PROCESS CONTROL SYNTHESIS IMPROVED BY STRUCTURAL MODEL PROPERTIES

Hans-Christian Lapp and Hans-Michael Hanisch
Martin Luther University of Halle-Wittenberg, Halle, Germany

Keywords: Discrete Event Systems, Process Control Synthesis, Net Condition/Event Systems, Invariants.

Abstract: This contribution presents a novel approach to synthesize discrete process control on shop-floor level. It takes advantage of the modular composition of the used modeling formalism. The synthesis procedure grounds on the model of the uncontrolled plant behavior, as well as on specifications of forbidden states and desired process behavior. Thus, the presented approach also opens the door to more flexibility, compared to solely forbidden state specifications. The plant model structure is abstracted without loss of information into a novel representation called Transition Invariants Graph. This representation is utilized to extract admissible trajectories out of the reachable state space of the uncontrolled plant. Hence, the introduced approach reduces the complexity during control synthesis procedure significantly by limiting the reachability analysis. That makes it feasible to be used even in real-scale industrial systems.

1 INTRODUCTION

The purpose of formal control synthesis is to automatically generate a controller model, which is proven to be correct. Thus, it reduces the possibility of human errors and eases the design process. Figure 1 shows the scheme of the control synthesis procedure.

Control synthesis grounds on a model of the uncontrolled plant behavior and a specification of desired or undesired behavior, mostly given in terms of plant states (e.g. forbidden states for safety reasons).

Generally, complexity of control synthesis is a crucial factor depending on the size of the plant model. So, much research work has been done to handle the costs and effectiveness of the synthesis procedure. Therefore, many of the existing approaches aim on synthesizing on process control level, assuming that there are underlying controllers on shop-floor level which perform the control tasks of single components.

The approach of Feng et al. (Feng et al., 2009) takes into account the modular component-based product-structure of manufacturing systems to derive a hierarchy of decentralized supervisors and coordinators.

Uzam and Wonham (Uzam and Wonham, 2006) introduced a hybrid approach. They coupled Ramadge and Wonham supervisors (automata) to DES modeled with Petri nets (PN), to close the gap be-

![Figure 1: Scheme of the control synthesis procedure.](image)
properties called siphons and mixed integer programming.

Iordache and Antsaklis gave a survey (Iordache and Antsaklis, 2006) over Supervision based on Place Invariants (SBPI). There, also structural properties of PN, namely place invariants, are used to generate a supervisor.

Another structure-based approach was introduced by Missal and Hanisch, (Missal and Hanisch, 2008a) and (Missal and Hanisch, 2008b). They analyze the pre-regions of forbidden states in plant model’s state space, to synthesize distributed control. This work takes advantage of a modular modelling formalism, the so-called Net Condition/Event Systems (NCES), which have extensions of PN as building blocks.

To reduce complexity during supervisor synthesis, different techniques are used, amongst others, model reduction (Uzam and Wonham, 2006), model abstraction (Feng et al., 2009) or utilization of model structure/properties and mathematical model representation (Iordache and Antsaklis, 2006).

The approach presented in this contribution is about synthesizing distributed controllers on shop-floor level. It extends commonly used specifications in terms of states by specifications of desired recurring process behavior. A lossless abstraction of the plant model structure is generated. This so-called Transition Invariants Graph (TIG) is utilized to extract admissible (in the sense of the specification) causal dependencies from the plant model. Based on both kinds of specifications and the extracted causal dependencies, an admissible subspace of the whole reachable state space is computed. The boundaries of this subspace are the starting points for determining a controller model. Hence, the controller model is determined for meaningful behavior regarding the specifications. The reduction of complexity is in the partial computation of the state space, which is therefore called partial reachability analysis.

This contribution is structured as follows. Section 2 introduces the used modular modeling formalism. The steps of the process control synthesis procedure are presented in Section 3. An example is presented in Section 4, followed by the conclusions.

2 MODELING

For DES and specification modeling, NCES are used, a 1-bounded (and therefore called safe) subclass of Net Condition/Event Systems, which were introduced in (Hanisch and Rausch, 1995). The crucial advantage of NCES is the modular model structure. Recommendations for a well-formed modular plant mod-
Definition 3. If an $\text{sNCE}$ Module has no inputs or outputs it is called safe Net Condition/Event System $\text{NCE}_M$.

An input state is defined for the signal inputs of $\text{sNCE}$ Modules as follows:

Definition 4. The input state is of an $\text{sNCE}$ is a mapping is : $C^\text{in} \cup E^\text{in} \rightarrow \{0,1\}$ assigning a value of $\{0,1\}$ to each signal input.

The semantics of $\text{sNCE}$s are given in terms of steps.
Steps are sets of transitions interconnected via event signals. An event signal synchronizes two transitions in one direction ($t_i \rightarrow t_j$) under the enabling conditions.

To enable a transition or a step, only the marking of places and the input state of a module are of interest.

Definition 5. A transition $t \in T$ of an $\text{sNCE}$ is:
1. marking-enabled at a marking $m$ iff
   $$(\forall p \in P \text{ with } (p,t) \in F : m(p) = 1) \land (\forall p \in P \text{ with } (t,p) \in F : m(p) = 0)$$
2. condition-enabled at a marking $m$ and an input state is iff
   $$\forall p \in P \text{ with } (p,t) \in C : m(p) = 1$$
   $$\forall e^\text{in} \in E^\text{in} \text{ with } (e^\text{in},t) \in C^\text{arc} : is(e^\text{in}) = 1.$$ 

A transition is marking-enabled if all pre-places are marked and all post-places are unmarked. A transition is condition-enabled if all places that are connected via condition arcs are marked and all connected condition inputs have the value one.

With these terms, we can define sets of event-interconnected transitions that are called steps in general and enabled steps in particular.

Definition 6. Let $M$ be an $\text{sNCE}$ with the marking $m$, the input state is and $\xi \subset T$ a non-empty set of transitions within $M$.

$\xi$ is a step iff
1. $|\xi \cap (T_E)\rangle = 1$,
   while $T_E := \{t \in T | (t',t) \in E^N\}$ and
2. for every transition $t \in \xi$ with $t \notin (T_E)$ holds:
   - $\text{em}(t) = \bigvee (\exists e^\text{in} \in E^\text{in} \text{ with } (e^\text{in},t) \in E^\text{arc} : is(e^\text{in}) = 1)$ or
   - $\text{em}(t) = \bigwedge (\forall t' \text{ with } (t',t) \in E^N \text{ and } \exists e^\text{in} \in E^\text{in} \text{ with } (e^\text{in},t) \in E^\text{arc} : is(e^\text{in}) = 1)$
   and
3. all transitions are free of conflicts to each other.

$\Xi$ is the set of steps within $M$.

$\xi$ is called enabled step under $m$ and is iff $\xi$ is marking and condition-enabled under $m$ and is and there is no set of transitions with $\xi' = \xi \cup \{t\}$, which is also a step and marking and condition-enabled under $m$ and is.

The defined enabled steps are always maximal steps and contain exactly one trigger transition. Conflict transitions must not be part of the same step.

The effect of firing an enabled step on the marking of the net is defined as follows.

Definition 7. Let $M$ be an $\text{sNCE}$ with the marking $m$ and the input state is.

If $\xi$ is an enabled step under $m$ and is, then $\xi$ is enabled to fire. The successor marking $m'$ is determined for $p \in M$ to:
\[ m'(p) = \begin{cases} 
  1 & \text{if } \exists t \in \xi : (t, p) \in F \\
  0 & \text{if } \exists t \in \xi : (p, t) \in F \\
  m(p) & \text{else.} 
\end{cases} \]

An enabled transition is forced to fire by an incoming event signal.

### 2.2 The Specification Model

Next to the formal plant model, two kinds of formal specifications are used in the presented approach. The first kind are forbidden states, which are defined in terms of state predicates (Missal and Hanisch, 2006). In (Missal and Hanisch, 2006) and (Missal and Hanisch, 2008a), they are the starting points for a backward search for controllable steps. This previous work is integrated in the approach presented in this contribution.

**Definition 8.** Let \( N \) be an \( sNCES \), \( p \in P \) a place of \( N \) and \( m \) a marking of \( N \). A state atom \( ZA \) of \( N \) at \( m \) and \( p \) is a declaration:

\[ ZA = \{ m(p) = a; \ a \in \{0;1\} \} \]

A state predicate \( ZP \) of \( N \) on \( m \) is a function of state atoms:

\[ ZP = ZA_1 \land ZA_2 \land \cdots \land ZA_n. \]

A state attribute \( ZE \) of \( N \) on \( m \) is a function of state predicates:

\[ ZE = ZP_1 \lor ZP_2 \lor \cdots \lor ZP_n. \]

The second kind are formal specifications of the plant’s desired process behavior, or more precisely, the desired transformation of work piece (WP) properties during a process cycle. Work piece properties can be geometric properties or work piece positions, etc. These specifications are given in terms of \( sNCES \) as partial orders over the desired process behavior. Figure 4 shows an example cutout of such a specification module. It reflects in general a partial order over the work piece presence at different plant locations (positions 1...4). This exemplary order is partial because it only contains the work piece positions. The plant components at each position as well as their activation sequences, which are necessary for work piece processing are not included.

These modules are connected with event arcs to the model of the uncontrolled plant behavior. For example, the lower left event input \( WP \ to next station \) in the module in Figure 4 is intended to receive an event from a corresponding plant component, which signals the transfer of a work piece to the next plant station.

The modularity of \( sNCES \) supports the supposed distributed control synthesis in such a way, that specification modules can be assigned to each relevant plant component or assembly group. This mapping is used during the supposed distribution of the synthesized control. Currently, the \( sNCES \) specification modules are derived manually. But this step can be automated if an unambiguous name assignment from a design framework, e.g. a SysML-based specification (Hirsch, 2010), to the \( sNCES \) model and specification modules is guaranteed.

### 3 PROCESS CONTROL SYNTHESIS

The basis for process control synthesis are formal models of the uncontrolled plant behavior and a formal specification, both modeled in \( sNCES \) and interconnected via event arcs. When determining the control model, the computation of the whole reachable state space of the plant model is avoided. Instead, specification-compliant paths are identified in a new abstracted representation of structural properties of the plant model. These paths are used to limit the reachability analysis to the specified behavior. Thus, we do some kind of partial reachability analysis to determine specification-compliant trajectories through the state space of the plant model. Afterwards, the control model is distributed regarding the plant model architecture.

The formal process control synthesis follows four
steps:
1. Create superset of feasible paths (in model structure abstraction),
2. partial reachability analysis, based on specifications and previously determined path set,
3. determine controllable trajectories,
4. distribution and code generation.
These steps are described in detail in the following subsections.

3.1 Create Superset of Feasible Paths

The specified process is cyclic. Thus, one can assume that every relevant plant component will return into its initial state. This fact is reflected in t-invariants of the PN parts of the $sNCES$ model. Therefore, every single t-invariant of the plant model can be considered as “specification-compliant” in that sense, that it does not contradict to a cyclic process behavior. T-invariants are a well known property of PN (Starke and Roch, 2002). In the following this property will be extended by the signal interconnections of $sNCES$. Thus, it is possible to determine an abstract representation of causal dependencies within the uncontrolled plant model. This representation is called Transition Invariants Graph ($TIG$) (Winkler et al., 2011).

Examples for the transformation of $sNCES$ Modules into $TIG$ representation are given in Figure 5. In detail, Figure 5(a) shows the transformation of nested t-invariants. There, one event arc ($t_2$, $t_1$) is transformed into two arcs in $TIG$ because $t_2$ is part of two nested t-invariants.

Figure 5(b) depicts the transformation of event-synchronized places into $TIG$ representation. Contrary to the foregone example, the two shown event arcs are transformed into one arc in the $TIG$, because the two event arcs are rectified and got identical source and sink t-invariants.

For example, the causal dependencies of an uncontrolled behavior model of a rotary table allow it to rotate from one table position to the next. The causal dependencies representing this behavior would be reflected in the $TIG$. In the sense of the process behavior specification, this would be only a part of the desired behavior. Thus, an admissible path within the $TIG$, in the sense of the process behavior specification, contains the causal dependencies which are reflecting the whole desired process behavior. In the case of the rotary table this might be a complete rotation passing all table positions.

Thus, after determining the $TIG$, a set of admissible paths within the $TIG$ is determined. This path set contains causal dependencies representing the desired process behavior. This extraction of $TIG$ paths grounds on the given specification of desired process behavior. Remember, this specification is given in terms of $sNCES$ and is connected via event arcs to the plant model. Causal dependencies of interest as well as their order can be determined by analyzing the interconnections of process behavior specification and plant model. These interconnections are also reflected in the $TIG$. Hence, they are utilized to extract $TIG$ paths which are complying with the given specification. That means:

- All causal dependencies of interest are included in the extracted paths,
- and they are included in the specified order.

This set of extracted $TIG$ paths is the superset of feasible paths. During the following step, this set is used to limit computational complexity when analyzing the state space of the uncontrolled plant model.

3.2 Partial Reachability Analysis

The purpose of this step is to create a set of specification-fulfilling trajectories through the state space of the plant model, which realizes the desired plant behavior. Dependent on the model of the uncontrolled plant behavior, the state space could be very large and strongly interconnected. Thus, the specifications of forbidden states and desired process behavior are used to restrict the state space computation to an admissible specification-compliant subspace. That

![Figure 5: Examples for transforming $sNCES$ into $TIG$.](image-url)
is why this step is called partial reachability analysis (pRA).

In the previous section, the process behavior specification was used to determine a set of feasible TIG paths representing some kind of desired causal dependencies. Because every t-invariant contains a set of transitions, the transitions along the specification-compliant paths, which were extracted in the previous step, can be determined easily. Furthermore, the specification of forbidden states represents a set of partial markings, which must not be reached.

Thus, the outcome of these two kinds of specifications are criteria to restrict the computation of the reachable state space of the uncontrolled plant model. Therefore, the resulting partial state space is much smaller than the whole reachable state space. Hence, subsequent analysis steps are eased, especially regarding large-scale models of real industrial size.

The execution of the pRA is similar to the commonly known reachability analysis. Starting at an initial marking, every reachable step is calculated according to the possible state transitions given by the current enabled step $\xi$. The pRA does not examine state transitions, if:

1. They lead into a forbidden state, according to the forbidden states specification.
2. They lead into a state, which implies an enabled step $\xi$ which does not contain at least one transition which is also contained in a specification-compliant TIG path.

The rejected state transitions, which contradict to the specifications, mark the boundaries of the state space of the desired plant behavior. This boundaries are the starting points for determining a control model in the following step (Section 3.3).

The following little example, depicted in Figure 6, should illustrate the intention of the pRA. The left module in Figure 6(a) represents the model of an uncontrolled plant behavior. The module on the right is the corresponding specification of the cyclic process behavior. The reachable state space for the $\gamma$NCE Modules in Figure 6(a) is given in Figure 6(c). The solid lines represent the result of the partial reachability analysis, respectively the desired process behavior. The undesired behavior, constituted by the state transitions, which were rejected in the pRA during the previous step, are represented by the open circles in the state space. Each of them is analyzed if it could be avoided by a control intervention during a previous step in the trajectory through the state space.

Figure 6: Partial reachability analysis for a simple example.

for subsequent modules. This assumption is possible because of the modularity of $\gamma$NCE.

The TIG in Figure 6(b) is not very meaningful because it is a small example to show the intention of the pRA. The recurring behavior of the left module in Figure 6(a) is represented by its t-invariant. This t-invariant is transformed into the first node of the TIG, which has the number 1. The TIG nodes 2 and 3 are corresponding to the two t-invariants of the specification module in the right of Figure 6(a). The event interconnections of the $\gamma$NCE Modules are represented by the two arcs in Figure 6(b).

### 3.3 Determine Controllable Trajectories

A trajectory is said to be controllable if control interventions by the process control are able to ensure the desired plant behavior in every state of the plant. These interventions are represented by the open condition inputs in the model of the uncontrolled behavior of the plant. The state transitions, which were rejected in the pRA during the previous step, are representing deadlocks in the specification-fulfilling state space. Each of them is analyzed if it could be avoided by a control intervention during a previous step in the trajectory through the state space.

The backward search described in (Missal and
Hanisch, 2009) is utilized for this. It explores the uncontrollable pre-regions of such a deadlock and tries to find a controllable step. At this controllable step, a control intervention could avoid the reaching of the deadlock. Figure 7 illustrates the principle of this backward search. The results will be the basis for generating the model for the process control. Finally, if several controllable and specification-fulfilling trajectories are found, the one with the lowest possible count of state transitions is chosen. The presented approach in this contribution is based on a completely observable state space. Future works will consider an incompletely observable state space.

### 3.4 Distribution and Code Generation

The result of the previous section is a set of monolithic control functions. For a distributed process control, this set needs to be distributed into local controller models. Based on the modular modeling, the determined controllable and specification-fulfilling trajectories are divided into local trajectories. The steps of each local trajectory are related to only one (composite) module of the plant model. The control model for each plant component control is represented by the corresponding local trajectory, under the assumption of complete observability. Currently the distribution and code generation have to be done semi-automatically. But in future works it is intended to be automated.

### 4 EXAMPLE

Figure 8 shows the processing station of a modular production system in laboratory-scale. This processing station consists of a rotary table with four positions. At positions ② and ③ there are a drilling module and a testing module. The purpose of this station is to transport an incoming ① work piece from one table position to the next. At ② the work piece is processed (drilled). The drilling result is checked at ③. If the processing result is positive, the work piece is ejected at position ④. Otherwise it stays on the table for reprocessing.

The plant architecture is reflected in the structure of the 3NCES model depicted in Figure 9. The module in the middle represents the rotary table. The four modules around represent the four table positions including the modeling of the work piece properties (presence and with hole) as well as the plant modules for drilling and testing. The upper module on the right side of the image is the specification, modeled as 3NCESM. This specification is about the work piece position, respectively the property presence, which is modeled within the work piece properties at the different table positions.

The whole model consists of 73 places and 92 transitions which are organized in a modular hierarchy with 32 basic modules and 17 composite modules.

Figure 10 illustrates the results of the structural analysis, respectively the TIG (Section 3.1), of the plant model linked with the specification. For clarity, the t-invariants covering the work piece properties are diamond-shaped, whereas the t-invariants covering the property presence are gray-filled. The TIG in Figure 10 illustrates the causal dependencies within the plant model in Figure 9. Regarding the specification, only TIG paths passing all table positions are of interest. These paths are represented by the thick solid edges in Figure 10.
This set of feasible paths and the specification of forbidden states is used to perform the pRA (Section 3.2). The result of the pRA is shown in principle in Figure 11. The highlighted part on the left of the whole reachable state space represents the admissible state space determined by the pRA. A more detailed representation would not be readable because of more than 9,000 states of the whole state space, which are strongly interconnected. The boundaries of the highlighted admissible subspace, consisting of 256 states, are the starting points for the backward search for controllable steps (Section 3.3).

With the presented approach, the complexity of the state space analysis is reduced to an admissible subspace of the whole reachable state space of the model of the uncontrolled plant behavior. Thus, the complexity of subsequent analysis steps is reduced too, which is beneficial especially if regarding real industrial-scale models.

5 CONCLUSIONS

Complexity has always been a crucial issue for formal synthesis methodologies regarding real-size models. Therefore, in this contribution a novel approach for discrete process control synthesis was presented, which is based on a modular modeling formalism, namely safe Net Condition/Event System. Structural properties of this formalism were used to generate an abstracted representation of the causal dependencies within the plant model. These causal dependencies were examined concerning their relevance with respect to the process behavior specification. The incorporation of these causal dependencies of the plant model into a common state space analysis allow an
efficient limitation of its computational complexity. This reduces the complexity at a point which is crucial for control synthesis, especially regarding large-scale models of industrial size.

Accomplishing the research for the last step of the presented approach is part of current and near future work. The incorporation of timing issues is a goal of subsequent research projects. In spite of this, the current results are promising regarding an application to industrial size systems.

ACKNOWLEDGEMENTS

This work is supported by the Deutsche Forschungsgemeinschaft (DFG) under reference numbers HA 1886/17-1 and HA 1886/17-2.

REFERENCES


