} \cdot \mathbf{D}^{\mathrm{H}}$, where $\mathbf{S}$ and $\mathbf{D}^{\mathrm{H}}$ are unitary matrices and $\mathbf{V}$ is a real-valued diagonal matrix of the positive square roots of the eigenvalues of the matrix $\mathbf{H}^{\mathrm{H}}\mathbf{H}$ sorted in descending order[1]. The SDM MIMO data vector $\mathbf{c}$ is now multiplied by the matrix $\mathbf{D}$ before transmission. In turn, the receiver multiplies the received vector $\mathbf{u}$ by the matrix $\mathbf{S}^{\mathrm{H}}$. Thereby neither the transmit power nor the noise power are enhanced. The overall transmission relationship is defined as

$$\mathbf{y} = \mathbf{S}^{\mathrm{H}}\left(\mathbf{H} \cdot \mathbf{D} \cdot \mathbf{c} + \mathbf{w}\right) = \mathbf{V} \cdot \mathbf{c} + \tilde{\mathbf{w}} . \quad (39)$$

Here, the channel matrix $\mathbf{H}$ is transformed into independent, non-interfering layers having unequal gains. When applying the proposed system structure, the SVD-based equalization leads to different weighted AWGN channels, where the weighting factor $\sqrt{\xi_{\ell,k}}$

[1] The transpose and conjugate transpose (Hermitian) of $\mathbf{D}$ are denoted by $\mathbf{D}^{\mathrm{T}}$ and $\mathbf{D}^{\mathrm{H}}$, respectively.

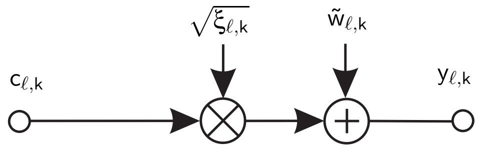

Figure 6: Resulting system model per MIMO layer ℓ and transmitted data block k.

represents the positive square roots of the eigenvalues of the matrix $\mathbf{H}^{\mathrm{H}}\mathbf{H}$ for the transmitted SDM data block k (Fig. 6). The number of readily separable layers is limited by $\min(n_{\mathrm{T}}, n_{\mathrm{R}})$.

4 RESULTS

For the performance analysis two different MIMO configurations are studied: Within the (2×2) MIMO system, the transmitter-side correlation matrix $\mathbf{R}_{\mathrm{TX}}$ is given according to (30), whereas in the (4×4) MIMO system it is assumed that correlation appears only between neighbouring antennas. The four antennas at the transmitter side are linearly disposed and uniformly distributed with a separation of d_{λ}. Fig. 7 shows the physical layout of the transmit antennas with respect to one of the receive antennas where antennas are numbered in increasing order. The discussion developed below can be applied to any receive antenna.

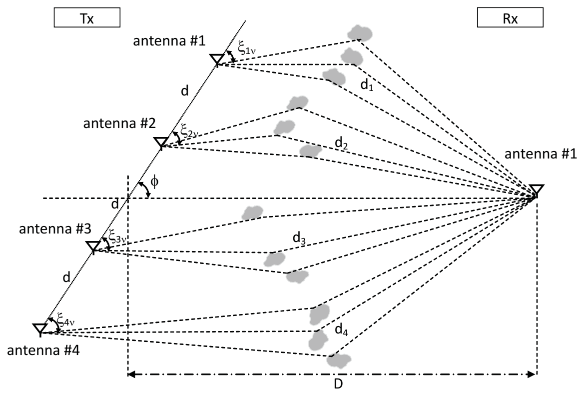

Figure 7: Antennas' physical disposition: (4×4) MIMO with neighbour transmit antennas correlation.

From Fig. 7 various equations can be obtained for each pair of transmit antennas. For antennas #1 and #2 and the antennas physical layout it is obtained:

$$d_1 \cdot \cos(\alpha_1) = D - \frac{3}{2} d \cdot \cos(\phi) \quad (40)$$

and

$$d_2 \cdot \cos(\alpha_2) = D - \frac{d}{2} \cdot \cos(\phi) \ . \quad (41)$$

By combining (40) and (41) the following relation is obtained:

$$d_2 \cdot cos(\alpha_2) - d_1 \cdot cos(\alpha_1) = d \cdot \cos(\phi) \quad . \qquad (42)$$

For neighbour antennas #2 and #3 it is obtained equation (41) and:

$$d_3 \cdot \cos(\alpha_3) = D + \frac{d}{2} \cdot \cos(\phi). \qquad (43)$$

By combining (41) and (43) the following relation is obtained:

$$d_3 \cdot \cos(\alpha_3) - d_2 \cdot \cos(\alpha_2) = d \cdot \cos(\phi) \quad . \qquad (44)$$

Finally, for neighbour antennas #3 and #4 it is obtained equation (43) and:

$$d_4 \cdot \cos(\alpha_3) = D + \frac{3}{2} d \cdot \cos(\phi) \quad . \qquad (45)$$

Finally, in a similar way than in previous antennas pairs, by combining (43) and (45) the following relation is obtained:

$$d_4 \cdot \cos(\alpha_4) - d_3 \cdot \cos(\alpha_3) = d \cdot \cos(\phi) \quad . \qquad (46)$$

Once the physical description and relations of the antennas layout has been described the next step is computing the neighbour antennas correlation following the steps described in the (2×2) MIMO set-up. Considering the separation between transmit and receive antennas is large compared with the transmit antennas separation, i.e., $D >> d$ then it can be assumed that $\alpha_1 \approx \alpha_2 \approx \alpha_3 \approx \alpha_4 \approx \psi/4$ where ψ is called the *spread angle*. In consequence (42), (44) and (46) can be respectively expressed as:

$$(d_2 - d_1) \cdot \cos(\psi/4) = d \cdot \cos(\phi) \quad , \qquad (47)$$

$$(d_3 - d_2) \cdot \cos(\psi/4) = d \cdot \cos(\phi) \quad , \qquad (48)$$

and

$$(d_4 - d_3) \cdot cos(\psi/4) = d \cdot cos(\phi) \quad . \qquad (49)$$

Furthermore, if $D >> d$ then $cos(\psi/4) \approx 1$ and the equations above can be further simplified. Let's consider antennas #1 and #2. The correlation coefficient is given according to (6) by:

$$\rho_{12}^{(\mathrm{TX})} = \frac{E\{s_{\mathrm{r}1}(t) \cdot s_{\mathrm{r}2}^*(t)\}}{\sqrt{E\{|s_{\mathrm{r}1}(t)|^2\}} \cdot \sqrt{E\{|s_{\mathrm{r}2}(t)|^2\}}} \quad . \qquad (50)$$

where $s_{\mathrm{r}1}(t)$ and $s_{\mathrm{r}2}(t)$ are the signals received from antennas #1 and #2 respectively. The received signals covariance, e. g. $E\{s_{\mathrm{r}1}(t) \cdot s_{\mathrm{r}2}^*(t)\}$ is given by

$$E\{s_{\mathrm{r}1}(t) \cdot s_{\mathrm{r}2}^*(t)\} = A^2(D) \cdot \mathrm{e}^{-\mathrm{j}2\pi(d_1-d_2)/\lambda} \quad . \qquad (51)$$

where it was considered that $d_1 \approx d_2 \approx D$ and hence $A(d_1) \approx A(d_2) \approx A(D)$. Besides it was considered that antennas are isotropic with unity gain. The signals standard deviations are given respectively by

$$\sqrt{E\{|s_{\mathrm{r}1}(t)|^2\}} = \sqrt{A^2(d_1) \cdot G_1^2} = A(D) \qquad (52)$$

and

$$\sqrt{E\{|s_{\mathrm{r}2}(t)|^2\}} = \sqrt{A^2(d_2) \cdot G_2^2} = A(D) \quad . \qquad (53)$$

It was assumed that the same signal $s(t)$ is a zero mean unitary power signal and it was transmitted from each transmit antenna signal and hence $s_{\mathrm{r}1}(t)$ and $s_{\mathrm{r}2}(t)$ are zero mean valued variables. Substituting (51), (52) and (53) in (50) it is finally obtained:

$$\rho_{12}^{(\mathrm{TX})} = \frac{A^2(D) \cdot \mathrm{e}^{-\frac{\mathrm{j}2\pi(d_1-d_2)}{\lambda}}}{A^2(D)} = \mathrm{e}^{-\frac{\mathrm{j}2\pi(d_1-d_2)}{\lambda}} \quad . \qquad (54)$$

The result obtained in (54) can be extended to any pair of neighbour antennas. Finally, considering the result in equations (47), (48) and (49), the correlation coefficients between neighbour antennas are given by:

$$\rho_{12}^{(\mathrm{TX})} = \rho_{23}^{(\mathrm{TX})} = \rho_{34}^{(\mathrm{TX})} = \mathrm{e}^{\frac{-\mathrm{j}2\pi d_\lambda \cdot \cos(\phi)}{\cos(\psi/4)}} \quad . \qquad (55)$$

where $d_\lambda = d/\lambda$ is the antennas separation in wavelength units. Further simplifications in can be performed considering that in practice $D >> d$ and finally $\cos(\psi/4) \approx 1$. By assuming a far field communication, the term (i. e. $\cos(\psi/4)$ can be approximated to unity assuming ψ is close to zero). Under this assumption, the correlation coefficients result in:

$$\rho_{12}^{(\mathrm{TX})} = \rho_{23}^{(\mathrm{TX})} = \rho_{34}^{(\mathrm{TX})} = \mathrm{e}^{-\mathrm{j}2\pi d_\lambda \cos(\phi)} \quad . \qquad (56)$$

The transmit antennas correlation matrix is then given by:

$$\mathbf{R}_{\mathrm{TX}}^{(4\times4)} = \begin{pmatrix} 1 & \rho_{12}^{(\mathrm{TX})} & 0 & 0 \\ \rho_{21}^{(\mathrm{TX})} & 1 & \rho_{23}^{(\mathrm{TX})} & 0 \\ 0 & \rho_{32}^{(\mathrm{TX})} & 1 & \rho_{34}^{(\mathrm{TX})} \\ 0 & 0 & \rho_{43}^{(\mathrm{TX})} & 1 \end{pmatrix} \quad . \qquad (57)$$

The obtained results are so far focussed on line-of-sight propagation. However, wireless channels require scattering conditions to be taken into consideration. Following the same procedure, as introduced earlier with the (2×2) MIMO link, the neighbour antennas correlation coefficients are given by

$$\rho_{(k,\ell)}^{(\mathrm{TX})}(\phi,\xi) = \mathrm{e}^{-\mathrm{j}2\pi d_\lambda \cos(\phi)} \cdot \mathrm{e}^{-\frac{1}{2}(2\pi d_\lambda \sin(\phi)\sigma_\xi)^2}. \qquad (58)$$

where (k,ℓ) takes the values $(1,2)$, $(2,3)$ and $(3,4)$ corresponding to the four transmit antennas. Furthermore, reciprocity can be assumed, i. e. $\rho_{(k,\ell)}^{(\mathrm{TX})} = \rho_{(\ell,k)}^{*(\mathrm{TX})}$.

In this paper it is analysed how the antennas correlation impact the MIMO link performance focussing on the transmitter-side antennas correlation effect. The correlation coefficients depends on the antennas spacing, the signal departure angle respect to the array axis and the spread angle concerning the scatters dispersion. It has been shown (see Fig. 3– 5) that the correlation effect increases as the antennas' separation diminishes. Furthermore, the correlation effect diminishes as the departure angle increases. On the other hand, the larger the spread angle the lower the correlation coefficient.

As the singular values decomposition is used to pre- and post-processing of the system signals in order to avoid inter-antenna interferences, the impact of antennas correlation on the singular values has been analysed (see Fig. 8–9). By applying the SVD, the MIMO channel can be described as multiple independent SISO channels (so called layers) with different gains (given by the corresponding singular values). For the same noise power at the receive antenna, the larger the singular value the higher is the SISO channel reliability. The ideal situation is when all singular values are equal. The apparition of predominant layers (high valued singular value) is accompanied by weak layers (low valued singular value). As highlighted in Fig. 8 and 9, the difference between the the smallest and the largest layer-specific singular value becomes smaller as the antennas correlation increases. The antennas correlation effect increases the probability of having predominant layers.

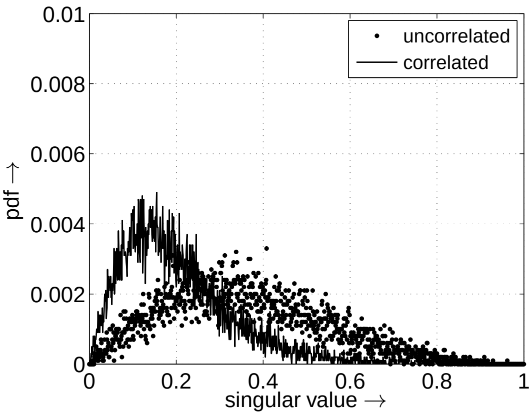

Figure 8: PDF (probability density function) of the ratio ϑ between the smallest and the largest singular value for correlated (solid line) as well as uncorrelated (dotted line) frequency non-selective (2×2) MIMO channels ($d_\lambda = 1/32$, $\phi = 30^\circ$ rad and $\sigma_\xi = 1{,}0$).

In order to show the distribution of the layer-specific characteristic properly, the CCDF (complementary cumulative distribution function) is used (see

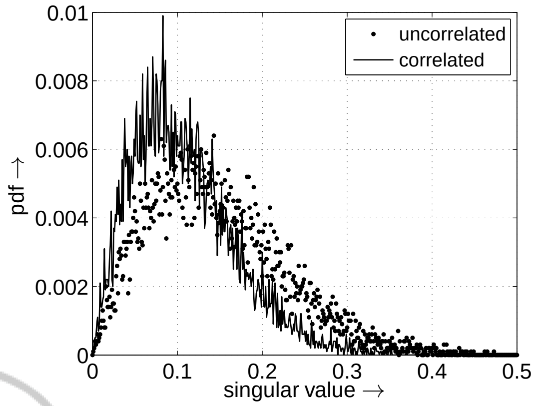

Figure 9: PDF (probability density function) of the ratio ϑ between the smallest and the largest singular value for correlated (solid line) as well as uncorrelated (dotted line) frequency non-selective (4×4) MIMO channels ($d_\lambda = 1/32$, $\phi = 30^\circ$ rad and $\sigma_\xi = 1{,}0$).

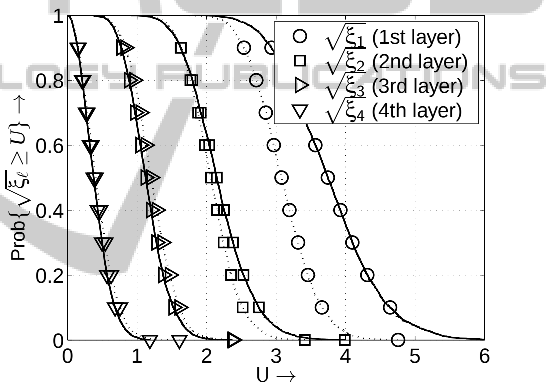

Figure 10: CCDF of the layer-specific distribution for correlated (solid line) as well as uncorrelated (dotted line) frequency non-selective (4×4) MIMO channels ($d_\lambda = 1/32$, $\phi = 30^\circ$ rad and $\sigma_\xi = 1{,}0$).

Fig. 10). It can be noticed the different effect on strong and weak layers. The antennas correlation increases the probability of having layers with larger values (see layers $\sqrt{\xi_1}$ and $\sqrt{\xi_2}$) and increases for weak layers the probability of having lower values (see layers $\sqrt{\xi_3}$ and $\sqrt{\xi_4}$).

5 CONCLUSIONS

Due to the proximity of transmitter and receiver side antenna arrays the theoretically possible potential of MIMO is significantly reduced by correlation. As shown by computer simulations, under the effect of correlation, the influence of layers with high weighting factors becomes even stronger whereas the influence of layer with low weighting factors diminished.

Our results show that not necessarily all layers might be used for the data transmission even when the wave-propagation between the different pairs of transmit and receive antennas is affected by correlation.

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