COULD TYRANNOSAURUS RUN FAST?
Mechanical Power Calculation for 15.7 m/s Tyrannosaurus Running

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Abstract: Running ability of large bipedal theropod Tyrannosaurus is studied with the use of evolutionary computational method. In 2002 Hutchinson and Garcia published a paper titled as "Tyrannosaurus was not a fast runner" (Hutchinson and Garcia, 2002). They postulated an arbitrary mid-stance posture in running motion, and calculated required muscle mass of the hind limb. This method can not tell information on running speed, because it is a static method. Then, running speed of Tyrannosaurus could not be evaluated quantitatively. We accomplished numerical simulation to obtain whole running motion of Tyrannosaurus with the use of evolutionary computation method. As a result, we have obtained running motion of Tyrannosaurus in a speed of 15.7 m/s within allowed parameters range. We have discussed on mechanical power output of the running motion of Tyrannosaurus for the first time in this research area. As for a problem of the simulation algorithm, there is room to improve simple evolutionary computation method applied in the present work. Generally, a solution of evolutionary computation method falls into a local minimum. However, finding the global minimum of the evaluation function i.e., velocity and vertical acceleration are needed for this problem. Then, developing such an algorithm is left as the future problem.

1 INTRODUCTION

A problem of running ability of dinosaurs has not been solved for many years. Both of the numbers of theoretical researchers, and evidence of foot print that show running motion have been quite limited. Early estimation on running speed of Tyrannosaurus was accomplished by Bakker and Paul in 1986 and 1988, respectively, based on morphological consideration of muscle and limb structure (Bakker, 1986; Paul, 1988). Bakker and Paul proposed 45 mph and 40 mph running, respectively, which correspond to the speed of 20 m/s and 17.9 (≈ 18) m/s. These estimates are almost two times faster than one of the humans at best. A monumental qualitative study was achieved by Alexander with the use of Froude number $F_R = \frac{v^2}{g \cdot h}$, where $v$, $h$, and $g$ are velocity, characteristic length of leg and gravitational constant, respectively. He employed dynamic similarity hypothesis such that animal walk or run in a dynamically similar fashion at the same Froude number (Alexander 1976, 1985, 1989, 1996, 2006; Alexander and Jayes1983). With comparing extant animal data, he found that relative stride $L_s/h$ has a relation with Froude number as,

$$\frac{L_s}{h} = 2.3\left(\frac{v^2}{g \cdot h}\right)^{0.3}$$

(1)

where $L_s$, $h$, $v$ and $g$ are the stride length which is a distance between two successive falls of the same foot, and hip height, velocity, and gravitational constant, respectively (Alexander 1976). Alexander proposed that the hip height $h$ can be estimated from foot print of dinosaurs as $h=4FL$, where $FL$ is foot length. Since then, many improvements have been added to this expression. (Gatesy and Biewener 1991; Russell and Beland 1976; Thulborn 1981, 1989, 1990; Rainforth and Manzella 2007; Wallace and Brooks, 2003). Recently, Rainforth and Manzella re-analyzed this factor using 24 specimens from different dinosaurian groups, and concluded that speed estimation could be incorrect by a factor of two (Rainforth and Manzella, 2007).

From fossilized foot print remains two evidence of large dinosaurs running motion have been reported. Farlow and Day et al. reported that 11.1 m/s and 8.11 m/s running evidence of medium and large size bipedal theropod is obtained, which are
calculated using Eq.(1) (Farlow, 1981; Day et al. 2002). Although the absolute value of running speed has to be re-considered with care, foot prints show that bipedal theropod was a fast runner with comparable to human.

In 2002 Hutchinson and Garcia published a paper titled as "Tyrannosaurus was not a fast runner" (Hutchinson and Garcia, 2002). They accomplished static calculation in which required muscle mass is calculated for an arbitrary chosen mid-stance posture in running motion. The result yields that 43 % muscle mass is required for one hind limb of Tyrannosaurus. On the other hand, computer aided mass property study revealed that the value is 16.0 % for MOR555 (Bates et al. 2009), 14.2 % for MOR 555 (Hutchinson et al. 2007), 14.4 % for BH13033 (Bates et al. 2009). The abbreviation stands for Museum of Rockies, and Black Hills Institute, respectively. Then, Hutchinson concluded that Tyrannosaurus was not a fast runner.

In 2004 Hutchinson re-evaluated this value, and obtained a result that 21 % muscle mass is required for one hind limb of *Tyrannosaurus* (Hutchinson 2004b). Recently, Gatesy et al. calculated the value for various mid-stance postures (Gatesy et al. 2009). The methodology is the same with Hutchinson and Garcia’s, then, the speed estimation is heuristic. The results are considered as medium speed running is possible, because 18.3 (~1.87 × 9.8) m/s² vertical acceleration is allowed in their estimation for static postures.

In our recent studies, re-consideration of involved parameters leads to almost two folds error in the estimation of required muscle mass (Usami, 2011a-c). Figure 1 displays 3D bone structure of BH13033 specimen taken by using 3D laser scanner FARO LS. Recently, Sellers and Manning accomplished numerical simulation study for this problem. They obtained that about 8–9 m/s running is possible for *Tyrannosaurus* (Sellers and Manning, 2007).

![3D bone structure of BH13033 specimen taken by using FARO co. LS.](image)

Instead of this methodology, this paper presents a result of calculation of mechanical power in running motion of *Tyrannosaurus*. This methodology excludes uncertainty of many biomechanical parameters. Then, this evaluation is superior to the static evaluation in evaluating running speed of *Tyrannosaurus*.

## 2 DYNAMICAL CALCULATION OF THE LOCOMOTION OF *TYRANNOSAURUS*

This section describes the method of numerical simulation of *Tyrannosaurus* running motion.

### 2.1 A Solid Object Approximation and Calculation of the Moment of the Force

Running motion is a periodic one, hence, expressing time change of each joint angle by Fourier series is appropriate. Validity of this method was checked in advance on human locomotion. The motion capture of human running motion was accomplished by the combination of optical measurements and the use of force plate on the ground. These data were analyzed by the reliable system VICON (Vicon Motion Systems). Next, time change of each joint angle is expressed by Fourier expansion series. Convergence within 1 % accuracy is checked by taking into account of 5th order Fourier expansion series. Thus, an expression of 5th order Fourier expansion is a good method to describe the periodic motion of each joint. For \( i \)-th joint angle \( \theta_i(t) \) the expansion is expressed as follows,

\[
\theta_i(t) = a_i^0 \sin(\omega t + \delta_i^0) + a_i^1 \sin(\omega t + \delta_i^1) + \cdots \quad (2)
\]

where \( a_i^j \), \( \delta_i^j \) are the amplitude and the phase of \( j \)-th order of expansion series for \( i \)-th angle, respectively. The \( \omega \) is angular velocity. The segment structure of *Tyrannosaurus* is the same of Hutchinson and Garcia’s model shown in Fig. 2(a) (Hutchinson and Garcia, 2002). To study time-dependent dynamics a solid object model is used to describe the motion of *Tyrannosaurus* limb. Namely, the model *Tyrannosaurus* moves as one solid object for the external force \( \vec{F}(\vec{r}) \) as the following equations,

\[
m_{body} \frac{d^2 \vec{X}}{dt^2} = -m_{body} \vec{g} + \vec{F}(\vec{r}) \quad (3a)
\]

\[
\int \frac{d^2 \Phi}{dt^2} = \vec{r} \cdot \vec{F}(\vec{r}) \quad (3b)
\]
where $\vec{X}$ and $\Phi$ are the position vector of the center of mass and the rotational angle of the object, respectively. The calculation is achieved in the sagittal plane, i.e., two-dimensional space $x$ (horizontal) and $y$ (vertical). $I$, $g$ and $\vec{r}$ are the momentum of inertia of Tyrannosaurus, gravitational constant, and the position vector to the point of the force, respectively. The term $-m_{\text{body}}g\vec{j}$ expresses that gravitational force acts in vertical direction $y$. The value of inertia $I$ is chosen as $I=19000 \text{ kg} \cdot \text{m}^2$ in our work with the references (Hutchinson et al. 2007; Bates et al. 2009). A solid object model is simple, however, it is known to express dynamics of moving object with many degree of freedom (Usami et al. 1998).

For calculating joint torque, or moment of the force about joint, a free-body diagram analysis is applied as shown in Fig.2(b). For example, let us call the foot segment as segment “1”, and define the mass and the moment of inertia as $m_1$ and $I_1$, respectively. Then, the equations of motion for translation and rotation become as follows in $(x, y)$ plane,

$$\vec{F}_1 - \vec{F}_2 - m_1g\vec{j} = m_1\vec{a}_1$$  \hspace{1cm} (4a)

$$\vec{x}_{1g} \times \vec{F}_1 - \vec{x}_{2g} \times \vec{F}_2 + M_1 - M_2 = I_1\vec{\omega}_1$$  \hspace{1cm} (4b)

where $\vec{F}_1$, $\vec{F}_2$ and $\vec{a}_1$ are the force from downside segment, the force from upper segment and acceleration, respectively. For rotational motion, $\vec{x}_{1g}$ and $\vec{x}_{2g}$ are the vectors from the center of mass of 1-th segment to the point acting force $\vec{F}_1$ and $\vec{F}_2$. $M_1$ and $M_2$ are the moment of force between the 0-th and 1-th, 1-th and 2nd segments, respectively. For the case of 1-th segment $\vec{F}_1$ corresponds to the ground reaction force. $\vec{\omega}_1$ is time derivative of angular velocity. Putting known terms $\vec{F}_1$, $\vec{a}_1$, $\vec{\omega}_1$ and $M_1$ into Eq.(4) yields unknown terms $\vec{F}_2$ and $M_2$. Thus, the moment of force acting to upper segment is obtained, subsequently. In the usual motion the term $\vec{\omega}_1$ and $m_1\vec{a}_1$ are negligible. Then, these terms are set as zero in our calculation. The parameter for ground reaction is described in our paper in detail (Usami, 2011c).

### 2.2 Evolutionary Computation Method

#### Solid Object Approximation for the Motion of the Whole Body

Searching the optimal Fourier coefficient $\delta_i$ in Eq.(2) for running motion is the next task. The other parameters are fixed in the simulation. Computational method for obtaining the optimal solution in many degree of freedom is usually not an easy task. So, a variety of approximation methods have been proposed in many research areas. One of the famous and well studied methods is the genetic algorithm (GA) (Fraser 1970; Holland 1975; Goldberg 1989). A vast number of studies has been published in many research areas concerning to GA. This method is based on an idea of gene evolution observed in the actual life system. In this method digitized virtual genes are introduced, and its evolution is simulated. The virtual gene falls into the stable state in which a value of evaluation function has the local minimum. On the contrary, the introduction of virtual genes is not necessary in the present study. So, looking for another convenient approximation method is appropriate. Another approximation method for obtaining near optimal solution is the evolutionary computational method (Sellers and Manning 2007; Usami 1998; Fogel 1995). This method is known as well as genetic algorithm method on searching near optimal solution. This method does not introduce digitized virtual gene, but change the parameters of the system directly. A value of the evaluation function converges into the local minimum rapidly, and the result is usually satisfactory. Then, we use evolutionary computational method for this problem.
At first, we create several typical patterns of running motion at hand using 3D software 3dsmax (Autodesk co.). The typical patterns include various motions from flexed one to upright one. Next, we apply dynamical simulation described above. Our method does not introduce virtual gene as appeared in the usual genetic algorithm, but, changes parameters itself in the Eq. (2).

Let us call the initial set of parameters as the initial parent. The initial parent makes 2000 children whose parameters are slightly different from the initial parent. The different parameters are generated by a random algorithm. The running simulation is applied for the 2000 children, and the best performance child is selected among those. Then, the best performance child becomes the parent in the next generation. Thus, near optimal solution on running motion is obtained as a result of the evolutionary computation scheme. The term “best performance” is determined by the introduction of the evaluation function. It is discussed in the next section in detail.

3 MECHANICAL POWER IN TYRANO-SAURUS RUNNING MOTION

In this section result of calculation of mechanical power output for Tyrannosaurus running motion is presented.

3.1 Moment of Force, Angular Velocity and Mechanical Power

In the simulation, we have carried out calculation in a range of cycle time $T=0.5$–$0.7$ (s). Many different running patterns are appeared in the simulations with different initial conditions. Let us define ground reaction force multiplier factor $G$ as $F = -m_{\text{body}}Gg$, where $F$ and $m_{\text{body}}$ is ground reaction force in the vertical direction and the total mass of Tyrannosaurus. Then, biomechanical knowledge tells that moderate speed running motion is realized when $1 < G < 2$, and fast running is realized when $2 < G < 4$. If $G$ exceeds 4, the running motion is said to be beyond reality.

As for the evaluation function in the optimization process, we have tried several types of evaluation function such as vertical acceleration, running velocity and the product of velocity and vertical acceleration, etc. Figure 3 displays a successful case that local minimum of vertical acceleration appeared in the simulation, which is shown as A and B. A and B give the same value of the local minimum in the simulation. However, corresponding velocities are different as shown by the circle in Fig.3. Then, B’ gives a slightly better solution in the criteria of velocity times vertical acceleration.

Hamming distance for subsequent generations is also plotted in Fig.3. This data shows that small change of the parameters affects pattern of locomotion, which leads to the change of vertical acceleration and running velocity. In the computer simulation most of the examples yielded bad results, namely, there was no appropriate local minimum. Or, in many cases vertical acceleration increased in the simulation of the evolution algorithm. Figure 4 displays such an example. It started from a slightly different initial condition from the one of Fig.3. Thus, a result of simulation depends on the choice of the initial condition. Then, we have carried out many running simulations for searching a solution which gives the lowest vertical acceleration and the fastest running velocity.

Generally, an evolution algorithm is able to obtain a local minimum of evaluation function. However, we have to obtain the global minimum of the evaluation function, i.e., running speed and vertical acceleration. Thus, searching efficient algorithm, which brings an appropriate solution for this study is left for us as the future problem.

Figure 3: An example of simulation of $4.4 \times 10^5$ generations. Vertical acceleration, velocity, velocity times vertical acceleration, and Hamming distance of subsequent generations are plotted.
Furthermore, it is noted that the present evolutionary algorithm is rather primitive. Developing better algorithm to search the global minimum would shed light on this problem. However, it is also noted that running motion in the simulation is very sensitive to a small change of parameters. The choice of the evaluation function, i.e., running velocity, vertical acceleration, or the product of those is also not a simple problem.

Then, as a whole finding the global minimum of the system is said to be a difficult problem.

In the range of cycle time $T=0.5~0.7$ (s) we have searched solution with $1< G<4$. In this range of $G$ we have obtained solution of running speed $v=8.0-15.7$ (m/s). Then, $v=15.7$ (m/s) is the maximum speed appeared in our numerical simulation with $G=3.2$, which is valid in biomechanical knowledge. Figure 5 displays stick diagram of running motion of $v=15.7$ (m/s). The cyclic time is $T=0.69$ (s) in this case, and the stride length is 10.9 (m) as appeared in Fig.5. Figure 6 shows vertical acceleration $a_y$, and moment of the force of the hip joint $M_{hip}$, knee joint $M_{knee}$ and ankle joint $M_{knee}$. The contribution of the toe joint is omitted because the ankle extensors could have been producing most of the required toe joint moments (Hutchinson 2004a, 2004b). We observe that the maximum of vertical acceleration is 31, which yields multiplier factor $G=3.2$. The maximum of the moment of force is $5.1 \times 10^4$ (N·m). Note that the center of mass is located at the hip joint in this calculation. It is known that the center of mass of *Tyrannosaurus* is located 0.3-0.7m cranial based on mass property study (Bates et al., 2009; Gatesy and Hutchinson, 2009).

It should be noted that these may change the value of the moment of force by a factor of two based on our works (Usami 2011b, 2011c). Corresponding angular velocity of each joint is plotted in Fig.7. Then, mechanical power output of each joint $P_i$ is calculated using the relation.

$$P_i = \omega_i \cdot M_i$$

$P_{\text{total}} = P_{\text{hip}} + P_{\text{knee}} + P_{\text{ankle}}$, and vertical acceleration is plotted in Fig.8. The moment of force of the hip joint $M_{hip}$ changes its sign from plus to be minus in Fig.6, then, $P_{\text{hip}}$ and $P_{\text{total}}$ shows the double peak.

In the data of biomechanical study, vertical acceleration usually shows the sine curve with time.

It is observed that $a_y$ is slightly different from sine curve in Fig. 6 and Fig.8. This is mainly due to the fact that each part of the body is smoothly connected. And, it prevents sudden change of acceleration.

Our *Tyrannosaurus* model is only made by leg part, then there is no room to absorb large change of acceleration. Each body part such as neck, trunk and tail would absorb such large change of acceleration.
in *Tyrannosaurus*. Then, we correct this discrepancy by giving sine function for vertical acceleration in the calculation of Fig.8. The result is shown in Fig.9, in which the maximum value of acceleration is set equal to the one of Fig.6. Namely, we give the sine function for vertical acceleration, and re-calculate $P_i$. The maximum power of $P_{\text{total}}$ is obtained as $3.89 \times 10^5$ (Watts) in the stance phase of 15.7 (m/s) running. Contribution of the power of the toe joint is omitted as the same reason discussed in the evaluation of the moment of force in Fig.9.

### 3.2 Mechanical Power per Kilogram of Muscle Mass, and Comparison with the Data of Extant Animals

For the evaluation of running ability, the mechanical power per kilogram muscle is calculated, and compared to the other data. The value of $P_{\text{total}}$ in Fig.9 is divided by the muscle mass of a leg. As for muscle mass of leg, 16% of the total mass are employed. 16% muscle mass per leg is the maximum ratio derived from recent mass property studies (e.g., Bates et al. 2009, Hutchinson et al. 2007).

The result is shown in Fig.10. For the comparison, the data of extant animal is introduced. Pontzer et al. recently suggested that endothermy is plausible for large bipedal dinosaurs (Pontzer et al. 2009). And at present, birds are widely recognized as flying theropod dinosaurs. These may imply high activity in locomotion for such dinosaurs. The mechanical power output of muscle has been reported in many articles. Askew et al. reported that a power output for one individual of blue-breasted quail (*Coturnix chinensis*) was 530 W/kg (Askew et al. 2001a, 2001b). They also calculated the power requirements during takeoff flights in four other species in the family Phasianidae, one species of hawk, several species of hummingbird and two species of bee. Remarkably, they concluded that, over a broad range of body size (0.0002–5 kg) and contractile frequency (5–186 Hz), the myofibrillar power output of flight muscles during short maximal bursts was very high as 360–530W/kg (Askew et al. 2001a, 2001b). On measurements on bundles of muscle fibres *in vitro* Askew and Marsh reported that the highest was 433 W/kg (Askew and Marsh, 2001a).

Smaller values were reported by Dial and Biewener as 119 W/kg for pigeon during takeoff (Dial and Biewener, 1993). For Australian rocket frogs (*Litoria nasuta*) James and Wilson reported that an average power output during takeoff in jumping varied between individuals within a range from 318 to 747 W/kg (James and Wilson, 2008).
Wilson et al. reported high power output of 900 W/kg (Wilson et al., 2000). The others are in a range of 225-550 W/kg by Marsh and Alder 1994; 360 W/kg by Lännergren et al. 1982). On mammals, high power output was observed in kangaroo jumping as 495-640 W/kg by McGowan et al. (McGowan et al. 2005). They suggested that back, trunk and tail musculature likely play a substantial role in contributing power during jumping. Inclusion of this musculature decreased the value to 452 W/kg for the power output.

To sum up, animal’s mechanical power output high values of several hundreds W/kg have been widely observed in different taxa. Relation with the result obtained in contractile property study of bundles of muscle fibre is left as future works. Currently, it is not solved yet how an animal can generate such high mechanical power output. Remarkably, Lappin et al. reported that mechanical power of mandibulae muscles of frog reached to 9600 W/kg for (Lappin et al., 2006). It is noted that lower values in different taxa have been widely reported as 107 W/kg for mouse (James et al. 1995), 110-122 W/kg for dolphin (Gray, 1936; Weis-Fogh et al., 1977). As a conclusion, these values permit for *Tyrannosaurus* to run in a speed of 15.7 (m/s).

### 4 CONCLUSIONS

In 2002 Hutchinson and Garcia doubted fast running ability of *Tyrannosaurus* for the first time (Hutchinson and Garcia, 2002). Since then, large bipedal theropod such as *Tyrannosaurus* has been considered as a slow runner. Sellers and Manning studied this issue in 2007, and published a result that 8–9 (m/s) running is possible. However, parameters used in the simulation was not well described. Then, it is difficult to check their result. Quite recently, we have studied this issue in detail (Usami, 2010a-c). In the study, it is revealed that parameters used in Hutchinson et al.’s studies (Hutchinson and Garcia, 2002; Hutchinson 2004b, Gatesy et al. 2009) have uncertainty of approximately two folds. In addition, we have obtained a result that 15.7 (m/s) running is possible even in the framework of Hutchinson et a's work. In this paper, we have studied on this issue from a different angle, namely, based on a calculation of mechanical power output. A calculation of this quantity has never been done in this research field. This methodology is superior than Hutchinson et al.’s methodology. Because, Hutchinson et al.’s methodology relied on static calculation. There is no explicit relation of running velocity with the other quantity in the theory. The discussion of Froude number is employed for speed evaluation, however, it is primitive, and known that it may contain two folds error (Rainforth and Manzella, 2007). There is no such uncertain parameter in the calculation of mechanical power output.

For the final evaluation for running ability only the quantity of power per kilogram of muscle mass is a parameter in the calculation. In addition, there has been detailed studies on how much muscle mass *Tyrannosaurus* has for hind limb. It tells that *Tyrannosaurus* has 15–16 % muscle mass for one hind limb. Then, comparatively accurate evaluation can be achieved. Note that approximately two folds error can be contained in our mechanical power calculation because of uncertainty of the position of the center of mass. Even considering this factor the data of power output of extant animal allows fast running motion of *Tyrannosaurus*. The speed is 15.7 (m/s), which is faster than human. Then, we can conclude that a possibility of fast running of *Tyrannosaurus* can not be excluded.
Figure 11: Running speed v.s. Froude number. Hutchinson and Garcia’s data is located for \( v = 20 \) (m/s) and \( Fr = 16 \) with hip height \( h = 2.5 \) (m). They denied such fast running of Tyrannosaurus. However, the other data permit fast running of Tyrannosaurus from the one that is comparable to human by Sellers et al. (Sellers et al. 2007) to the present result and Paul and Bakker (Bakker 1986; Paul, 1988). The \( h \) represents hip height in the figure.

Figure 12: How it looks like if Tyrannosaurus runs in a speed of 15.7 (m/s) in the present world.

As it is noted in Section 3, searching the global minimum of the evaluation function of running velocity and vertical acceleration is a difficult problem. Small difference of parameters changes pattern of locomotion greatly as appeared in Fig.3 and Fig.4. We have carried out many running simulations to obtain a good solution, i.e., the lowest vertical acceleration and the fastest running speed. Evolution algorithm is able to obtain a local minimum of the evaluation function. However, it is needed to obtain the global minimum of the evaluation function, i.e., vertical acceleration and running velocity for this problem. Then, developing more sophisticated algorithm to bring an appropriate solution easily is left as a future problem. If a better algorithm was developed, it would be of great help for the study of this research area.

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**ERRATUM**

"COULD TYRANNOSAURUS RUN FAST? MECHANICAL POWER CALCULATION FOR 15.7 m/s TYRANNOSAURUS RUNNING“ by Y.Usami.

Numerics of the vertical left hand axis in Fig.8 and Fig.9 are 1~5 instead of 10~50. These are simple notation mistakes. Fig.6 and Fig.7 yield to these results. And, it matches Fig.10.