

CMAC STRUCTURE OPTIMIZATION WITH Q-LEARNING APPROACH AND ITS APPLICATION

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Abstract: Comparing with other neural networks based models, CMAC is successfully applied on many nonlinear control systems because of its computational speed and learning ability. However, for high-dimensional input cases in real application, we often have to make our choice between learning accuracy and memory size. This paper discusses how both the number of layer and step quantization influence the approximation quality of CMAC. By experimental enquiry, it is shown that it is possible to decrease the memory size without losing the approximation quality by selecting the adaptive structural parameters. Based on Q-learning approach, the CMAC structural parameters can be optimized automatically without increasing the complexity of its structure. The choice of this optimized CMAC structure can achieve a tradeoff between the learning accuracy and finite memory size. At last, the application of this Q-learning based CMAC structure optimization approach on the joint angle tracking problem for biped robot is presented.

1 INTRODUCTION

The Cerebellar Model Articulation Controller (CMAC) is a neural network (NN) based model proposed by Albus inspiring from the studies on the human cerebellum (J. S. Albus, 1975). Because of the advantages of simple and effective training properties and fast learning convergence, CMAC has been used in many real-time control systems, pattern recognition and signal processing problems successfully. However, besides its attractive features, the main drawback of the CMAC network in realistic applications is related to the required memory size. For the input space greater than two, on one hand, in order to increase the accuracy of the control the chosen quantification step must be as small as possible which will cause the CMAC's memory size become quickly very large; on the other hand, generally in real world applications the useable memory is finite or pre-allocated. Therefore, for high-dimensional input cases we often have to make our choice between accuracy and memory size.

To solve the problem relating to the size of the memory, generally these efforts can be classified into three main approaches. The first theoretical

aspect is developed on how to modify the input space quantization (Hung-Ching Lu et al., 2006); (S. D. Teddy et al., 2007). This is based on the idea of the quantization method of input space is a decisive factor of the memory utilization and the more intervals we quantized, the more precise learning we will obtain. However, not only the quantization step but also number of layers determines the learning preciseness and the required memory size. The second approach involves the use of multilayered CMACs of increasing resolutions, demonstrating the properties of generating and pruning the input layers automatically (A. Menozzi and M. Chow, 1997); (Chih-Min Lin and Te-Yu Chen, 2009). Nevertheless they lack the theoretical proof of the system's learning convergence, which is a desirable attribute for control and function approximation tasks. The third orientation which is most popular focuses on incorporating fuzzy logic into CMAC to obtain a new fuzzy neural system model called fuzzy CMAC (FCMAC) to alleviate the required memory size (M. N. Nguyen et al., 2005); (Daming Shi et al., 2010). Yet, it rises new problem on how to design an optimal fuzzy sets.

In the above CMAC literatures, there is no one related to the tradeoff problem of limited memory

size and learning quality. It is traditionally thought that the more exquisitely the input space is divided, the more accurately the output results of CMAC can be obtained. However, this will certainly cause quickly increasing of memory size, if we do not develop more complex CMAC structure, since the simplicity of structure play an important role in the on-line application of neural network. In fact, by experimental study of approximation examples, in which several high-dimension functions were selected and several combinations of structural parameters were tested, we found that the learning preciseness and the required memory size are determined by both of the quantization step and number of layers. Thus, adaptive choice of these structural parameters may overcome the above primary limitation. Our goal is that a CMAC structure can be optimized automatically for a given problem. In this way, it is possible to decrease the memory size according to the desired performance of the CMAC neural network.

The paper is organized as follows: In section 2, CMAC model and its structure parameters are concisely overviewed. Section 3 presents the experimental study of the influence of structural parameters on the memory size and approximation quality. In section 4, a Q-learning based structure optimized approach is developed. The proposed approach is applied on the desired joint angel tracking for biped robot in section 5. Conclusion and further works are finally set out.

2 CMAC NN STRUCTURE AND STRUCTURAL PARAMETERS

The output Y of the CMAC NN is computed using two mappings. The first mapping ($X \rightarrow A$) projects the input space point $X = [x_1, x_2, \dots, x_n]$ into a binary associative vector $A = [a_1, a_2, \dots, a_{N_c}]$. Each element of A is associated with one detector. When one detector is activated, the corresponding element in A equals to 1, otherwise it equals to 0. The second mapping ($A \rightarrow Y$) computes the output Y as a scalar product of the association vector A and weight vector $W = [w_1, w_2, \dots, w_{N_c}]$ according to relation (1), where $(X)^T$ represents the transpose of the input vector.

$$Y = A(X)^T W \quad (1)$$

The weights of CMAC neural network are updated by using equation (2). $w(t_i)$ and $w(t_{i-1})$ are respectively the weights before and after training at each sample time t_i . N_i is the generalization number of each CMAC and β is a parameter included in $[0 \ 1]$. Δe is the error between the desired output Y^d of the CMAC and the computed output Y of the corresponding CMAC.

$$W(t_i) = W(t_{i-1}) + \frac{\beta \Delta e}{N_i} \quad (2)$$

Due to its structure, CMAC is preferable to be used to approximate both linear and non-linear functions. If the complexity of its structure is not increased additionally, there are essentially two structural factors ruling the function approximation quality. The first one, called "quantization step" Δq , allows to map a continuous signal into a discrete signal. The second parameter, called "generalization parameter" N_i , corresponds to the number of layers. These two parameters allow to define the total number of cells N_c .

3 IMPACT OF STRUCTURAL PARAMETERS ON CMAC NN

We try to show the relation between the structural parameters of CMAC neural network, the quality of the approximation and the required memory size for a given function. Our study is based on an experimental enquiry, in which several high-dimension functions are used in order to test the neural network's approximation abilities. In this section, take FSIN and two dimension GAUSS functions as examples, simulations for several step quantization Δq are carried out, when the number of layers increases from 5 to 50 for FSIN function, and from 5 to 450 for two dimension GAUSS function. For each of the aforementioned functions, a training set including 100×100 random values, selected in the corresponding two-dimensional space, has been constructed. Weights of CMAC are updated using equation (2). When CMAC is totally trained, three modeling errors: mean absolute error E_{mean} , mean squared error E_{square} and maximum absolute error E_{max} are carried out. The overview of the obtained results for only three step quantization is shown in Figure 1 and 2 respectively.

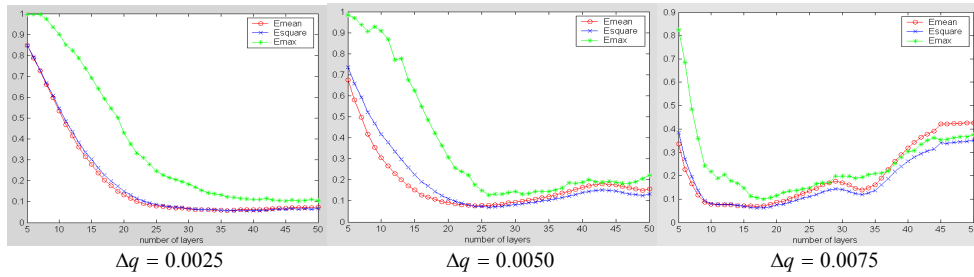


Figure 1: Approximation error according to the number of layers for FSIN function with different step quantization.

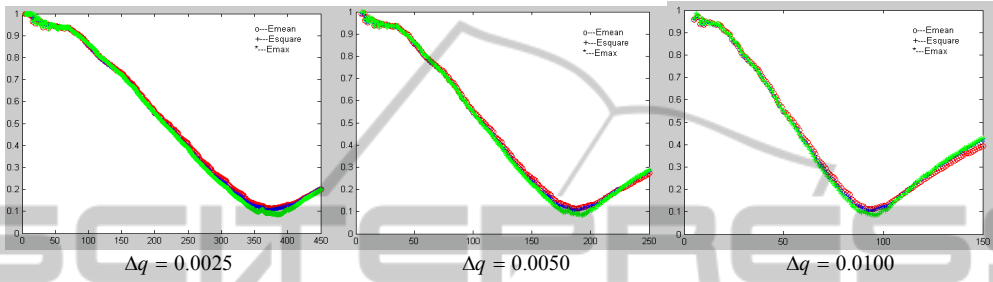


Figure 2: Approximation error according to number of layers for GUASS function with different step quantization.

When Δq is relatively small (for example $\Delta q = 0.0025$), errors converges toward a constant value close to the minimum error. But, when the quantization is greater, results show there is an optimal or near optimal structure when the modeling errors can achieve minimum. However, it must be noticed that for each quantization step, the minimal errors are quasi-identical but for different number of layers. As the curve trends of the mean absolute error E_{mean} , mean squared error E_{square} and maximum absolute error E_{max} are same, only take E_{square} as an example.

The mean squared error E_{square} for FSIN function equals to 5.81% and 6.21% in the case where $\Delta q = 0.0025$ and $\Delta q = 0.0075$ respectively. These chosen results show that the approximation abilities of the CMAC are similar in these two cases, however, in the points of view of memory size, for $\Delta q = 0.0025$ the required memory size is 4940, 3.5 times greater than when $\Delta q = 0.0075$ ($N_c = 1441$). The experimental enquire simulation results show that by optimizing CMAC structure, a nearly minimal modeling error with much smaller memory size can be achieved.

4 CMAC NN STRUCTURAL PARAMETERS OPTIMIZATION

4.1 Structural Parameters Optimized with Q-learning Approach

In this section, our goal is to design an optimizing strategy allowing to adjust automatically the structural parameters of CMAC NN in order to make a tradeoff between the desirable approximation quality and the limited memory size.

Q-Learning, proposed by Watkins (1992), is a very interesting way to use reinforcement learning strategy and is most advanced for which proofs of convergence exist. It does not require the knowledge of probability transitions from a state to another and is model-free. Here, the Q-Learning based on the temporal differences of order 0 is introduced, while in our structure optimized approach only considering the following step. Take the number of layers and quantization step $[N_l, \Delta q]$ as two dimension states of the world, while regarding the discrete actions as the increment of these two scalars. There are four possible actions when the agent searching the world:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} N_l & \Delta q + \delta_q \\ N_l & \Delta q - \delta_q \\ N_l + 1 & \Delta q \\ N_l - 1 & \Delta q \end{bmatrix} \quad (3)$$

where δ_q is the incremental quantity of quantization step and the variation of layer is 1 for each step. Change of the layer number and quantization step is supposed to be alternated. Each discrete time step, the agent observes state $[N_i^t, \Delta q^t]$, take action $a_i^t \in A^t$ ($i=1, \dots, 4$), observes new state $[N_i^{t+1}, \Delta q^{t+1}]$, and receives immediate reward r^t . Transitions are probabilistic, that is, $[N_i^{t+1}, \Delta q^{t+1}]$ and r^t are drawn from stationary probability distributions. In our approach, Pseudo-stochastic method is chosen to describe the probability distributions.

The reinforcement signal r^t provides information in terms of reward or punishment. In our case, on one hand, the reinforcement information has to take into account the approximation quality of network. On the other hand, the required memory size needs to be minimized within the limitation. Taking these considerations, we designed the reinforcement signal as three cases:

- $E_{square}^{t+1} < E_{square}^t$, the chosen of structural parameters is towards the correct direction. if E_{square}^t and N_C^t achieve the desirable value

$$r^t = 1$$

else

$$r^t = \frac{1}{\alpha N_C^t / 1000 + 10(1 - \alpha) E_{square}^t} \quad (4)$$

- $E_{square}^{t+1} > E_{square}^t$, the trend of the chosen action is not appropriate.

$$r^t = -1$$

- $E_{square}^{t+1} = E_{square}^t$, appropriateness of the trends of the chosen action is not clear.

$$r^t = 0$$

In equation (4), factor 1000 and 10 are designed only to balance the order of magnitude for memory size and approximation quality. α indicates the weight of these two structural parameters.

The Q matrix updates its evaluation of the value of the action while taking in account the immediate reinforcement r^t and the estimated value of the new state $V^t(N_i^{t+1}, \Delta q^{t+1})$, that is defined by:

$$V^t(N_i^{t+1}, \Delta q^{t+1}) = \max_{b \in A^{t+1}} Q(N_i^{t+1}, \Delta q^{t+1}, b) \quad (5)$$

where b is the action chosen within A^{t+1} . If there is enough learning, the update equation could be written in the following form:

$$Q^t(N_i^t, \Delta q^t, a^t) = (1 - \beta)Q(N_i^t, \Delta q^t, a^t) + \beta[r^t + \gamma V^t(N_i^{t+1}, \Delta q^{t+1})] \quad (6)$$

where γ is discount factor and β is the learning rate. If there comes up the end of a period, there is not a following state and the agent restarts a new sequence of training. The updating equation is:

$$Q^t(N_i^t, \Delta q^t, a^t) = (1 - \beta)Q(N_i^t, \Delta q^t, a^t) + \beta r^t \quad (7)$$

When the mean squared error satisfies the desirable approximation (refers to equation (8)), and the memory size is within the allocated rang as well (presented in equation (9)), the goal state is achieved.

$$|E_{square}^t| < |E_{square}^d| \quad (8)$$

$$N_C^t < N_C^d \quad (9)$$

4.2 Simulation Results and Convergence Analysis

Also, take FSIN function approximation as an example. Suppose that the finite number of usable memory size is 1500, and the approximation error less than 6.00% is favorable in order to maintain the approximation quality. In this case, we choose $|E_{square}^t| < 6.00\%$ and $N_C^t < 1500$ as the goal state in the training phase. The initial state of number of layer N_i^0 is set to be 20 and the quantization step can be chosen randomly within $[0.0000 \ 0.0100]$, every 0.0002 as the incremental quantity δ_q . In this example, the discount factor γ is set to be 0.9.

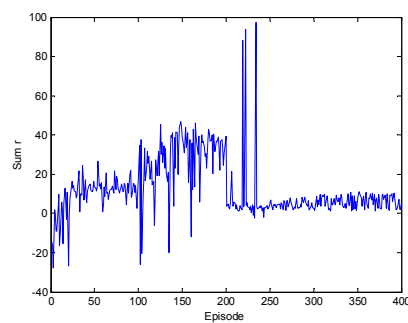


Figure 3: Sum of $\Delta Q(t)$ for each episode.

Figure 3 shows the sum of the computing value $\Delta Q(t)$ for each episode according to the number of episode. This updating value, which depends directly on the reinforcement signal, converges toward 3 within 250 episodes. The

stability of our CMAC structure learning approach is theoretically guaranteed by the proof of the Q-learning convergence. After the training phase, the CMAC with optimized structure ($N_l = 12$, $\Delta q = 0.0084$) can guarantee both of the desirable approximation quality and limitation required memory size, referring to Table 1.

Table 1: CMAC structure with minimum mean squared error for FSIN function.

Structure Optimized	Δq	N_l	E_{square}	N_C
NO	0.0025	41	5.81%	4940
YES	0.0084	12	5.94%	1431

5 THE APPLICATION OF CMAC STRUCTURE OPTIMIZATION

In order to increase the robustness of control strategy for robot, CMAC neural network has been applied to learn a set of articular trajectories with popularity. However, the CPU of the robot has to do many intricacies tasks at the same time, therefore the useable memory size is often allocated with restriction or fixed number, and always the precise control output is favorable. In this case, the structural parameters optimization problem is needed to be considered if we do not increase the complexity of the CMAC NN, since the simplicity of structure for network is always desirable. On the basis of our previous work on the gait pattern planning strategy of biped robot (C.Sabourin et al, 2008), CMAC structural parameters optimization with Q-learning approach is applied to learn the joint angle trajectories of biped robot.

Usually, after footstep planning strategy, the position of the two stance feet can be calculated. Therefore, it is not difficult to derive the trajectory of the joint angle by inverse kinematics or bio-inspired approach. The geometrical relationship between stance leg and swing leg of biped robot is described in Figure 4. Based on our control strategy, each reference gait is characterized by both of the step length and step height. Joint trajectory associated to one gait is memorized into one CMAC neural network. The biped robot is walking with a weighted average of several reference trajectories. Regarding the coordinate of swinging foot (P_{3x}, P_{3y}) as the two inputs, two CMAC neural networks are utilized for training hip joint angle θ_2 and knee joint angle θ_3 separately for each gait pattern. The weights of CMAC are updated based on

the difference between the output of CMAC and reference hip or knee joint angle of swinging leg.

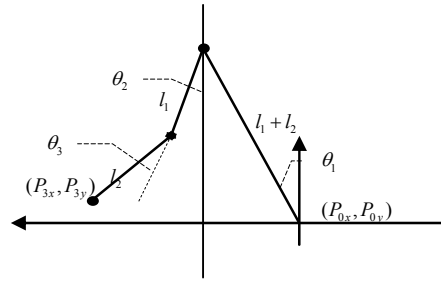


Figure 4: Geometrical relationship between stance leg and swing leg.

Figure 5 shows the results of swinging leg joint angle approximation with CMAC, in which blue curve stands for the reference joint angle profile, red and green curves represent the output of CMAC approximating hip and knee joint angle respectively. In the first two simulations the structural parameters are chosen randomly and we do not know if they are appropriate. In the third experiment, the CMAC structural parameters are learned based on the developed Q-learning approach. We hope that the approximation error of reference gait less than 1.10% is better and the pre-assigned memory size for each CMAC NN is 1000. $|E_{square}^d| = 1.10\%$ and $N_C^d = 1000$ are set to be the goal state according to equation (8) and (9). After the learning phase, the optimized parameters are $N_l = 20$, $\Delta q_1 = \Delta q_2 = 0.0051$ (refers to Figure 5(b)). The required memory size and approximation errors for both of the hip and knee angles are listed in Table 2 for these three experiments. In the first experiment, the calculated mean squared error ($E_{square}^{\theta_2} = 1.16\%$, $E_{square}^{\theta_3} = 1.19\%$) is very near to the desirable value, but the utilized memory size $N_C = 3589$ is 3.5 times bigger than the structure optimized example. Since in this biped robot application case, several reference pattern gaits have to be stored, the total number of memory size becomes quickly very large. In the second example, the memory size is desirable, however, the approximation quality of CMAC neural network ($E_{square}^{\theta_2} = 1.52\%$, $E_{square}^{\theta_3} = 1.56\%$) is much worse than the structure optimization case ($E_{square}^{\theta_2} = 1.07\%$, $E_{square}^{\theta_3} = 1.03\%$). However, the precise desired gait tracking is important in the case of biped robot walking in the unknown environment.

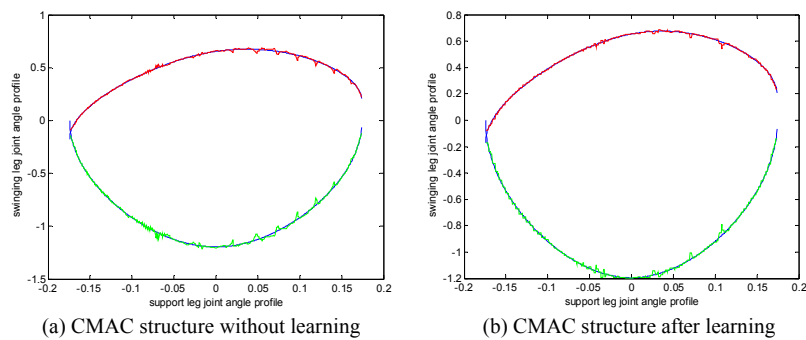


Figure 5: Joint trajectory tracking with CMAC Neural Network.

Table 2: Memory size and mean square error with randomly chosen and after learning CMAC structural parameters.

CMAC structure	N_1	$\Delta q_1 = \Delta q_2$	$E_{square}^{\theta_1}$	$E_{square}^{\theta_2}$	N_c
Randomly chosen	30	0.0021	1.16%	1.19%	3598
Randomly chosen	10	0.0080	1.56%	1.52%	795
After learning	20	0.0051	1.07%	1.03%	993

6 CONCLUSIONS

Besides the appealing advantages of CMAC NN, such as simple and effective training properties and fast learning convergence, a crucial problem to design CMAC is related to the choice of the neural network’s structural parameters. In this paper, we have shown how both the number of layers and step quantization influence the approximation qualities of CMAC neural networks. The presented simulation results show that by optimizing CMAC structure, a nearly minimal modeling error with much smaller memory size can be achieved. Consequently, a CMAC structure optimization approach which is based on Q-learning is proposed. The stability of our proposed approach is theoretically guaranteed by the proof of the learning convergence of Q-learning. This Q-learning based structure optimization method is applied on generating the joint angle of biped robot. Simulation results show that the choice of an adaptive structure of CMAC allows on one hand decreasing the memory size and on the other hand achieving the desirable approximation quality.

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