Keywords: Skimming pricing, Non-linear programming, New product development.

Abstract: This article presents a new model for pricing a new product considering skimming pricing strategy in the presence of the competition. We consider two periods for price setting including skimming and economy period. The problem is deciding on a skimming price as well as an economy price in order to maximize the total profit. The derived model is a non-linear programming model and we analyzed the structure and properties of optimal solution to develop a solution method. Analytical results as well as managerial insights are presented by mathematical analysis and numerical analysis.

1 INTRODUCTION

Pricing is a main step in the marketing planning that generates revenue. Besides the other factors such as product quality and performance, brand image, distribution channels, and promotion plans, price plays a main role to encourage the customers to buy the product. So the companies have to consider many factors and analyze them to set the price. Hence, developing pricing models to get some managerial insights are of interest to the marketing managers. A company can consider any of five major objectives for its pricing as survival, maximum current profit, maximum market share, maximum market skimming, or product-quality leadership (Kotler, P., Armstrong, G., 2008).

The objective of price skimming involves a relatively high price for a short time where a new, innovative, or much-improved product is lunched to the market. The objective is to skim off consumers who are willing to pay more to have the product sooner. Prices are lowered later when demand from the early customers falls or competitors introduce the same product with lower price. A company may decide to be the product-quality leader in the market. Price skimming is used by many companies especially in the automobile, mobile phones, TV, laptop, and many other luxury industries. For example Sony Company is a frequent practitioner of skimming pricing, where prices start high and are lowered over time (Kotler, P., Armstrong, G., 2008). The Apple inc., introduced its new mobile phone named “iphone”, in June 2007 at a top price of $599 in the united states. Despite its high price, consumers across the country stood in a long line to buy the iphone on the first day of sales. After two months later, Apple cut the price from $599 to $399 (D. Sliwinska, and et al., 2007-2008).

For the first time Nancy L. Stokey (1979), develop a model to consider the price discrimination policy to enter a new product to the market. It is assumed a monopolist and the customer’s reservation price to buy the product is considered as a probability function. There is no competition and the monopolist wants to maximize the present value of profit over the time. D. Besanko and W L. Winston (1990), consider rational customers and analyze the optimal skimming price. The price discrimination is considered over the time. The objective of seller is to maximize its profit over the time. L. Popescu and Y. Wu (2007), consider the reference price and analyze the pricing strategy using dynamic pricing. The consumers at each time decide to buy the product based on their reference prices. The reference price is shaped by the past prices. So in a long run the monopolist can decide to have a constant steady state price or skimming price strategy. They investigate these situations using dynamic programming method and show the optimal policy.
Xuanming Su (2008), develops a model of dynamic pricing with endogenous inter-temporal demand. He assumes the finite inventory over a finite time horizon. A. Haji and M. Asadi (2009), develops a fuzzy expert system to new product pricing. This fuzzy expert system includes practical rule bases to analyze the appropriate price of new product in fuzzy environment. A. Dolgui and J. Proth (2010), discusses the pricing strategies and models. They discuss the benefits price skimming strategy for a company in the monopolistic market and they recommend that the high price cannot be maintained for a long time. A good review of pricing models and their coordination with inventory decisions can be found in Chan, L., et al. (2004).

In this paper, we develop a new model considering price skimming and economy pricing in the presence of competitor’s effects and customers demand elasticity. The objective of the model is to maximize the total profit of company at both skimming and economy phases. In this article we try to analyze the structure of the problem as well as optimal solution properties to derive a solution approach as well as managerial insights.

2 PROBLEM FORMULATION

Consider a market which can be segmented to two segments: \(A\) and \(B\). Segment \(A\), contains the customers who are willing to purchase the product sooner with a higher price. Conversely in segment \(B\), the customers will purchase the product when the market price is lower than their reservation price. We set skimming price for segment \(A\) and economy price for segment \(B\). But at the first time that a new product is introduced to the market, the skimming phase is considered and the company set a higher price to skim the segment \(A\) to achieve more profit. The length of the skimming phase depends on competitor’s ability and the profit margin of the skimming price. The skimming phase will end when the demand falls or a competitor joined the market with a lower price. At this time, the economy phase starts and the company has to decrease its price based on its skimming price, the competitor’s price and customer’s elasticity.

We assume that it is possible to estimate the maximum volume of demand for each market segment and the penetration rate of company to capture the demand depends on its price. The problem objective is to determine the best price for skimming and economy phase in order to maximize its overall profit and market share. At first two definitions that are considered to model the problem are presented in the following.

**Definition 1.** Maximum Reservation Price (MRP), is the price above which none of the customers will buy the product. In other words it is the lowest price at which demand is equal to zero. Maximum Willing to Buy (MWB), is the lowest price which all of the customers will buy the product (Philips, Robert L., 2005).

**Definition 2.** A myopic customer is one who makes a purchase immediately if the price is below her reservation price without considering the future prices. Conversely, a strategic (or rational) customer takes into account the future estimated prices when making purchasing decisions (Philips, Robert L., 2005).

In this article we assume the myopic behavior for customers. The parameters and variables needed to formulate the problem are defined as follows.

2.1 Notations

**Parameters:**
- \(FC\): The finished cost of product.
- \(MRP\): Maximum reservation price.
- \(TV\): The maximum estimated of total market demand volume.
- \(V\): The maximum estimated of market demand volume for skimming phase.
- \(PR^s\): The penetration rate function at skimming phase.
- \(PR^e\): The penetration rate function at economy phase.

**Variables:**
- \(P^s(P^e)\): The skimming (economy) price for product.

2.2 Analysis of Penetration Rate Functions

The penetration rate at the skimming phase depends on the price of product. If the skimming price is high, then the penetration rate is low. In the real situation the relationship between penetration rate and price is non-linear and the negative exponential function is more consistent and was applied more than other functions in the literature. So we apply the negative exponential function to model the penetration rate at the skimming and economy phases. We propose the penetration rate at skimming phase as:

\[
PR^s = e^{-\alpha \left(\frac{P^s}{MRP-FC}\right)}
\]
It can be simplified by substituting the parameters as:

\[ PR^s = e^{-\alpha a \cdot (P^s - FC)} \]

(2)

Where, \( \alpha = \frac{1}{MRP - FC} \) and \( \alpha \geq 1 \). The parameter \( \alpha \) is the shape parameter which is estimated by historical data from market for previous product that presents the behavior of customers.

The penetration rate at economy phase depends on the skimming price, competitor’s price and also the economy price. We assume that the competitor will join the market with a lower price. So the competitor will set its price smaller than skimming price and larger than finished cost of product. We assume that the cost function of production for competitor is the same as company. We also assume that there is just one opportunity to set the price and the company cannot estimate the exact price of the competitor. A high skimming price increases the penetration rate of competitor and the company will lose its market share for economy phase and we assume that the price cannot be larger than maximum reservation price and the economy price also cannot be larger than skimming price.

The objective function (5), attempts to maximize the company’s profit over the skimming and economy phases. By substituting the penetration rate functions from relations (2) and (4) in the objective function we see that it has the non-linear. All the constraints are in the form of linear and constraint (6) states that the price cannot be larger than maximum reservation price and the economy price also cannot be larger than skimming price (7).

3 STRUCTURAL ANALYSES

3.1 Optimal Solution Analysis

In this section we are going to do some structural analysis on the model to find the properties of optimal solution. By replacing the penetration rate functions the objective function is as the form of:

\[ z(P^s, P^e) = V e^{-\alpha a \cdot (P^s - FC)} (P^s - FC) + (TV - V e^{-\alpha a \cdot (P^e - FC)}) (e^{-\alpha a \cdot (P^e - FC)}) \frac{\eta}{\eta + \frac{P^e - FC}{P^e - FC}} \]

Theorem 1. The optimal economy price is derived based on the optimal skimming price by the following equation:

\[ P^e = \frac{\eta FC + P^s - FC}{\eta} \]

(8)

Proof: By taking the first derivative condition of objective function with respect to \( P^e \) we have:

\[
\frac{\partial Z}{\partial P^e} = -\frac{1}{P^e - FC} \left[ TV - V e^{-\alpha a \cdot (P^e - FC)} (e^{-\alpha a \cdot (P^e - FC)}) \frac{\eta}{\eta + \frac{P^e - FC}{P^e - FC}} \right]
\]

\[ = (TV - V e^{-\alpha a \cdot (P^e - FC)}) (e^{-\alpha a \cdot (P^e - FC)}) \frac{\eta}{\eta + \frac{P^e - FC}{P^e - FC}} \left[ 1 - \frac{\eta (P^e - FC)}{P^e - FC} \right] \]

\[ \frac{\partial Z}{\partial P^e} = 0 \Rightarrow P^e = \frac{\eta FC + P^s - FC}{\eta} \]
The second derivative of $Z$ with respect to $P^s$ ensures that the maximum value of $Z$ can reach at $P^{es}$ and hence it is the optimal economy price.

$$\frac{\partial^2 Z}{\partial P^s} = \frac{1}{(P^s - FC)^2} \left[ (TV - V e^{-a(p^s - FC)}) (e^{-a(p^s - FC)}) (\eta e^{a(p^s - FC)} (P^s - FC)) \right]$$

$$= \frac{1}{P^s - FC} \left[ (TV - V e^{-a(p^s - FC)}) (e^{-a(p^s - FC)}) (\eta e^{a(p^s - FC)}) \right]$$

$$= \left[ (TV - V e^{-a(p^s - FC)}) (e^{-a(p^s - FC)}) (\eta e^{a(p^s - FC)}) \right] \frac{(P^s - FC)}{P^s - FC} (1 - 2)$$

**Proposition 1:** The economy price is increasing in skimming price and the skimming strategy is reasonable for $\eta > 1$. Considering the constraint, $P^e \leq P^s$, if $\eta \leq 1$ then the skimming and economy prices are equal and it means that obtaining a single price is optimal and the skimming strategy is not acceptable. Therefore the economy price is always equal or less than skimming price and hence the constraint $P^e \leq P^s$ is surfaced in the model and can be eliminated.

**Observation 1:** Based on proposition 1, the company can estimate the parameter $\eta$ using the historical information about the price and demand of previous products and decide to apply the skimming strategy according to parameter $\eta$. If $\eta \leq 1$ then the price skimming strategy is not reasonable. The historical data show the behaviour of customers regarding to different values of price and if the value of parameter $\eta$ is equal or less than 1 it means the customers prefers to buy the product in one and lower price.

**Observation 2:** The economy price is the average of finished cost and skimming price in case of $\eta = 2$:

$$P^e = \frac{FC + P^s}{2}$$

The model can be modified by replacing the equation of optimal economy price in the objective function and transforming it to a function of single variable $P^s$. Therefore the model becomes:

$$\text{Max } Z = (P^s - FC) \left[ V e^{-a(p^s - FC)} + e^{-a(TV e^{-\beta ax} - V e^{-(\alpha + \beta ax)})} \right]$$

s.t.

$$P^s \leq MRP \quad P^s \geq 0$$

By replacing, $x = P^s - FC$, the objective function can be transformed as:

$$Z = x \left[ V e^{-ax} + e^{-a(TV e^{-\beta ax} - V e^{-(\alpha + \beta ax)})} \right]$$

(9)

In order to analyze the objective function and optimal solution, it can be simplified as:

$$\dot{Z} = x \left[ a_1 e^{-k_1 x} + a_2 e^{-k_2 x} - a_3 e^{-(k_1 + k_2)x} \right]$$

(10)

Parameters $k_1$ and $k_2$ are positive and,

$$k_1 = a_1, \quad k_2 = a \beta$$

The parameters $a_1$, $a_2$ and $a_3$ are as follows:

$$a_1 = V, \quad a_2 = \frac{e^{-a(TV)}}{\eta}, \quad a_3 = \frac{e^{-aV}}{\eta}$$

Since, $\eta \geq 1$ and $TV > V$, therefore we have $a_1 > a_3$ and $a_2 > a_3$.

Now by first derivative condition on equation (10) the optimal value of $x$ can be determined as:

$$x^* = \frac{a_1 e^{-k_1 x} + a_2 e^{-k_2 x} - a_3 e^{-(k_1 + k_2)x}}{a_1 k_1 e^{-k_1 x} + a_2 k_2 e^{-k_2 x} - a_3 (k_1 + k_2) e^x}$$

(11)

The above equation can be solved numerically by Maple software. If equation (11) has one unique solution then it is the optimal solution.

Now we are going to show the uniqueness of optimal skimming price. Recalling the equation (11), it can be transformed as equation (12). If we show that the equation (12) has just one solution therefore we can develop a procedure to obtain the optimal solution.

$$L(x) = a_1 e^{-k_1 x} (1 - k_1 x) + a_2 e^{-k_2 x} (1 - k_2 x) - a_3 e^{-(k_1 + k_2)x} (1 - k_1 + k_2) x$$

(12)

To show the uniqueness of solution for equation (12), we solved 560 example problems which the summary of their parameters are shown in table 1. In all sample problems there was just one solution for
equation (12). The behaviour of equation (12) with respect to $x$ is shown in figure 1.

Table 1: The summary of example problems parameters.

<table>
<thead>
<tr>
<th>FC</th>
<th>TV</th>
<th>$V$</th>
<th>MRP</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
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<td>15</td>
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<td>1</td>
<td>1</td>
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Number of values | 4 | 5 | 4 | 7 |

Based on this observation we can propose a solution algorithm to solve the model which is presented in the next sub-section.

![Figure 1: The behaviour of relation (18) respect to $x$.](image1)

### 3.2 Solution Algorithm

**Step 1:** compute the value of $x^*$ by Maple software using the equation:

$$x^* = \frac{a_1 e^{-k_1 x} + a_2 e^{-k_2 x} - a_3 e^{-(k_1 + k_2)x}}{a_1 k_1 e^{-k_1 x} + a_2 k_2 e^{-k_2 x} - a_3 (k_1 + k_2) e^{-(k_1 + k_2)x}}$$

**Step 2:** The optimal solution is:

$$P^{s*} = x^* + FC$$

$$P^{e*} = \frac{\eta FC + P^{s*} - FC}{\eta}$$

### 4 EXPERIMENTAL RESULTS

By solving the 560 example problems we analyzed the sensitivity of each parameter on solution. We considered the distance between economy price and skimming price as a criteria to analyze the effects of each parameter to this criteria. The distance between economy and skimming price gives an insight to management about the importance of skimming strategy. The more distance between economy and skimming price, the more interest to apply the price skimming strategy. Our observations are as follows:

1. The distance between skimming and economy price ($P^s - P^e$) is increasing in $\eta$. Figure 2, presents the relation between skimming price and economy price in $\eta$. Skimming and economy prices are the same in $\eta = 0$. By increasing $\eta$, the economy price decreases to finished cost and its distance from skimming price increases.

![Figure 2: The behavior of skimming and economy price in $\eta$.](image2)

2. For parameters $\alpha$ and $\beta$ we observed that $P^s - P^e$ is decreasing in both parameters $\alpha$ and $\beta$. Figure 3 shows the behavior of skimming as well as economy price respect to $\alpha$ and $\beta$. The most distance between skimming and economy price is where $\alpha$ and $\beta$ are equal to one and by increasing theses parameters the distance between skimming and economy price decreases.

![Figure 3: The behavior of skimming and economy price in $\alpha$ ($\beta$).](image3)

### 5 CONCLUSIONS

In this article a new pricing model was developed considering skimming pricing strategy for introducing new product. We considered two periods for price setting: first the skimming period and the second one is economy period. In the skimming period the company faces a monopolistic market but in the economy period there is at least one competitor. We formulate the effect of competitor in
the economy phase by penetration function. The penetration rates at skimming and economy phases were formulated as an exponential function and the effect of competitor’s price was formulated as loss of the market share in economy phase penetration rate. The optimal economy price is calculated considering the skimming price. An algorithm was developed to solve the model based on the lower and upper bound derived in structural analysis. Many example problems were solved and some managerial insights presented by numerical analysis. As an extension, this problem can be analyzed by game theory to realize the competition in dynamic environment considering the reaction of competitor and company. The other functions to formulate the penetration rates and other solution methods can be of interest for future research.

REFERENCES


