

HYBRID ALGORITHM FOR FUZZY MODEL PARAMETER ESTIMATION BASED ON GENETIC ALGORITHM AND DERIVATIVE BASED METHODS

A. Lavygina and I. Hodashinsky

*Department of Data Processing Automation, Tomsk State University of Control Systems and Radioelectronics
40 Lenina Street, Tomsk, Russian Federation*

Keywords: Fuzzy Modeling, Parameter Estimation, Hybrid Algorithm, Genetic Algorithm, Gradient Descent Method, Kalman Filter, Least Squares Method.

Abstract: Hybrid method for estimation of fuzzy model parameters is presented. The main idea of the method is to apply gradient descent method or Kalman filter as a mutation operator of genetic algorithm for estimation of antecedent parameters of fuzzy “IF-THEN” rules. Thus, part of the individuals in the population mutate by means of gradient descent method or Kalman filter, the others mutate in an ordinary way. Once antecedents are tuned, consequents tuning is performed with the least squares method. The results of computer experiment are presented.

1 INTRODUCTION

Input-output representation mapping in fuzzy models is presented as a set of fuzzy “if-then” rules. Each rule consists of two parts: antecedent and consequent. An antecedent (conditional part) contains a statement regarding input variables values, while a consequent presents the value that output variable takes. The rules for single fuzzy model are as follow:

Rule i : IF $x_1 = A_{1i}$ AND $x_2 = A_{2i}$ AND... AND $x_m = A_{mi}$ THEN $y = r_i$,
where A_{ji} is a linguistic term to evaluate variable x_j , while output y is evaluated by real number r_i .

The model performs the mapping $F: \mathfrak{R}^m \rightarrow \mathfrak{R}$, substituting fuzzy conjunction operator by product, and fuzzy rules aggregation operator is replaced by addition. Mapping F for singleton type model is defined by the formula:

$$F(\mathbf{x}) = \frac{\sum_{i=1}^n \mu_{A_{1i}}(x_1) \cdot \mu_{A_{2i}}(x_2) \cdot \dots \cdot \mu_{A_{mi}}(x_m) \cdot r_i}{\sum_{i=1}^n \mu_{A_{1i}}(x_1) \cdot \mu_{A_{2i}}(x_2) \cdot \dots \cdot \mu_{A_{mi}}(x_m)} \quad (1)$$

where $\mathbf{x} = [x_1, \dots, x_m]^T \in \mathfrak{R}^m$, n is the number of fuzzy model rules, m is the number of input

variables in the model, $\mu_{A_{ji}}(x_j)$ is a membership function of j -th variable to term A_{ji} .

2 FUZZY IDENTIFICATION

The problem of fuzzy identification is the following: the results of observations of input and output variables of the system must be designed in optimal fuzzy model. Optimality criterion is the smallest error.

In the initial phase of fuzzy model tuning before using parameter estimation algorithms it is necessary to identify the fuzzy model structure and initial values for antecedent parameters and rules consequents.

Parameter estimation is carried out by two types of methods: 1) methods based on derivatives (least squares method, gradient descent method, Kalman filter), 2) metaheuristics (genetic algorithm, algorithm of ants colony, particle swarm method, simulated annealing method and search with bans). Derivative based methods are more accurate but can get stuck in local minimums. Metaheuristic methods are more stable, but require more time resources.

3 HYBRID ALGORITHM

Hybrid algorithm allows to combine advantages of both metaheuristics and derivative based methods. This association will enhance the quality of decisions compared to using methods individually.

Hybrid algorithm based on genetic algorithm and derivative based methods (gradient descent method, Kalman filter, least squares method) has been developed. The following hybridization technique is suggested. At the first stage, least squares method is used to adjust consequents parameters. Then, modified genetic algorithm is started. The essence of the modification is in applying gradient descent method or Kalman filter together with mutation operator of the genetic algorithm to tune antecedent parameters. In doing so, part of the individuals in the population change with gradient descent method or Kalman filter. Such mutation take place with a probability p' ($p' \in (0, p)$, where p – probability of individual mutation). The other individuals are subject to mutation with the probability $p-p'$ by means of random one-point or multipoint mutation. Once antecedents are tuned, consequents tuning is performed with least squares method.

4 SIMULATION RESULTS

The idea of the experiment was to use fuzzy model for approximation of the following test functions:

- a) $f(x_1, x_2) = \sin(2x_1/\pi) \cdot \sin(2x_2/\pi), x_1, x_2 \in [-5;5]$
- b) $f(x_1, x_2) = x_1 \cdot \sin(x_2), x_1, x_2 \in [-\pi/2; \pi/2]$.

Based on test functions, the tables of 121 lines were built and then, using these tables training of fuzzy models was carried out. Triangular-shaped membership functions are considered for each variable.

Figure1 shows the results of the suggested hybrid algorithm and separate methods used for selected test functions. In the left column of the histogram the mean-square error (MSE) of initial solution is given, the other columns correspond to the averaged values of MSE of the fuzzy model for each of the algorithms.

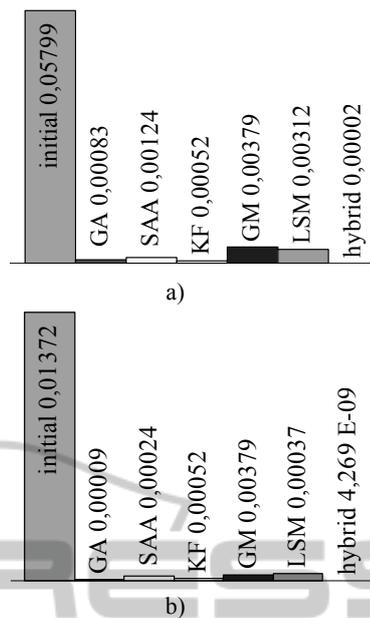


Figure 1: Experiment results for test functions a)-b) (GA – genetic algorithm, SAA – simulated annealing algorithm, LSM– least squares method, GM – gradient descent method, KF – Kalman filter, hybrid– suggested hybrid algorithm).

The results of the experiment allow to conclude that the suggested hybrid algorithm provides better results than each method separately.

To compare the developed hybrid algorithm with the existing methods of building fuzzy models, the study of approximation results was carried out for the nonlinear functions presented in table 1.

The values of mean-square approximation error, obtained with the developed algorithm and analogues for these functions are shown in table 2.

Considering the obtained results, it is possible to conclude that the suggested hybrid algorithm in most cases yields fewer errors compared to the existing analogues.

Table 1: Data sets considered in experimental analysis.

Test Function	Number of Observation
c) $f(x) = (1 + 10 \cdot \exp(-100 \cdot (x - 0,7)^2)) \cdot \left(\frac{\sin(125/(x+1,5))}{x+0,1} \right), x \in [0;1]$	100
d) $f(x_1, x_2) = (1 + x_1^{-2} + x_2^{-1,5})^2, x_1, x_2 \in [1;5]$	400
e) $f(x_1, x_2) = \sin\left(\frac{2x_1}{\pi}\right) \cdot \sin\left(\frac{2x_2}{\pi}\right), x_1, x_2 \in [-5;5]$	441

Table 2: MSE values obtained with the proposed hybrid algorithm and other authors' algorithms.

Function	Algorithm	Number of Rules	MSE
c	(Mitaim et al., 1996)	12	1.426
	(Lisin et al., 1999)	12	0.247
	Hybrid algorithm	12	0.0169
d	(Rojas et al., 2000)	9	0.146
		16	0.051
		25	0.026
		36	0.017
	(Sugeno et al., 1993)	6	0.079
	(Nozaki et al., 1997)	25	0.0085
	(Teng et al., 2004)	4	0.016
	(Lee, 2008)	3	0.0028
	(Wang et al., 2005)	3	0.0052
	(Tsekouras et al., 2005)	6	0.0108
		9	0.0168
	Hybrid algorithm	16	0.0018
		25	0.0002
25		less than 0.001	
e	(Lee, 2008)	25	less than 0.001
	Hybrid algorithm	25	0.00009

Lisin, D., Gennert M.A., 1999. Optimal Function Approximation Using Fuzzy Rules. In *Proc. Int. Conf. North American Fuzzy Information Processing Society*.

Rojas, I., Pomares, H., Ortega, J., Prieto, A., 2000. Self-organized fuzzy system generation from training examples. In *IEEE Transactions on Fuzzy Systems*, vol. 8 (1).

Nozaki, K., Ishibuchi, H., Tanaka H., 1997. A simple but powerful method for generating fuzzy rules from numerical data. In *Fuzzy Sets and Systems*, vol. 86.

Sugeno, M., Yasukawa, T., 1993. A fuzzy-logic-based approach to qualitative modeling. In *IEEE Transactions on Fuzzy Systems*. vol.1, no. 1.

Teng, Y., Wang, W., Chiu, C.H., 2004. Function approximation via particular input space partition and region-based exponential membership functions. In *Fuzzy Sets and Systems*, vol. 142.

Tsekouras, G., Sarimveis, H., Kavakli, E., Bafas G., 2005. A hierarchical fuzzy-clustering approach to fuzzy modeling. In *Fuzzy Sets and Systems*, vol. 150.

Wang, H., Kwong, S., Jinb Y., Wei, W., Man, K.F., 2005. Multi-objective hierarchical genetic algorithm for interpretable fuzzy rule-based knowledge extraction. In *Fuzzy Sets and Systems*, vol. 149.

5 CONCLUSIONS

The results of the experiment allow to conclude that:

- The suggested hybrid algorithm based on genetic algorithm and derivative based methods provides better results than each method separately;
- The suggested hybrid algorithm for fuzzy models tuning allows to achieve smaller error values in most cases compared to existing analogues.

ACKNOWLEDGEMENTS

This paper is supported by Russian Foundation for Basic Research (09-07-99008).

REFERENCES

Mitaim, S., Kosko, B., 1996. What is the best shape for a fuzzy set in function approximation? In *Proc. Fifth IEEE Int. Conf Fuzzy Systems*, vol. 2. New Orleans

Lee, Zne-Jung, 2008. A novel hybrid algorithm for function approximation. *Expert Systems with Applications*, vol. 34.