FLOW SHOP GROUP SCHEDULING WITH LIMITED BUFFER CAPACITY AND DIFFERENT WORKFORCE

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Keywords: Group scheduling, Workforce, Genetic algorithm.

Abstract: A permutational flowshop group scheduling problem (GSP) with sequence dependent set-up times, finite inter-operational buffer capacity and workers with different skills has been investigated in this paper. The set-up times are influenced by the sequence of groups and the worker skill level; the manufacturing tasks on a part are completely automated and the working times do not depend on the operator’s skill. The minimization of the completion time is the objective of the group scheduling. A Genetic Algorithm is proposed as an efficient tool to solve the investigated problem; a benchmark of problems has been generated to investigate the influence of the inter-operational buffer capacity and the worker skill level on the completion time.

1 INTRODUCTION

To be competitive in the worldwide market of goods, companies should produce small lots of different products at a convenient cost level and different quality standards; consequently, their manufacturing systems should be flexible and reconfigurable in a short time. In this changing environment, the workforce plays a strategic role: in particular, at the capacity planning level a team of operators should be correctly assembled who provide for the sufficient skills to manufacture a specific lot of production.

In the investigated manufacturing system, the jobs are grouped into families in accordance to the group technology principles and should visit an identical sequence of machines, whose set-up times are sequence dependent. The inter-operational buffers between the machines have finite dimension. In order to evaluate scheduling conditions as close as possible to the actual process configuration, the influence of workers is modelled, too. The operators are not a critical resource: for this reason, the job transfer time from one machine to its downstream one is considered as negligible. Conversely, the tasks related to the set-up of each group of jobs worked on each workstation are carried out by one operator randomly selected out from the currently available crew of workers; thus, the set-up times depend on the sequence of the groups visiting the workstation and the operators skills. Finally, the worker does not influence the processing time of each job because each working machine is automated. The objective of the scheduling is the minimization of the total completion time. This is a frequent scenario encountered in the manufacturing of mechanical parts by means of CNC centers.

Currently, at the best of our knowledge no optimization procedure is available from literature to solve the investigated scheduling problem. The aim of this paper is building the mathematical model and designing an optimization tool able to find efficient solutions to this problem. A proper genetic algorithm has been designed and a benchmark of process scenarios characterized by different numbers of machines, families and worker skills has been generated to study the proposed scheduling problem.

The remainder of the paper is organized as follows: in Section 2 a review of the literature is reported; Section 3 presents the problem statement and the mathematical notation; then, in Section 4 the optimization algorithm is described; the computational results are provided and discussed in Section 5. Conclusions and future research complete the paper.
2 LITERATURE REVIEW

As stated above, the investigated problem can be approached as a Group Scheduling Problem with sequence dependent set-up times and finite capacity of the inter-operational buffers in presence of a multi-skilled workforce.

The flowshop Group Scheduling problem has been challenged as an evolution of the traditional flowshop problem, where the effect of set-up and similarities among jobs grouped into families is considered into the mathematical model formulation. Several approaches to the Group Scheduling Problem (GSP) with sequence independent set-ups and unlimited buffer capacity have been proposed: Wemmerlov and Vakharia (1991) extended the implementation of constructive algorithms for the group scheduling problem by performing an external sequencing of families and then an internal scheduling of parts within families. A Simulated Annealing algorithm was proposed by Vakharia and Chang (1990). Genetic algorithms have been approached as an assignment problem (McDonald et al., 2009) to solve the multi-objective flow shop problem with sequence dependent set-up times, finite buffer capacity and skilled workforce. The jobs are clustered into $g \ (1,...,G)$ families, $(groups)$, to be worked within a line consisting of $i \ (i=1,..,M)$ working machines; for each group $g$, a job set $J_g=[j_g, j_g+1,.., n_g]$ is defined, where $n_g$ denotes the number of jobs within the group: thus, it holds $n_g = \sum_g n_g$. All the machines should be visited in the same order by the families of jobs within the mix; thus, a permutation group flow shop can be considered. On the machine $i$, the processing time of job $j_g$ clustered within group $g$ is denoted as $p_{i,j_g, g}$.

Between two machines $i-1$ and $i$ there is an inter-operational buffer having a finite capacity $f_i$; thus, there are $M-1$ finite capacity inter-operational buffers between machines; finally, two unlimited capacity buffers are positioned before machine $1$ and after machine $M$. The set-up time of the generic $g$-th scheduled group on machine $i$ is denoted as

\[ SS_i^{(g)} \text{ and } SS_i^{(g)} = SS_i^{(g)} + SS_i^{(g)} = 0 \]  

(1) and is sequence dependent.

The workers ($w=1...W$) employed in the manufacturing process have different skill levels, denoted as $SL_w$.

The influence of the generic worker $w$ on the set-up time is modeled as follows: $SS^{(g-1)}(\theta) \cdot SL_w$, when the worker $w$ is assigned to machine $i$ to develop the set-up of group $g$.

Within each group $g$, the vector $\Omega = \{\pi^{(g)}_1,\pi^{(g)}_2,..,\pi^{(g)}_{n_g}\}$ represents a permutation of jobs, whereas the external permutation of the groups is denoted by $\Omega = \{\pi^{(1)},..,\pi^{(G)}\}$.

To determine a feasible sequence of jobs, a set of constraints related to jobs, machines and buffer availabilities must be defined. Let us denote as $SJ_{i,g}(\theta)$ the starting time on machine $i$ for the $\pi^{(g)}_j$ scheduled job within $g$-th scheduled group.

The job routing constraint is expressed as:

\[ SJ_{i,\pi^{(g)}_j}(\theta) \geq SJ_{i-1,\pi^{(g)}_j}(\theta) + p_{i-1,\pi^{(g)}_j}(\theta) \]

\[ i=2,..,M \quad j_g = 1_g,.., n_g \quad g=1,..,G \]  

(1)

3 PROBLEM STATEMENT

A notation similar to that adopted by Nowicki (1999) for the flowshop scheduling problem with finite inter-operational buffer capacity is here proposed and extended to the group scheduling problem with sequence dependent set-up times, finite buffer capacity and skilled workforce. The jobs are clustered into $g \ (1,...,G)$ families, $(groups)$, to be worked within a line consisting of $i \ (i=1,..,M)$ working machines; for each group $g$, a job set $J_g=[j_g, j_g+1,.., n_g]$ is defined, where $n_g$ denotes the number of jobs within the group: thus, it holds $n_g = \sum_g n_g$. All the machines should be visited in the same order by the families of jobs within the mix; thus, a permutation group flow shop can be considered. On the machine $i$, the processing time of job $j_g$ clustered within group $g$ is denoted as $p_{i,j_g, g}$.

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\[ SJ_{i,\pi^{(g)}_j}(\theta) \geq SJ_{i-1,\pi^{(g)}_j}(\theta) + p_{i-1,\pi^{(g)}_j}(\theta) \]

\[ i=2,..,M \quad j_g = 1_g,.., n_g \quad g=1,..,G \]  

(1)

487
A machine is available to work a job after that the preceding job in the sequence belonging to the same group has been unloaded or at the completion of the set-up activities on the machine, if the currently scheduled job is the first within a group:

\[
\begin{align*}
S J_{i,x}(g)_0 & \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) \quad (2a) \\
S J_{i,x}(g)_0 & \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) + \text{SS}^{(g_{-k_1})}_i \cdot S L_w \\
\text{valid for} & \quad i = 1,2,...,M \quad j_g = 1,2,...,n_g \quad g = 1,2,...,G
\end{align*}
\]

With reference to the constraints related to the generic inter-operational buffer capacity \( f_{i+1} \), the following situations can occur:

i) the inter-operational buffer capacity \( f_{i+1} \) is saturated and the machine \( i \) is blocked by jobs belonging to the same group. The job to be loaded on machine \( i \) belongs to the same group too. Then, it holds:

\[
\begin{align*}
S J_{i,x}(g)_0 & \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) \quad (3a) \\
S J_{i,x}(g)_0 & \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) + \text{SS}^{(g_{-k_1})}_i \cdot S L_w \\
\text{valid for} & \quad i = 1,2,...,M \quad j_g = 1,2,...,n_g \quad g = 1,2,...,G
\end{align*}
\]

ii) the inter-operational buffer capacity \( f_{i+1} \) is saturated and the machine \( i \) is blocked by jobs belonging to the same group. The job to be loaded on machine \( i \) is the first of a new group. Then, it holds:

\[
\begin{align*}
S J_{i,x}(g)_0 & \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) \quad (4a) \\
S J_{i,x}(g)_0 & \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) + \text{SS}^{(g_{-k_1})}_i \cdot S L_w \\
\text{valid for} & \quad i = 1,2,...,M \quad j_g = 1,2,...,n_g \quad g = 1,2,...,G
\end{align*}
\]

iii) the inter-operational buffer capacity \( f_{i+1} \) is saturated by different groups of jobs. The job to be loaded on machine \( i \) is not the first of a new group. Then, it holds:

\[
\begin{align*}
S J_{i,x}(g)_0 & \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) \quad (5a) \\
S J_{i,x}(g)_0 & \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) + \text{SS}^{(g_{-k_1})}_i \cdot S L_w \\
\text{valid for} & \quad i = 1,2,...,m \quad g = 2,...,G
\end{align*}
\]

\[
\begin{align*}
& S J_{i,x}(g)_0 \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) \\
& S J_{i,x}(g)_0 \geq S J_{i+1,x}(g_{-k_1}) + p_{\lambda,x}(g_{-k_1}) + \text{SS}^{(g_{-k_1})}_i \cdot S L_w \\
\text{valid for} & \quad i = 1,2,...,m \quad g = 2,...,G
\end{align*}
\]

Equations (1) through (7) are valid under the following assumptions: \( \pi^{(0)}(j_g) = 0 \) if \( j_g \leq 0 \), \( \pi^{(0)}(j) = J_g \), \( S J_{i,x}(g)_0 = 0 \) and \( p_{\lambda,x}(g_{-k_1}) = 0 \) if \( f_{i+1} \).

Given a sequencing of groups and jobs within each group, a step-by-step evaluation of the starting inspection time \( S J_{i,x}(g)_0 \) for each job on machine \( i \) is obtained by computing the maximum value of \( S J_{i,x}(g)_0 \) with respect to the constraints (1)-(7b).

Then, the completion time

\[
C_{i,x}(g) = S J_{i,x}(g)_0 + p_{\lambda,x}(g)
\]

and each job on machine \( i \) can be easily computed.

The objective of the group scheduling problem is the minimization of the makespan of the entire mix to be worked.

\[
C_{\text{max}}(g) = \left[ S \frac{M_{x}(0)}{M_{x}(0)} + p \frac{M_{x}(0)}{M_{x}(0)} \right]
\]
4 THE GENETIC ALGORITHM

The genetic algorithm developed to solve the investigated Group Scheduling Problem requires a proper chromosome encoding able to work with standard genetic operators.

The chromosome encoding has been developed to map separately the sequence of groups and the sequences of jobs within each group: a chromosome is coded through a two dimensional decimal array \( \text{seq}(r,s), \quad (r=1,G+1; \quad s=1, \quad \max_g \left[ \begin{array}{c} g \\ N \end{array} \right] G) \). Rows \((1:G)\) contain the internal job sequence \( \rho^{(s)} \) within each group; the last row \( G+1 \) contains the external group sequence: each row of the array \( \text{seq}(r,s) \) is here considered as a sub-chromosome. The number of columns of a chromosome array coincides with the maximum number of jobs assigned to a single group. The chromosome fitness is equal to the makespan corresponding the schedule it encodes. With this notation, a population of chromosomes can be coded through a three dimensional array. In Figure 1 an example of population for a problem with \( G=4 \) and \( n=11 \) jobs, \((n_1=4, \quad n_2=2, \quad n_3=3, \quad n_4=2)\) is presented:

![Population and chromosome representation.](image)

Two different kinds of crossover operators with an equal probability to be selected have been implemented: the position based crossover (PBC) and the two point crossover (TPC).

The mutation operator can work with \( P_m \) probability on sub-chromosomes representing both sequences of jobs within a group or sequences of groups. Mutation is performed with an equal probability of selecting one among the following two operators: an allele swapping operator, which performs an exchange on a random number of alleles; and a block swapping operator, which performs a block exchange of alleles. To prevent from the elimination of the current best sequence, the survival of a copy of the current fittest individual within the population is ensured by an elitist strategy. Conversely, the premature convergence of the algorithm towards a sub-optimum solution due to a rapid increase of the number of copies of «the fittest individuals» within the current population can occur: thus, a population diversity control technique has been embedded in the developed GA. In the current population, a mutation operator is applied to those copies of chromosomes exceeding an assigned number \( D_{\text{max}} \). To perform consecutive intensification-diversification cycles of the evolutionary process, after \( N_{\text{op}} \) iterations without improvements in the fittest individual, the mutation probability is increased by \( \Delta P_m \). When a new minimum is found, \( P_m \) is reset to its initial value. Once a new optimal solution is found, the algorithm execution is continued for at least \( \Delta t \) further population generations.

5 COMPUTATIONAL RESULTS

The model investigation and the GA algorithm test have been performed by generating a benchmark of problems accordingly to the procedure proposed in Schaller et al. (2000) for the GSP with set-up times and adapted to cope with finite buffer capacity and differently skilled workers. The number \( n_g \) of jobs within a group randomly varies in the interval \([1,10]\); the job processing times \( p_{i,j,g} \) have been randomly generated from a uniform distribution \( U[1,10] \). The number of groups \( G \) and the number of machines \( M \) range between 3 and 10: totally, ten different line configurations have been considered. The sequence dependent set-up times are extracted out from three different uniform distributions to generate three classes of problems for each line configuration: the shorter set-up \( SS, \quad (U[1,20]) \), the medium set-up \( MS, \quad (U[1,50]) \), and larger set-up \( LS, \quad (U[1,100]) \) classes of problems. Each class of problems consists of 10 instances: a total number of 300 problems have been generated. Four available buffer capacities \( f_i=1, \quad 2, \quad 4, \quad 20, \) for \( i=1, \ldots, \quad M-1 \), between machines have been considered. Four skills levels \( SL_w = 1.0, \quad 1.1, \quad 1.3, \quad 1.5 \) have been assumed to model the workforce impact on the GS problem: this means that a worker with skill level 1.3 complete the set-up operation in a time 30% longer than a worker with skill 1.0. For each problem, a percentage \( PW=10\%, \quad 30\%, \quad 50\% \) of available workforce having skill level larger than 1.0 has been considered. Totally, 10800 scenarios have been investigated. The following GA parameters have been selected: number of population chromosomes \( N_{\text{pop}}=30 \); maximum number of duplicates \( D_{\text{max}}=2 \); equal probability for the two crossover operators to be selected \( PCR=0.5 \); equal probability for the two
mutation operators to be selected $PCM=0.5$; mutation probability $P_M=0.13$; improvement in the mutation probability $\Delta P_M=0.05$ after 500 iterations without a new improvement in the makespan. The maximum number of iterations is set equal to 15,000. The numerical analysis has been carried out as follows. First of all, the benchmark of 300 problems has been optimized by assuming the limiting condition of infinite inter-operational buffer capacity and uniform skill level equal to $SL_w=1.0$. For each problem, the obtained makespan can be interpreted as a lower bound $LB$ that can be used to quantify the effect of the limited buffer capacity, the skill level and the skill variety within the working team. Then, for each problem the optimal makespan $C_{max}$ in presence of the constraints related to limited inter-operational buffer capacity and skilled workforce has been found. Finally, the percentage increase of the makespan with respect to its corresponding lower bound has been calculated:

\[ \Delta C_{MK} = \frac{C_{max} - LB}{LB} \]  

Thus, a large value of $\Delta C_{MK}$ should be interpreted as an index of significant makespan increase with respect to the limiting condition. Table 1 shows the obtained results.

As expected, it turns out that a larger inter-operational buffer capacity reduces the influence of the workforce skill variability; for example, when the inter-operational buffer capacity varies from $f_i=1$ to $f_i=4$, with the same workforce allocation strategy the averaged makespan distance from its lower bound value decreases from 8.36% to 2.20%. In table 1 it is also possible to appreciate the effect of the workforce skill and the team mix: both of them have a positive effect on the makespan.

For example, when the workforce skill level is reduced from $SL_w=1.1$ to 1.5, the distance from the lower bound increases from 4.1% to 15.4% for $f_i=4$ and $PW=50%$. In the same way, the variety of team composition $PW$ influences the makespan: for example, when $PW$ varies from 10% to 50% the distance from the lower bound increases from 1.18% to 6.68% for $f_i=20$ and $SL_w=1.3$.

Finally, a second order interaction between the worker skill level $SL_w$ and the buffer capacity $f_i$ is evident: in fact, the larger is $f_i$ the lighter is the effect of $SL_w$ on the makespan deterioration. For example, given $PW=50%$, when $f_i=2$ and $SL_w=1.3$ it results $\Delta C_{MK}=12.49%$; otherwise, when $f_i=20$ and $SL_w=1.3$ it results $\Delta C_{MK}=6.68%$, that is $\Delta C_{MK}$ is halved. This happens because the available buffer capacity partially decouples the interactions between consecutive machines, thus reducing the probability of starving/blocking conditions due to delays in the set-up activities. Therefore, the worker skills need to be carefully accounted for those manufacturing scenarios characterized a finite inter-operational buffer capacity.

Table 1: Average results for each class of problems.

<table>
<thead>
<tr>
<th>$f_i$</th>
<th>$PW=10%$</th>
<th>$PW=30%$</th>
<th>$PW=50%$</th>
<th>$PW=50%$</th>
<th>$PW=50%$</th>
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<tr>
<td></td>
<td>$SL_w=1.1$</td>
<td>$SL_w=1.3$</td>
<td>$SL_w=1.5$</td>
<td>$SL_w=1.1$</td>
<td>$SL_w=1.3$</td>
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<tr>
<td>1</td>
<td>8.36%</td>
<td>9.41%</td>
<td>10.27%</td>
<td>8.36%</td>
<td>9.41%</td>
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<tr>
<td>2</td>
<td>4.78%</td>
<td>5.85%</td>
<td>6.72%</td>
<td>4.78%</td>
<td>5.85%</td>
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<td>4</td>
<td>2.20%</td>
<td>3.02%</td>
<td>4.04%</td>
<td>2.20%</td>
<td>3.02%</td>
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<tr>
<td>20</td>
<td>6.2%</td>
<td>7.3%</td>
<td>8.7%</td>
<td>6.2%</td>
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</table>

Table 2a: variable $SL_w$ and Table 2b: variable $PW$.
Table 3a: variable \( f_i \) 3b: all factors are variable.

<table>
<thead>
<tr>
<th>PW=10% SLw=1.1</th>
<th>PW=10% PW=30% PW=50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_i=4 )</td>
<td>( f_i=2 )</td>
</tr>
<tr>
<td>1 2 1</td>
<td>1.2% 2.1% 3.3%</td>
</tr>
<tr>
<td>3 3 1</td>
<td>0.3% 1.5% 3.5%</td>
</tr>
<tr>
<td>4 4 1</td>
<td>1.5% 3.3% 5.9%</td>
</tr>
<tr>
<td>( f_i=6 )</td>
<td>( f_i=5 )</td>
</tr>
<tr>
<td>5 5 1</td>
<td>1.4% 3.6% 6.2%</td>
</tr>
<tr>
<td>6 6 1</td>
<td>1.6% 2.8% 6.2%</td>
</tr>
<tr>
<td>5 5 2</td>
<td>1.5% 4.9% 8.2%</td>
</tr>
<tr>
<td>6 6 2</td>
<td>2.3% 4.7% 8.0%</td>
</tr>
<tr>
<td>8 8 2</td>
<td>2.3% 4.5% 8.6%</td>
</tr>
<tr>
<td>8 10 2</td>
<td>3.2% 6.3% 10.8%</td>
</tr>
<tr>
<td>10 10 2</td>
<td>2.7% 6.6% 10.7%</td>
</tr>
<tr>
<td>average</td>
<td>2.2% 4.8% 8.4%</td>
</tr>
</tbody>
</table>

the three factors is significantly higher and ranges between 2.2% and 22.3%. The averaged results presented Tables 2 and 3 are graphically shown in Figure 2.

![Image](image.png)

Figure 2: Influence of factors.

6 CONCLUSIONS

A permutational flowshop group scheduling problem (GSP) with sequence dependent set-up times, limited interoperational buffer capacity, workers with different skills and different mix of the working crew has been taken into account. In the model, the set-up times depend on both the sequence of groups and the worker skill level; the working times have been considered independent by the skill of the operator because the working operations are completely automated. A Genetic Algorithm has been proposed as an efficient tool to solve the investigated problem with respect to the minimization of the total completion time. A sensitivity analysis has been carried out on a benchmark of problems to show the relevant influence of all factors considered in the model. A future development of this research will treat the scheduling of jobs as well as the workers assignment strategy to each machine as independent variables of the optimization problem.

REFERENCES


