# ON THE INTEGRATION OF KNOWLEDGE IN A PROPOSITIONAL LOGICAL LAYER

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Abstract: In general knowledge is complex which means that it is not an isolated entity, but a phenomenon which is coming (or can be derived) from different sources of knowledge. The position of the present paper is that it can be highly beneficial to study the integration of knowledge coming from different knowledge sources as an explicit propositional logical layer in knowledge engineering. Here the term of *proposition* is to be understood in the understanding of Frege as the inherent sense of a formal expression. We discuss a certain family of propositional logics – the so-called $\in_T$-logics – which allow for an explicit interpretation of formulas as propositions. We argue that the family of $\in_T$-logics and their models offer a very expressive logical setting which is able to realize a certain scenario of the integration of knowledge from different knowledge sources.

## 1 MOTIVATION

The initial observation on which the position of the present paper is based is that in the general case knowledge is complex and thus not appearing in an isolated form, but in a form of combination and integration of information entities coming from different sources. The thesis of this paper is that an explicit study of the propositional logical conditions of combinations and integrations of knowledge can be highly beneficial for achieving a *sense* (in the Fregean understanding of the term) related form of integration and to reproduce natural ways of combining and integrating knowledge adequately.

Most approaches to the integration of knowledge and information in today's existing information systems include the use of certain ontological concepts. An integration of heterogeneous information or knowledge entities is realized by breaking down the entities to the smallest concepts of the domain of discourse given by the ontology. The integration then takes place on this level of the ontology by the identification of similarities or relationships between the single parts of the information entities (compare for example with the ideas of ontology-based data integration). The uses of ontologies include, beyond others, single ontology and multiple ontology approaches. In the first case one single ontology is used for the description of concepts, while in the second case a combination of multiple ontologies is used which requires for the definition of adequate mappings between the involved ontologies (see (Wache et al., 2001)). It is one important observation for the idea of the present paper that such a definition of mappings between ontologies must include certain logical considerations to guarantee a consistent cooperation of the involved ontologies.

It is the position of this paper that the described ontology based approaches of breaking down entities to their smallest concepts does not reflect the natural way in which integrations of knowledge take place. Furthermore ontologies are very fixed and thus rather inflexible concepts which do not allow for sudden context dependent changes of the meanings of certain concepts. However, such context dependent changes in the conceptions of things are very natural in the human way of knowledge exchanges, negotiations, or discussions.[1] The position of this paper is that due to that naturality it is of interest and necessity to study ways for formal describing and reasoning[2] about integrations of knowledge which first of all take the inherent senses of entities and their relationships on a semantical level into account.

In computer science one can observe two conceptions of integration which can be characterized as *bottom-up* or *top-down* approaches. The first case

[1] Compare for example with the explanations on *conceptions* of Mahr in (Mahr, 2010a).

[2] Compare with (Davis et al., 1993, Role 3).

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corresponds to the described approach of ontologies where one defines the domain of discourse first as a fine granulated basis for the understanding of entities and where integration of heterogeneous entities originates from this fine granulated basis. The second case of *top-down* approaches to integration is more close to the human thinkings, as it takes the relationships between complex concepts of different entities into account first, and only afterwards breaks down the relationships to more granular units.

If one attempts to integrate entities of a different form from a logical point of view one does not have to specify the nature of every single information unit in the first place, but is more interested in the logical characteristics of the integration, that means the relationships between complex concepts. The similarity to the concept-driven thinking of the humans gives a justification for an additional logical consideration of the integration of knowledge in knowledge engineering.

Consider for example the development of a new car model. Here the variety of knowledge ranges from aspects of the molecular nature of the tires of the car to questions of the included sound system. Thus the variety of knowledge is so huge and divers and provided with multiple different languages and logical principles that it cannot be adequately covered in one view. It is an obvious observation that in the industrial development an integration of the different views of the developers on the car does not start from breaking down their concepts to the smallest bits of information, but in fact by integrating their views on a logical level of concepts. Although a multiple ontology approach to this example can be successful and as well must include logical considerations in the definitions of the mappings between the ontologies, these logical considerations are not explicitely given and it cannot be reasoned about the logical characteristics of the integration of concepts of the several views. Thus there is a need for a logical layer in the field of knowledge integration.

In this paper we discuss a formal treatment of knowledge integration in such a logical layer which describes integration over different sources. As mentioned before such a logical layer must take the inherent senses of entities (or concepts) explicitely into account. To do this we understand the term of knowledge as *propositional knowledge* in the sense of Searle (see (Searle, 2008)). Here the notion of proposition can be defined as follows[3]:

A *proposition* is the inherent *sense* of a formal expression. Propositions can be either true or false.

[3] For detailed essays on the notion of propositions see for example (Mahr, 2010b) and (Robering, 2011).

Here the notion of *sense* is used in the understanding of Frege as the inherent idea of a formal expression (see for example (Frege, 2008; Frege, 2007)). In the following sections we will define a *scenario of integration of knowledge and information* which can be seen as a setting allowing for the studying of certain kinds of integrations. We discuss a certain family of *propositional logics*, the family of $\in_T$-logics. In the understanding of the present paper a propositional logic is not to be understood in the classical way where expressions are directly evaluated to truth values, but as a logic in which formulas are explicitely interpreted as propositions and where these propositions are explicitely available as entities in the semantics of the logics.

It is the position of the present paper that the family of $\in_T$-logics and their models offer a very expressive logical tool which can be used to realize the scenario and thus a wide range of information and knowledge integrations. Here the integration over different sources of knowledge is realized by a representation of the knowledge sources as models of certain propositional logics. These models are integrated into an $\in_T$-logic which acts as a meta-level language for the description of and reasoning about the underlying sources of knowledge, their relationships and their integration. More practical questions like for example questions of a representation of concrete existing sources of knowledge in certain propositional logics shall not be dealt with in the present paper.

## 2 SCENARIO OF INTEGRATION OF KNOWLEDGE AND INFORMATION

Part of the position of this paper is that any real knowledge integration can be adequately comprised by the following scenario of integration which is a generalization and advancement of the work of (Mahr and Bab, 2005). The goal of the following scenario is not to give a definition of the notion of integration in knowledge engineering, but to state a formal setting which allows for the studying and reproduction of the results of knowledge integrations in a logical fundament.

For a studying of knowledge integrations in a logical setting two different cases are conceivable which both shall be represented in our scenario. The first is to assume that we have given a complex integrated knowledge object and that we logically describe and reason about the integration of the knowledge sources which has taken place and which led to the integrated object. The second is to assume to have given a certain set of knowledge sources on an object, but not the integrated knowledge object itself. The task in this second case is not only to describe an already existing integration of different sources of knowledge, but to actually *perform* the integration in the logical setting. In this case questions of consistency arise which have to be dealt with in the integration, while in the first case with a given integrated knowledge object the different knowledge sources on the object can be considered as consistent as there already has taken place a successful integration of the different sources of knowledge.

The scenario of integration is defined as follows:

> **Scenario of Integration of Knowledge and Information.** Given different *views* $V_1, \ldots, V_n$ on a complex knowledge object $A$. Each view $V_i$ represents a source of knowledge for the object $A$. Each view can be identified by a model of a certain propositional logic. Each of these models includes a set of propositions by explicitely interpreting any formula of the logics as true or false propositions. Each of these sets of propositions represents the knowledge of the corresponding view on the object $A$.
>
> The goal of integration is to define a meta-level propositional logic which offers means of expression to describe and reason about that certain set of propositions which is representing the *integrated knowledge* of the single sets of propositions given by the single models of the views on the object $A$.

It is the position of the present paper that the proposed scenario of integration of knowledge and information is adequate to cover most of the concepts of integration in knowledge engineering. An explicit interpretation of the sources of knowledge on a complex object $A$ as certain sets of propositions and the construction of new propositions stating any relationships between the single propositions results in such a set of propositions which represents the whole integrated knowledge on the object $A$. The construction of a propositional logic which allows for the description of and reasoning about the resulting set of propositions has the advantage that even complex concepts (represented in certain complex formulas) do not have to be broken down to their atomic parts, but do themselves denote certain propositions in the models of the propositional logic.

We will state a possible realization of the scenario of integration of knowledge and information based on the concepts of propositional logics and $\in_T$-logics in Section 4.

# 3 A SHORT OUTLINE OF $\in_T$-LOGICS

In the following section we want to discuss a possible realization of the scenario given in the previous section using the logics of the family of $\in_T$-logics. Every $\in_T$-logic has a modeltheoretic semantics where the semantic entities are given by propositions. To get a better understanding of the ideas of $\in_T$-logics we will now give a short outline of the most important works in this field.

$\in_T$-logics are propositional logics offering means for formulating self-referential and explicit truth statements while preserving a total truth predicate and thus being free from antinomies. The first logic in this field is the *classical $\in_T$-logic* by Sträter (see (Sträter, 1992)) which forms a theory of truth and propositions in the context of the re-construction of natural language semantics by means of self-referential structures (see for example (Mahr et al., 1990; Mahr, 1993; Bab et al., 2008)). The expressions of classical $\in_T$-logic are built over propositional variables, constants, classical connectives, means for the quantification over propositional variables, as well as means for propositional equivalence and predicates for truth and falsity.

The works of Sträter were picked up by Zeitz in (Zeitz, 2000) where he extends Sträters concept in the way that every $\in_T$-logic can extend an arbitrary underlying object-level logic (represented in a certain abstract form) whose components become constants in the corresponding $\in_T$-logic. In this context Zeitz' $\in_T$-logics can be interpreted as a logic theory for the meta-level reasoning over propositions originating from underlying arbitrary logics.

In (Bab, 2007) Bab generalized and newly interpreted the work of Zeitz in the way that his class of $\in_\mu$-logics subsumes the concepts of Zeitz $\in_T$-logics, but furthermore allows for the integration of modalities from arbitrary modal logics as means of expression in a Kripke-like semantics. Thus, until this point, $\in_\mu$-logics are the most expressive and general logics in the field of $\in_T$-logics allowing for the reasoning about situation dependent propositions. Moreover, Bab gave a new interpretation of $\in_T$-logics as a theory of propositions.

Sträter, Zeitz, and Bab stated different model existence theorems which prove that all $\in_T$-logics are free from antinomies despite their total truth predicates and their ability to model self-referential sentences and impredicative quantification. $\in_T$-logics have been proven to be a suitable concept for truth and reference, because they avoid antinomies which necessarily appear with logics having total truth predicates and at the same time allow for representations of decidable relations and computable functions (see (Tarski, 1935)). Furthermore $\in_T$-logics allow for an implicit representation of antinomies like the *liar paradox* over unsatisfiable propositional equations (see for example (Sträter, 1992; Zeitz, 2000; Bab, 2007)).

The theory of intensional models of $\in_T$-logics was widely extended by the work of (Bab and Wieczorek, 2010) which shows that $\in_T$-logics offer a wide range of different intensional models. Furthermore there exist sound and complete calculi for the $\in_T$-logics by Sträter and Zeitz.

The interpretation of formulas as propositions in $\in_T$-logics is achieved by a *sense function* which is part of every model of every $\in_T$-logic. Here the interpretation of formulas as propositions is not arbitrary, but must comply with certain natural rules and concepts of logic. The models of an $\in_T$-logic define the range of propositions which can be denoted by formulas. However, the interpretation of formulas as propositions is free in the sense, that it can differ from one model to the other. Thus $\in_T$-logics offer logical concepts which allow for the studying of the inherent senses of formulas in different contexts. By using the situation dependency of, for example, the class of $\in_\mu$-logics, one gets a rich logical tool for a formal treatment of integrations.

Another logic in the family of $\in_T$-logics, but of a different nature, was defined by Lewitzka in (Lewitzka, 2009). His $\in_I$-logic offers a non-Fregean intuitionistic logic with a truth predicate and a falsity predicate as intuitionistic negation. $\in_I$-logic can be seen as an extension of the works of Zeitz, but without using the concept of quantification.

## 4 DESCRIBING AND REASONING ABOUT KNOWLEDGE INTEGRATIONS IN $\in_T$-LOGICS

In the following we want to state a possible realization of the scenario given in Section 2. It is the position of the present paper that the family of $\in_T$-logics and their models offer a very expressive logical setting which is able to realize the scenario and thus a wide range of information and knowledge integrations. The proposed realization of the scenario can be achieved in the following way:

1. In a first step the different views on the object $A$ must be represented by models of certain propositional logics which offer sets of true and false propositions describing the propositional knowledge which can be said about $A$ from the point of the corresponding views. These model constructions can be accomplished by models of certain $\in_T$-logics of appropriate expressive power.
2. In a second step the propositions of the single views have to be integrated to form an overall set of propositions representing the integrated knowledge on $A$. This integration of propositions can take place in a certain model of an $\in_T$-logic acting as an *integration logic* in which single propositions are combined to more complex propositions according to logical connectives. The $\in_T$-logic here acts as the *integrator* performing the integration depending on certain specifications of the aspired method of integration.

   In this step inconsistencies between the single views become directly obvious if there exists for example a proposition which occurs to be a true proposition in the one view and a false proposition in the other. In this case we have a proposition which is true in the context of one view and false in the context of another view which cannot be the case in any consistent integration process. In the following we assume that the different sources of knowledge resp. their representation as sets of propositions of the corresponding views are consistent.
3. The goal of integration can then be accomplished in a third step by defining a certain $\in_T$-logic which offers means of expression to state and to reason about any of the propositions of the integrated knowledge on $A$.

When looking at the realization one could argue that the integration logic of the second step would be sufficient to act as the one logic which meets the goal of integration as stated in the scenario of integration of knowledge and information. In the sense of this paper, however, there is an important difference between the logic which *performs* the integration and a logic which can *describe* or *reason about* the integrated range of propositions from a meta-level perspective. A logic of the latter sense can represent an integration in a much more general and common way which is due to the observation that in certain integration situations there is a need to regard an integrated object in a language which is completely independent from the languages of the single views to be integrated.

The existence of sound and complete calculi for the $\in_T$-logics by Sträter and Zeitz and the proofs of the existence of certain intensional models for every $\in_T$-logic can be seen as an indicator that the proposed realization of the scenario using the family of $\in_T$-

logics is a reasonably approach to an explicit logical handling of integrations.

## 5 CONCLUSIONS AND FURTHER WORK

In the present paper we argued that a study of integrations of knowledge from different knowledge sources in an explicit propositional logical layer is a natural approach which can be highly beneficial to enrich the studies of the integration of knowledge in knowledge engineering.

The proposed scenario of integration of knowledge and information is not limited to the field of knowledge engineering, but can be taken as a basis for a general understanding of integrations in computer science. The family of $\in_T$-logics allows for the meta-level reasoning about integrations like knowledge integration by integrating the sets of propositions which represent the states of affair of the underlying different sources of knowledge. The explicit interpretation of formulas as propositions in the $\in_T$-logic layer thus allows for an integration which explicitely takes the *senses* of the objects and the logical characteristics of the integration, that means the relationships between complex concepts, into account. The wide range of different models of $\in_T$-logics allows for representing and comparing arbitrary different ideas of integration as any model includes its own set of denotable propositions and its own sense function which interprets the formulas as propositions.

The presented ideas of this work can be seen as a basis for a treatment of knowledge integrations which extends the established regarded concepts by an explicit propositional logical layer. However, the proposed ideas of a scenario of integration and its realization in propositional logics like $\in_T$-logics does have to be elaborated in detail, which is part of ongoing and soon to be published work of the author. Beyond that studies on a theoretical logical framework which can be seen as the main attention of the work of the author more questions emerge from a more practical point of view. These include questions of the practical representation of specific knowledge sources as models of certain propositional logics or the elaboration of a real application in form of a case study.

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