Improving Network Performane
Management of Nonlinear Dynamics

Seyed Shahrestani
School of Computing and Mathematics, University of Western Sydney, Sydney, Australia

Keywords: AQM/RED, Bifurcation Analysis, Delay Control, Internet, Nonlinear Dynamics.

Abstract: To manage the huge amount of traffic that is to be carried using the limited bandwidth and other resources, large networks and the Internet are heavily dependent on the use of protocols, and in particular, on TCP/IP protocol suite. While the utilization of TCP/IP is of significant practical value, for most large complex networks it can be inefficient, as it fails to fully take into consideration the importance of the major parts of the system. To overcome this, more complicated congestion control mechanisms, such as AQM/RED are widely utilized. However, these complex mechanisms exhibit nonlinear dynamics, which are not well understood and are usually unaccounted for. As a result of this, to avoid oscillatory behavior or loss of stability, the parameters of these systems are often set too conservatively. In turn, this will lead to unnecessary underutilization of the network resources. On the other hand, through the analysis and management of nonlinearities, the operability regions for the networked systems can be expanded, while its performance is also improved. This paper presents our visionary works of applying these ideas to networked systems, resulting in higher loading and throughput, and avoiding oscillatory or unstable behavior.

1 INTRODUCTION

The explosive growth of the Internet has provided modern societies with many new opportunities and challenges. Apart from the two end-systems, invariably for communications outside a LAN, routers play dominant roles in establishing the communication paths. In general, given the store-forward nature of the Internet, queuing and delay of packets are inevitable.

Obviously, the utilized transport protocols play a major role in delay and reliability of communications of the packets. However, it is well established that for most large networks this simplistic utilization of TCP/IP can result in severe inefficiencies (Fan, 2010). Such approaches tend to ignore to fully take into consideration the importance of the major parts of the system, namely the routers and the links or the network itself.

To overcome such inefficiencies, many approaches have been proposed and implemented. Of the well established approaches, among these, is the so-called Active Queue Management (AQM). Random Early Detection (RED) is the most widely used AQM scheme (Hollot, 2002). While the concepts behind RED mechanism are very straightforward, its interfaces with TCP can lead to nonlinear dynamics that are not well understood. The rather complex Internet behavior arising from the existence of inherent nonlinearities can cause instability and oscillatory dynamics. To avoid such undesirable dynamics, in practice the parameters of AQM/RED mechanisms may be set more constrained, compared to what the system is actually capable of. In other words, underutilization of the networks and resources has been a common practice as the system is set to operate below its operation limits.

Nonlinearities of TCP/AQM and the drastic changes of the system behavior that they can cause, even with small loading changes for example, are well established (Chen, 2005). Such variations in behavior and the existence of chaos can in turn be linked to bifurcations in the networked systems (Liu, 2007). Consequently, rather than ignoring the nonlinearities, it is advantageous to exploit them and employ strategies to manage and control the chaotic behavior and bifurcations. This must be based on a global view of the system and can improve performance of the underlying networks, leading to higher loading possibility of the Internet without the oscillatory or unstable behavior (Shahrestani, 2000).
To facilitate the discussion of these points, this paper is organized as follows. The bifurcation and structural stability analysis are introduced in the next section. Such analysis is used to study the performance of the AQM/RED based networks. The results along with strategies to manage such networks to improve their performance are presented in Section III. The conclusions and potential future works are given in the last section.

2 MANAGEMENT OF COMPLEX NETWORKS

Many complex systems, such as large networks, exhibit multiple equilibrium points leading to several potential steady-state operating states. Such systems require a management scheme capable of administration over a wide range of anticipated operating conditions. In developing such schemes, the qualitative changes that occur in the behavior of the system in different operating regions must be taken into account.

The limits, at which qualitative changes in complex system behavior occur, may be related to the structural stability of the system. The structural stability limits of a nonlinear system can in turn be related to bifurcation points in the mathematical model of the system. The bifurcations refer to qualitative changes in the system behavior as some of the system parameters vary quasi-statically (Seydye, 2010).

Complex nonlinear dynamical systems can generally be described by a number of coupled differential-algebraic equations

\[ x = f(t, x, a, w), \]
\[ 0 = \xi(t, x, a, w), \]
\[ y = h(t, x, a, w); \quad x(t_0) = x_0. \quad (1) \]

Where \( t \) is time, \( x(t) \) and \( a \) are the dynamic and instantaneous states of the system and \( w \) represents external influences, such as system input, time varying parameters, disturbances, and the like. Now, consider a single-output case where the variations of the control input \( u(x, a, t) \) and another one of the system parameters \( v(t) \in [v_{\text{min}}, v_{\text{max}}] \), dominate other variations in the system. For example, in networked systems there is a rather clear separation between the time-frames involved in the analysis (and design of the required management actions) for transient congestion and delay and longer-term bandwidth and capacity management consideration (Shahrestani, 2011). To emphasize these points, the model (1) can be put in the following form

\[ x = f(x, a, u, v), \]
\[ 0 = \xi(x, a, u, v), \]
\[ y = h(x, a). \quad (2) \]

A typical network management problem can be considered as identification and setting of system parameters such that a point \( X_d \) is a secure operating point of the system (2). Additionally, there may be constraints on the manageable parameters or the specification of transient characteristics of the network. Naturally, if with \( x = X_d \), no proper manageable and controlled parameters satisfying (2) can be identified, then the network will not be operable steadily at that point.

More generally though, the system state may be considered to be constrained to a certain operating region \( X_d \), containing steady operating points (or in some cases, to some operating region containing secure oscillatory solutions with restrained amplitudes). Consequently, with properly identified parameters at the operating points, the state space region of interest for system (2) can be considered as

\[ X_d = \{ x \mid x_{\text{min}} \leq x \leq x_{\text{max}} \}, \quad (3) \]

while \( v \) varies slowly with time within a certain range of interest

\[ \Omega_d = \{ v \mid v_{\text{min}} \leq v \leq v_{\text{max}} \}, \quad (4) \]

The value of the managed parameters with the network operating at some steady-state, is to be chosen such that the existence of some \( x \in X_d \) for some \( v \in \Omega_d \) is assured. Therefore, with a particular and proper value of \( u_e \), say \( u_e = u_{e,j} \), the network operation constrained to region \( X_d \) may happen to exist for only a range \( \Omega_{e,j} \subset \Omega_d \), defined by

\[ \Omega_{e,j} = \{ v \mid v_{\text{min}} \leq v \leq v_{\text{max}} \} \quad (5) \]

That is, to cover the complete range of interest \( \Omega_d \) in general, multiple sub-ranges may need to be considered, with each sub-range corresponding to a different set of managed parameters with the system at steady-state. These points are illustrated in Figure 1. It can also be noted that each value of \( u_{e,j} \)
corresponds to some reference segment on the parameter dependent steady-state operation manifold of a nonlinear system such as (1).

As pointed out earlier, in general, the region $X_d$ may contain steady-state operating points with different characteristics. That is, as $V$ varies within its limits, the system with the managed parameters being simply set as $u = u_{c,j}$, may exhibit qualitatively different behavior for different levels of $V$. To account for the manager capability in handling the system dynamics appropriately, the managed parameters are now considered as $u = u_{c,j} + \delta u_f$.

The critical values of the slowly changing parameter $V$ that are associated with the boundaries of operating regions with distinct dynamic behaviors, correspond to bifurcation points of the system (2).

In some cases, the management of bifurcating systems is achieved by ensuring that the system operation is such that the bifurcation parameter is always below its critical values, and the bifurcations are ultimately avoided (Shahrestani, 2000). While this approach solves part of the operation problems, it can result in a conservative design with examples of loading margins larger than what is really required. For a bifurcating system exhibiting regions with non-identical structural stability behavior, the bifurcation points may be used to establish the bounds on segments of the state space of the system with different management requirements. Consequently, depending on the ranges of the bifurcation parameter, several regional management schemes will be needed, while each scheme may pursue a different objective. For example, in some range of the bifurcation parameter the manager may force the system to track the existing steady-state operation points while in some other range, the stabilization of the bifurcated solutions may be the primary objective of the managing scheme. These points are further illustrated through management of delay and congestions to improve the network performance, in the next section.

3 SHAPING THE NETWORK BEHAVIOR

To reduce the delay and to achieve improved throughput, many adaptive Random Early Detection (RED) algorithms have been developed and studied. As discussed before, to overcome the shortcomings associated with the linear dropping probability functions originally used by adaptive RED, the utilization of nonlinear adaptive approaches have found widespread acceptance. These approaches are mainly based on Active Queue Management (AQM). As mentioned before, utilization of AQM/RED introduces complex nonlinearities. Such nonlinearities can in turn induce several bifurcated solutions.

Effectively, the RED controller output that provides the feedback to sender, is a probability of drop rate $p(t)$. This probability is a function of average queue length $q(t)$. The nonlinearities of the AQM/RED model are essentially a consequence of the multiplicative characteristics of packet loss and are represented through describing TCP window control mechanism (Rezaie, 2007).

Ignoring secondary effects and with $D$ denoting the propagation delay, it can be shown that $R(t)$, the round trip delay, will be $(\omega/c + D)$. The packet drop probability, as a function of queue length $q(t)$, can then be put in the following form (Raina, 2005).

\[
p(q) = \begin{cases} 
0, & q \leq q_{Min} \\
\frac{q(t) - q_{Min}}{q_{Max} - q_{Min}}, & q_{Min} < q < q_{Max} \\
1, & q \geq q_{Max}
\end{cases}
\]  

(6)
The TCP/AQM and the router queuing dynamics can be described by the following set of differential equations (Misra, 2000) and (Ranjan, 2004).

\[
\omega = \frac{1}{R(t)} - \frac{\omega(t)\omega(t - R(t))}{2(t - R(t))} p(t - R(t))
\]

\[
q(t) = \begin{cases} 
  n(t) + R(t) & \text{if } c > 0 \\
  \max[0, c - n(t)/R(t)] & \text{if } c = 0.
\end{cases}
\]

In this model, \(p(t)\) is the probability of packet drop within the closed interval \([0, 1]\), \(R\) is the round trip delay in seconds, \(\omega\) is the average TCP window size in packets, \(q(t)\) is the average length of the queue in packets, \(c\) is the link capacity in packets per second, and \(n\) is the load factor or the number of TCP sessions.

In most network management analysis, it is assumed that the load \(n(t)\) and the round trip delay \(R(t)\) are time-invariant. Given their slow variations compared to other network parameters, this is usually a reasonable assumption. But it is more realistic to consider them varying slowly with time. This is in line with bifurcation analysis approaches.

With these considerations, the second set of differential equations (7) can be normalized by noting that the dropping probability \(p(t)\) is proportional to the queue length. The normalization can be carried out by using the following substituting in those equations (Rezaie, 2007).

\[
t_{current} = t_{old} / R,
\]

\[
Q = \frac{q}{n},
\]

\[
p(t) = \frac{Kq(t)}{n},
\]

\[
C = \frac{Rc}{n}.
\]

These substitutions will result in the following normalized equations.

\[
\omega = 1 - \frac{\omega(t)\omega(t - 1)}{2} KQ(t - 1)
\]

\[
\dot{Q}(t) = \begin{cases} 
  \frac{\omega(t) - c}{\max[0, \omega(t) - c]} & \text{if } Q > 0 \\
  0 & \text{if } Q = 0.
\end{cases}
\]

The system described by the normalized differential equations in (9) has a unique equilibrium point, \((c, 2/(Kc^2))\). Due to space limitations, only the results of bifurcation analysis of this model are presented here. The eigenvalue analysis of this system shows that a zero eigenvalue occurs for \(K = 0\). This corresponds to one of the bifurcation points, at which the system behavior changes and bifurcated solutions are expected to emerge.

It can be noted that generally speaking, most of the RED parameters are set based on network manager experiences or at best based on experiential data. This may lead to very conservative set-ups to avoid instability or cyclic behavior similar to those discussed so far and portrayed in Figure 2.

The choice of the RED parameters, selected by the network administrator, dictates the value of \(K\). In other words, if the network manager sets the RED parameters in a way that \(K > 0\), growing oscillatory solutions can be expected. Obviously the growing oscillations

![Figure 2: Unstable oscillatory behavior, at network loading near but after a bifurcation point.](image1)

![Figure 3: Growing oscillatory behavior, at network loading near a bifurcation point.](image2)
lead to unstable operation of the system and collapse of the network. The expected system behavior for values of $K$ just after $K_S$ will be similar to that shown in Figure 2. $K_S$ corresponds to a subcritical Hopf bifurcation of the original nonlinear system describing AQM/RED. In the Hopf bifurcation a branch of stable periodic solutions originates, which again becomes unstable after a period doubling bifurcation. The sequence of period doubling bifurcations ultimately leads to chaos. For more details see (Shahrestani, 2000). For the chaotic situation, the unstable oscillatory behavior that the network exhibits will be dependent on its initial state. The behavior will be similar to those depicted in Figure 4.

Clearly, more analytical approaches for selection of RED controller parameters and analysis of their effects on the network performance will be advantageous. In our previous works, we have developed a framework for design of management schemes and control laws for a parameter dependent complex nonlinear system (Shahrestani, 2000) and (Shahrestani, 2008).

![Figure 4: Unstable oscillatory behavior.](image)

The result is a global multilevel management and control scheme, where the first level depending on signal and loading levels switches the parameters, so that the system states are confined to the neighborhood of some desired reference segment. For each reference segment, the information gained through bifurcation analysis is used for further segmentation of the state space of the system, similar to that shown in Figure 1.

To move the eigenvalues, $\lambda$, $\lambda_{j,k}^i = -\alpha + j\beta$ to $\hat{\lambda}_{j,k}^i = -(\alpha + \varepsilon) + j\beta$ (10)

to locations, $\hat{\lambda}$ corresponding to more desirable behavior and operations, the so-called modal control approach will require the feedback

$$\delta u^i = -\sum_{n=1}^{2} \left( k_n^i (l_n^i)^T \right) \delta x;$$

(11)

where the gains are

$$k_j^i = \frac{(\lambda_j^i - \hat{\lambda}_j^i)(\lambda_j^i - \lambda_j^0)}{(l_j^i)^T b(\lambda_j^i - \lambda_j^0)} = \frac{c}{(l_j^i)^T b} \left( 1 - \frac{\varepsilon}{2\beta} \right),$$

(12)

$$k_j^i = \frac{c}{(l_j^i)^T b} \left( 1 + \frac{\varepsilon}{2\beta} \right).$$

With such modal control, the response of the system with the situation the same as that shown in Figure 4 will be improved to the stabilized behavior shown in Figure 5.

![Figure 5: Stabilized behavior through modal control, corresponding to the response shown in Figure 4.](image)

In our previous works we have also shown that only the feedback of critical variables up to cubic terms may have any effect on the existence of a Hopf bifurcation or changing the stability behavior of the bifurcated solutions. Obviously, even for a system with controllable modes only linear terms have any effect on the location of the eigenvalues. Consequently, to change a subcritical bifurcation to a supercritical one, quadratic and/or cubic (critical) state feedback can be identified. These relate to improving the network behavior, when the dynamics exhibit oscillatory behavior. Figure 6 for instance, shows the effect of cubic state feedback for the same situation depicted in Figure 4.

![Figure 6: Stabilized behavior by cubic state feedback, corresponding to the response shown in Figure 4.](image)
4 CONCLUDING REMARKS

In this paper, some approaches to improve the performance of complex networks are proposed. These are based on the analysis and management of the system nonlinear dynamics and bifurcations. By managing the bifurcations, performance of these networks can be improved while their operability region can also be expanded. We also reported the works in-progress towards applying these ideas to establish a more analytical management scheme for networked systems. Choosing and regulating the parameters, based on these types of analysis and management and their utilization in communication systems and the Internet can result in expanding their stability and operability regions, for instance over a wide range of loading, throughput, delay and congestion levels over TCP connections. In our future works, we aim to expand these ideas and validate the analytical results through more experimental works.

REFERENCES