SEMANTIC GRAPHS AND ARC CONSISTENCY CHECKING
The Renewal of an Old Approach for Information Extraction from Images

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Abstract: The aim of this paper is to show that symbolic computation based on constraint satisfaction can be useful for information extraction from images. It presents how some limitations of this approach have been overcome by the development of new conceptual tools: arc-consistency with bilevel constraints, weak arc-consistency, a system of complex qualitative spatial relations. The application of these tools to images of various domains (medical images, high resolution satellite images) shows its effectivity.

1 INTRODUCTION

Animals are able to perform complex visual discrimination tasks and decision making. It means that processing visual input does not require a high degree of symbolic reasoning. The good results obtained by many computer vision algorithms based on statistical/physical models tend to prove it. On the other hand, it is difficult to believe that image understanding will never draw benefit from symbolic reasoning which is a so powerful tool for human intelligence. Now, if we want to translate this human specific ability in a computer algorithm, how to do it? Graph formalism has long been used in the field of artificial intelligence to represent the conceptual knowledge (ontologies, semantic graphs) because it is a convenient way to represent logical constraints between different concepts (textual or visual). These graphs can be used to solve Constraint Satisfaction Problem (CSP). The resolution of such problems consists in checking the consistency of a graph. Values are assigned to a set of variables (which are represented by nodes of the graph and can be seen as a way of symbolizing concepts) constrained by binary relations (represented by the arcs of the graph). This kind of problem is NP-complete. To get a solution in a reasonable amount of time, fast algorithms of arc-consistency checking have been proposed (Waltz, 1975), (Mohr and Henderson, 1986), (Mackworth, 1977),(Mackworth and Freuder, 1985), (Hentenryck et al., 1992), (Freuder and Wallace, 1992), (Bessière, 1994). Even if some authors have proposed applications of constraint satisfaction checking on images (Benmouffek et al., 1991), (Mahonney and Fromherz, 2002), (Nempont et al., 2008), these approaches have been little used so far in image interpretation.

However, converting an image interpretation problem into a problem of constraint satisfaction is fairly easy: the nodes of the graph correspond to objects or part of objects that we look for in the image and the arcs symbolize the spatial constraints between objects. Then, the image interpretation consists in finding regions of the image that can be assigned to each node in the graph and that satisfy the spatial constraints imposed by the graph.
The two limiting factors of these approaches are:

- Classical arc-consistency checking algorithms assume that with one node of the graph is associated only one value (a node of the graph is associated with only one region of the image). It is a great limitation because images are in practice often over-segmented.
- The local constraints are often too general to describe many objects made up of over-segmented regions in unpredictable ways.

Our work sought to overcome these limitations. We created the concept of arc-consistency checking with two levels of constraints. This allows to associate several regions of the image with a node in the graph model (for example, several nodes in the adjacency graph representing the segmented image). With this multivalent graph-matching by constraint satisfaction checking, it becomes possible to interpret over-

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segmented images and then to apply the principle of arc-consistency in common situations. We then developed various tools to build a more powerful constraint language:

- A richer lexicon of constraints (mainly spatial constraints).
- The possibility of combining these constraints with logical expressions, which requires the introduction of the notion of weak arc consistency. We will see the algorithmic solutions to implement this concept.

The aim of this article is to show that symbolic approach based on the notion of constraint satisfaction can bring interesting solutions to image interpretation problem. This article presents the basic concepts of this work and describes briefly the different tools developed in this framework. Some applications in medical images and high resolution satellite images are described.

2 SEMANTIC GRAPHS AND ARC CONSISTENCY CHECKING

In a semantic graph, the binary constraints represented by the arcs and the unary constraints associated with the nodes are supposed to be known at the beginning of the matching process. The hypothesis is that some specific constraints exist in the image and the aim is to find the set of regions satisfying these constraints.

2.1 Problem with Two Levels of Constraint

We can say that solving an image interpretation problem is similar to solving a constraint satisfaction problem. To define such a problem we use the following conventions:

- Variables are represented by the natural numbers $1, \ldots n$. Each variable i has an associated domain D_i. We use D to denote the union of all domains and d the size of the largest domain.
- All constraints are binary and relate two distinct variables. A constraint relating two variables i and j is denoted by C_{ij}. $C_{ij}(v,w)$ is the Boolean value obtained when variables i and j are replaced by values v and w respectively. Let Rc be the set of these constraining relations.

A Finite-Domain Constraint Satisfaction Problem (FDCSP) consists of finding all the sets of values

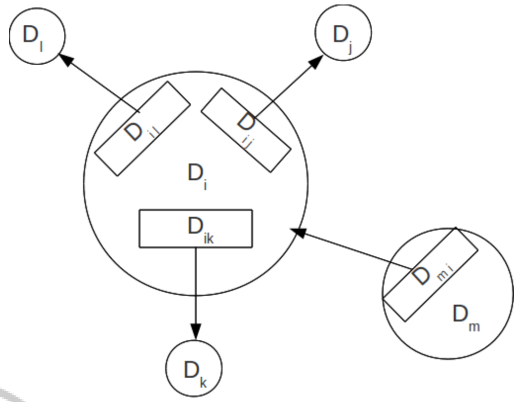


Figure 1: Structure of a node used to manage the arc-consistency checking with bilevel constraints. In this example the node i of the semantic graph is constrained by the nodes j, k and l and it constrains the node m. The rectangles inside the node i represent the sets "interface".

$\{a_1, \ldots, a_n\}$, a_1 x ... x $a_n \in D_1$ x ... x D_n, for(1, ..., n) satisfying all relations belonging to Rc.

A graph G is associated with a constraint satisfaction problem as follows: G has a node i for each variable i. One oriented arc (i, j) is associated with each constraint C_{ij}. Arc(G) is the set of arcs of G and e is the number of arcs in G. Node(G) is the set of nodes of G and n is the number of nodes in G.

In this classical definition of FDCSP, one variable is associated with one value. This assumption cannot hold for some classes of problems where we need to associate a variable with a set of linked values as described in (Deruyver and Hodé, 1997) and (Deruyver and Hodé, 2009b). It is the case when several segmented regions have to be associated with a node of the semantic graph.

To cope with this difficulty, the structure of the nodes is modified. A node becomes a multi-set made up of a kernel and several interfaces (each interface is associated with each arc leaving the considered node, see Figure 1). Moreover, two levels of constraint are introduced: The first one, called inter-node, links two interfaces of two different nodes. (it is similar to the previous constraint C_{ij} between two nodes i and j but in our case it is reduced to a part of each node). The checking of constraints on sub-sets of values associated with each node generates a constraint relaxation between the nodes and can produce a dramatic increase of the number of solutions. This drawback can be removed if the intra-node constraint is efficient enough to balance the inter-node relaxation. However, additional computation may increase the time complexity of the constraint satisfaction checking. Then this checking has to be possible in practice in term of time complexity of the algorithms.

We propose to use two types of intra-node constraint. The first type corresponds to global constraints $Cmpg_i$ linking a set of values of the node i. The second type corresponds to constraints linking two values of a node with a compatibility constraint Cmp_i associated with each variable i (spatial relations between sub-parts of an object associated with a node of the graph). Then, the notion of constraint satisfaction problem with two levels of constraint ($FDCSP_{BC}$) can be defined as follows:

Definition 1.

- *Let Cmp_i be a compatibility constraint such that (a,b) satisfies $Cmp_i \Leftrightarrow$ a and b are compatible, where $a \in D_i$ and $b \in D_i$.*
- *Let $Cmpg_i$ be a global compatibility constraint on a set of values $S_i \subset D_i$ such that S_i satisfies $Cmpg_i$.*
- *Let C_{ij} be a constraint between i and j.*
- *Let S_i and S_j be a pair such that $S_i \subset D_i$ and $S_j \subset D_j$, $S_i, S_j \models C_{ij}$ means that (S_i, S_j) satisfies the oriented constraint C_{ij}.*
  $S_i, S_j \models C_{ij} \Leftrightarrow \forall a_i \in S_i, \exists (a'_i, a_j) \in S_i \times S_j$, such that (a_i, a'_i) satisfies Cmp_i and (a'_i, a_j) satisfies C_{ij}.

Definition 2. *Sets $\{S_1 ... S_n\}$ satisfy $FDCSP_{BC} \Leftrightarrow \forall C_{ij}$, S_i, $S_j \models C_{ij}$ and S_i satisfies $Cmpg_i$ and S_j satisfies $Cmpg_j$.*

2.2 Arc Consistency Checking Problem with Bilevel Constraints

Solving the constraint satisfaction problem with bilevel constraints with the arc consistency checking implies to define a class of problems called arc consistency checking problem with bilevel constraints (AC_{BC}). Solving this type of problems allows to solve a multivalent matching problem. This is what we want to do when we want to match an adjacency graph and a semantic graph. This class of problem is associated with $FDCSP_{BC}$. Let P(D_i) be the set of sub parts of the domain D_i.

Definition 3. *Let $(i, j) \in arc(G)$. Arc(i,j) is arc consistent with respect to $P(D_i)$ and $P(D_j) \Leftrightarrow \forall S_i \in P(D_i), \exists S_j \in P(D_j)$ such that $\forall v \in S_i \exists t \in S_i$, $\exists w \in S_j$, $Cmp_i(v,t)$ and $C_{ij}(t,w)$ and $Cmpg_i(S_i)$ and $Cmpg_j(S_j)$ (v and t could be identical).*

Definition 4. *Let P= $P(D_1) \times ... \times P(D_n)$. A graph G is arc consistent with respect to P $\Leftrightarrow \forall (i, j) \in arc(G) : (i, j)$ is arc consistent with respect to $P(D_i)$ and $P(D_j)$.*

The purpose of an arc-consistency algorithm with bilevel constraints is, given a graph G and a set P, to compute P', the largest arc consistent domain with bilevel constraints for G in P.

3 A SYSTEM OF QUALITATIVE SPATIAL RELATIONS: THE CONNECTIVITY-DIRECTION-METRIC FORMALISM (CDMF)

In order to better describe complex spatial relations between two objects made up of several segmented regions, we developed a system of topological and directional relations (Deruyver and Hodé, 2009c). Some of these relations are calculated thanks to the well known notion of minimum bounding box of a region and to the notion of minimum bounding boxes of border interfaces between two regions.

3.1 Minimum Bounding Boxes of Border Interfaces between Two Regions

The notion of minimum bounding box of border interfaces is introduced to improve the description of distance relationship between two objects. We mean by border interface the border part of a region which, given a cardinal direction, is in front of another region (Cf. Fig. 2).

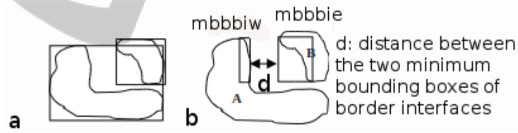


Figure 2: a) In this case the two regions are not overlapped but their minimum bounding boxes are overlapped. The analysis of the spatial relation between these two regions is not possible by using minimum bounding boxes. b) mbbbiw is the minimum bounding box of the border interface which is on the left of region A. mbbbie is the minimum bounding box of the border interface which is on the right of region B.

Definition 5. *Let R be a region (a set of connected pixels) and let p(x,y) be a pixel of R. E(R)=$\{$p(x,y) $\in$ R $\mid$ $\exists$ p(x',y') one of the 8 connected neighbors of p(x,y), p(x',y') $\notin$ R$\}$. Let A and B be two regions:*

- *The border interface Cw(A,B) is defined by $\{$p(x,y) $\in$ E(A) such that $\exists$ p(x',y) $\in$ E(B) and $\forall$ p(x",y) such that x < x" < x' p(x",y) $\notin$ A and p(x",y) $\notin$ B $\}$*
- *The border interface Ce(A,B) is defined by $\{$p(x, y) $\in$ E(A) such that $\exists$p(x', y) $\in$ E(B) and $\forall$ p(x", y) such that x> x" >x p(x", y) $\notin$ A and p(x", y) $\notin$ B $\}$*
- *The border interface Cn(A,B) is defined by $\{$p(x, y) $\in$ E(A) such that $\exists$p(x ,y') $\in$ E(B) and $\forall$ p(x,*

y”) such that y< y”<y' p(x, y”) ∉ A and p(x, y”) ∉ B }

- *The border interface Cs(A,B) is defined by {p(x, y) ∈ E(A) such that ∃p(x ,y') ∈ E(B) and ∀ p(x, y”) such that y> y”>y' p(x, y”) ∉ A and p(x, y”) ∉ B }*

Definition 6. *The minimum bounding box of a border interface in the direction d (mbbbid) is defined by* $(inf_x(Cd(A,B)), inf_y Cd(A,B))), (sup_x(Cd(A,B)), sup_y(Cd(A,B)))$

We can see on the example of Figure 2 that the two mbb of the regions A and B are overlapped. On the contrary the mbbbiw and the mbbbie are not overlapped. Then, thanks to the mbbbi, it is easy to deduce, on this example, that the region A is on the left side of region B.

3.2 Additional Relations between Two Regions

Minimum bounding boxes of border interface (*mbbbiw*, *mbbbie*, *mbbbin*, *mbbbis*) allow to describe additional relations. The four spatial relations between A and B linked to the corresponding *mbbbid* can be defined as follows:

- $A\ Ei\ B$ iff $sup_x(Cw(B,A)) \leqslant inf_x(Ce(A,B))$,
- $A\ Wi\ B$ iff $sup_x(Cw(A,B)) \leqslant inf_x(Ce(B,A))$,
- $A\ Ni\ B$ iff $sup_y(Cn(A,B)) \leqslant inf_y(Cs(B,A))$,
- $A\ Si\ B$ iff $sup_y(Cn(B,A)) \leqslant inf_y(Cs(A,B))$,

All these relations may be associated with the metric d defined as follows: $d(A,B)= inf_z(A)\text{-}sup_z(B)$ where $z = y$ for Ni or Si relationship, and $z = x$ for Ei or Wi relationship.

3.3 Elementary Relations in CDMF

CDMF allows to define very complex relationships by a combination of elementary relationships. An elementary relationship is a relation:

- (1) of connectivity or non connectivity
- (2) of directional constraint between *mbb* with none or one metric relation chosen among the metrics *dsi* and *dgi* (i=1 4) described in Figure 3 (with inferior and superior limits). In that case, we have four directional relationships: *N* (North), *S* (South), *W* (West) and *E* (East).
- (3) of directional constraint between *mbbbi* with one metric relation d defined before (with inferior and superior limits). In this case, we have four directional relationships: *Ni*, *Si*, *Wi* and *Ei*.

A concrete implementation of these relations is proposed to use it in the context of a constraint satisfaction problem with bilevel constraints. These relations are used as spatial constraints associated with the arcs of a semantic graph.

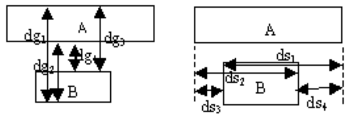


Figure 3: The 8 distances that can be defined on minimum bounding boxes.

3.4 Weak Arc-consistency to Combine Spatial Relations

To be able to describe more complex relationships between regions such as ”is surrounded by”, it is necessary to combine relations in a logical expression such that *Ei or Wi or Ni or Si* which means that a region has to be in the neighbourhood of another region in one of the four directions. To express such combination we use the notion of quasi-arc consistency described in (Deruyver and Hodé, 2009b). The idea is to associate a number of relaxation with a set of arcs. For example let $a1$, ..., an be a set of n arcs. With the notion of quasi-arc consistency it is possible to associate with this set, a number r of relaxation. If $r = n-1$, then at least one constraint associated with one of these arcs has to be satisfied. This notion of quasi-arc consistency introduces an operator *or* between each constraint associated with the arcs. The Figure 4 shows the structure of a node describing the logical expression (*A or B or C*) *and* (*C or D*) *and* (*D or E*) *and* (*E or F*) where *A*, *B*, *C*, *D*, *F* and *G* are spatial relations. The quasi-arc consistency can be defined as follows: Let Γ_i be the set of the nodes linked to the node i in the graph G such that: $\Gamma_i = \{j \in Node(G), (i,j) \in Arc(G)\}$

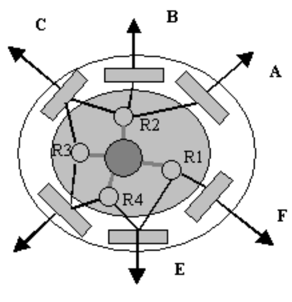


Figure 4: Node representing the expression (A or B or C) and (C or D) and (D or E) and (E or F). The relaxation numbers have the following values: R2=2, R1=1, R3=1, R4=1.

Let the function $Rl(i)$ be such that it maps to a node i the number nb of constraint relaxations in the graph G and it is such that:

$$\begin{array}{lll} Rl: & node(G) & \rightarrow \mathbb{N} \\ & i & \mapsto Rl(i) = nb \, with \, nb \geq 0 \\ & & and \, nb \leq Card(\Gamma_i) \end{array}$$

This function allows that nb binary constraints associated with the node i can be not satisfied. Let $\Gamma^{ac}_{i,l}$ the set of the nodes $j \in \Gamma_i$ making arc-consistent arc with a given node i and a given $l \in D_i$ such that:
$\Gamma^{ac}_{i,l} = \{j \in \Gamma_i \mid (i,j)$ is arc-consistent with respect to $l \in D_i \, and \, \mathcal{P}(D_j)\}$
With the function Rl, a new class of problems called quasi arc-consistency problem can be defined. The definition of a quasi arc-consistent graph is:

Definition 7. *Let $\mathcal{P}(D_i)$ be the set of sub parts of the domain D_i. Let P=$\mathcal{P}(D_1) \times .... \times \mathcal{P}(D_n)$. A graph G is quasi arc-consistent with respect to P iff $\forall i \in node(G)$ we have*

$$Card(\Gamma_i) - \min_{\forall l \in D_i}(Card(\Gamma^{ac}_{i,l})) \leq Rl(i)$$

4 EXPERIMENTS

We present three applications of image interpretation using semantic graphs and arc-consistency checking. The first one concerns the interpretation of anatomical cerebral images. The second one concerns the analysis of water meter images and the last one concerns the detection of residential areas in high resolution satellite images.

4.1 Interpretation of Anatomical Cerebral Images

Experiments have been made on a set of NMR images obtained on the web site "BrainWeb" (http://www.bic.mni.mcgill.ca/brainweb/). This web site contains a database of simulated images. In these experiments we focus our interest on six internal grey nuclei (small internal anatomical structures of the brain). Figure 5 shows the conceptual graph describing this part of the cerebral anatomy. In this application the white and the grey matter are labeled. The interpretation is made on the 10 slices containing these nuclei (Figure 6 shows 3 slices taken from this set of images). The semantic analysis is done directly on a segmentation provided by a watershed algorithm. The arc-consistency checking algorithm had to deal with a large number of segmented regions (between 500 and more than 1000 regions). On each slice, the 6 grey nuclei are correctly identified. As we had to work with

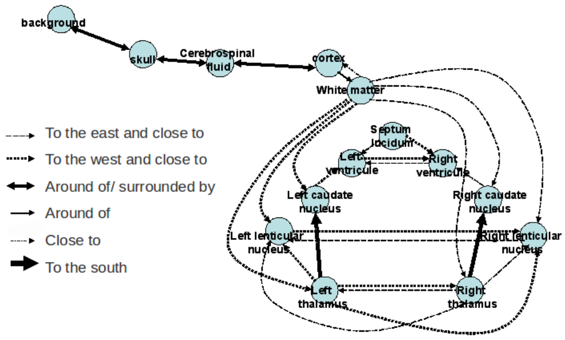
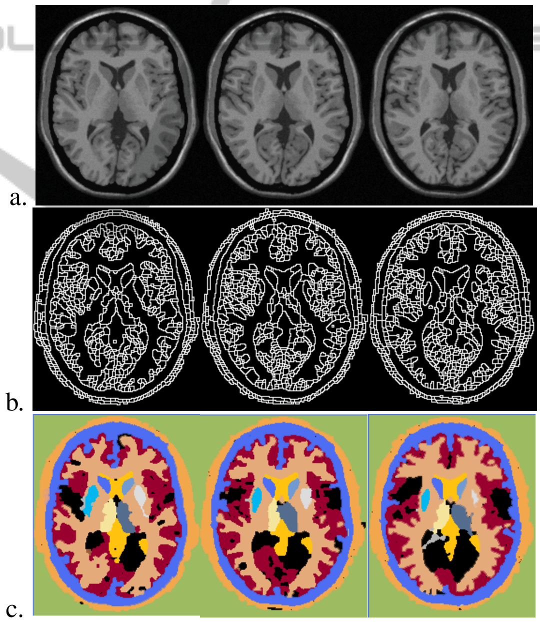


Figure 5: Structure of the semantic graph representing the anatomy of the brain.

a large number of regions, our analysis was applied to 2D data to avoid some problems of memory space. However, in this case, the definition of a conceptual graph in 3 dimensions may be easily achieved from anatomical book. Such 3D conceptual graph should lead to better results.



Number of regions in each image before applying the semantic analysis: 625, 552, 819,

Figure 6: Experiments on a set of NMR images obtain on the brain web. a) original images, b) segmentation obtained with a watershed algorithm c) interpreted images: Every anatomical structure is identified by a color. The 6 grey nuclei (small internal anatomical structures of the brain) are correctly identified.

4.2 Analysis of Water Meter Images

In this application, the aim is to localize in the image, the water meter in order to detect if it is not broken, to recognize its type (analogical or numerical) and to read the numerical value displayed on it if there is one. These images are very noisy and the grey level values are not a relevant information to recognize the frame and the center of the water meter. The only way to make a correct interpretation is to use the spatial relations and the morphological characteristics of the object sub-parts. Our approach has been applied to a set of 26 images to localize the frame and the center of the water meter. Among these images, 6 images do not contain any water meter but an object sharing some similarities with the water meter. This 6 images were correctly detected as not containing water meter. In this case the graph is not consistent. All the other images were correctly interpreted. Figure 7 presents the 26 test images. The following step will be to analyse the content of the center of the water meter. Figure 8 shows that the differences between the coordinates of the gravity center of the water meter's center detected manually and automatically remain small compared to the size of the water meter's centers which varies between 80 and 100 pixels.

4.3 Extraction of Residential Areas in High Resolution Satellite Images

The obtained segmentation is described by a region adjacency graph. A graph model describing houses belonging to a residential areas is build. Then, the AC_{BC} algorithm is applied to detect these houses.

4.3.1 The Model Graph

The model graph is made up of 5 nodes and 18 arcs. It uses 13 kinds of spatial relation: 4 relations corresponding to the four directions (north, south, east and west) imposing distance constraints, 4 relations corresponding to the four directions that verify that there is no region belonging to a given node (ex: the node associated with the concept of street) between the two considered regions (To use this relation, no binary constraint has to be imposed on the considered node), 4 relations corresponding to the four directions without any distance constraint and the identity relation.
A simplified version of the graph can be seen on figure 9.

4.3.2 Gathering of Houses belonging to Sub-parts of Residential Areas Surrounded by a Road

To gather houses belonging to sub-parts of a residential area surrounded by a road, we use an algorithm based on a region growing process. We first look for

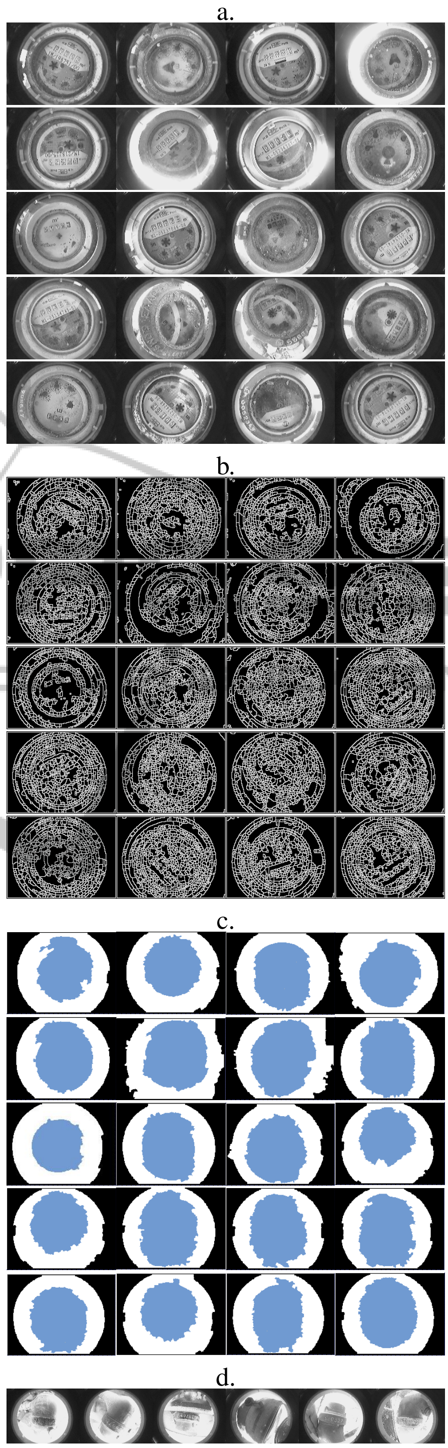


Figure 7: Interpretation of water meter images. a) original images, b) segmented images with a watershed algorithm c) detection of the frame and the center of the water meter d) images providing a non consistent graph. The object of these images does not have the morphological characteristics described by the conceptual graph.

region belonging to the node "houses of a residen-

	x coordinate (unit: pixels)	y coordinate (unit: pixels)
mean	6	3.35
median	5	3.5
standard deviation	4.93	2.64
min	0	0
max	16	9

Figure 8: Statistics concerning the differences between the coordinates of the gravity center of the water meter's center obtained manually and the coordinates of the gravity center obtained automatically.

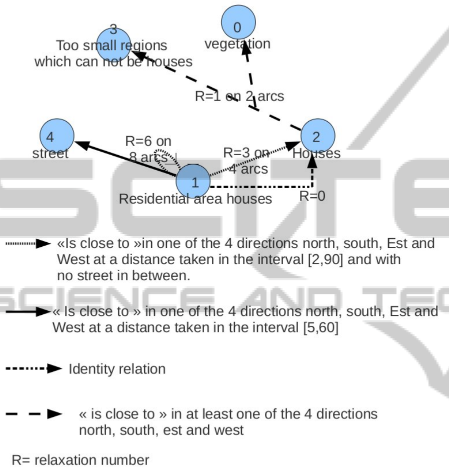


Figure 9: The model used to recognize residential areas

tial area". For each region we look for its supports belonging to the node "houses". If these supports belong to the node "houses of a residential area", they are marked and they are enqueued to process them recursively. If they do not belong to this node, these supports are simply marked in the result image.

4.3.3 Experiments

The experiments were made on images from the French satellite SPOT 5 representing the urban area of Strasbourg (France) at different spatial resolutions and scaling. The result can be seen on the figure 10. We can see that the approach is general enough to process different kind of configuration and different spatial resolutions.

5 DISCUSSION

This paper tries to show that symbolic approach based on constraints satisfaction is an interesting way to solve some problems of image interpretation and to extract information from images. A set of tools has

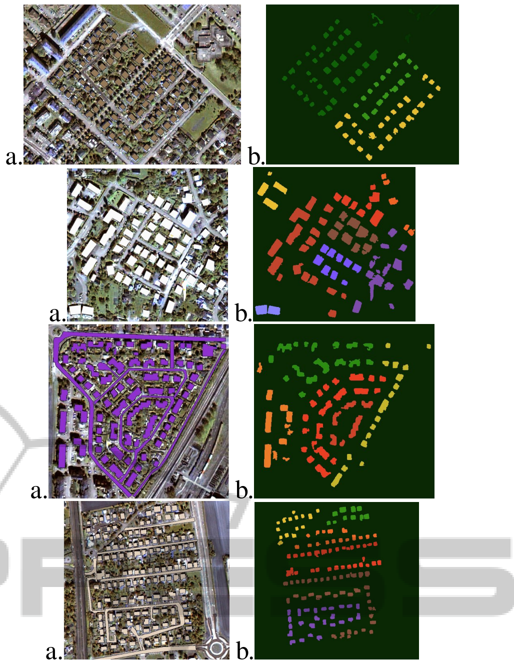


Figure 10: Test images: a) original images with annotation b) labelised images. Houses belonging to a same residential area (set of houses surrounded by a street) have the same color.

been developed in this framework and the applications of these tools on images from different domains shows their effectiveness. In this paper only binary relations have been used, however it is quite easy to introduce n-ary relations by using hyper-arc. In this case, the time-complexity will increase, however, the algorithm of arc-consistency checking with bilevel constraint can be parallelized (Deruyver and Hodé, 2009a), and the search domain (segmented regions) can be reduced by data partitioning. Moreover, it is possible to assign an order of priority to the arcs such that less time consuming arcs can be process before the others. All these points show that this approach is valuable and can be complementary to numerical or statistical approaches.

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