APPROXIMATE REASONING IN CANCER SURGERY

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Keywords: Approximate reasoning, Deterministic operation chance, Parametric membership functions.

Abstract: The compositional rule of inference, grounded on the modus ponens law, is one of the most effective fuzzy systems. We modify the classical version of the rule (Zadeh, 1973, 1979) to propose an original model, which concerns determining an operation chance for gastric cancer patients. The operation prognosis will be dependent on values of biological markers indicating the progress of the disease.

1 INTRODUCTION

One of the early systems, evolved by Zadeh as an approach to decision making in vague circumstances, was the technique of approximate reasoning (Zadeh, 1973, 1979). The compositional rule of inference found adherents who adapted the primary foundations of the theory to own models (Baldwin and Pilsworth, 1979; Mizumoto and Zimmermann, 1982; Zimmermann, 2002).

Some trials of technical use of approximate reasoning have been made, but it is still difficult to find a medical application based on the inference rule. The rule was once tested by the author in order to make decisions concerning operation chances for gastric cancer patients (Rakus-Andersson, 2009). The decisions were based on values of one biological marker C-reactive proteins CRP, regarded by physicians as the essential index of cancer progress.

In the current paper we wish to extend the number of clinical symptoms in the model. In practice we want to add a value of age to CRP-value (Do Kyong-Kim et al., 2009) to deduce a verbal evaluation of the operation chance for post-surgical survival in cancer diseases.

We discuss the approximate reasoning structures in Section 2. Fuzzy sets, taking place in the model, will be created in Section 3. Section 4 is added as a presentation of the algorithm prognosis made for an individual patient.

2 RULE OF INference

Surgical decisions are made with the highest thoughtfulness in the case of patients suffering from cancer. The physician wants to prognosticate the operation role positively; we therefore introduce the concept “operation chance” to determine the outcome of a surgery.

The most decisive clinical markers CRP and age found in an individual patient will constitute the input data in the approximate reasoning model to evaluate the operation chance for survival.

Let us state a logical compound tautology (Rakus-Andersson, 2009)

\[
\text{(IF}( p_1 \land \cdots \land p_p \land (\text{IF}( p_1 \land \cdots \land p_p ) \text{THEN } q) \text{ELSE}(\text{IF}(\text{NOT}( p_1 \land \cdots \land p_p ) ) \text{THEN} (\text{NOT } q)))) \text{THEN } q. \]

In accordance with the generalized law modus ponens (Zadeh, 1973) we interpret (1) as a statement

\[
\text{(IF}( p_1 \land \cdots \land p_p \land (\text{IF}( p_1 \land \cdots \land p_p ) \text{THEN } q) \text{ELSE}(\text{IF}(\text{NOT}( p_1 \land \cdots \land p_p ) ) \text{THEN} (\text{NOT } q) )) \text{THEN } q'. \]

provided that the semantic meaning of \( p_i \) and \( p_i' \), \( i = 1, \ldots, p \), (\( q \) and \( q' \) respectively) is very close.

In (2) let \( p_i \) and \( p_i' \) be mapped in fuzzy sets \( P_i \) and \( P_i' \) in the universes \( X_i \) and let \( q \) and \( q' \) be expressed by fuzzy sets \( Q \) and \( Q' \) in the universe \( Y \).

We now make a feedback to the medical task previously outlined to evaluate the operation chances as verbal expressions.
Let $S_1, \ldots, S_p$ denote clinical markers possessing the decisive power in the evaluation of the operation chance. We regard $S_i$, $i = 1, \ldots, p$, like the symptoms whose growth levels are assimilated with codes. Values of the codes form the universes $X_i = \text{"S"}_i$’s levels” = \{1, \ldots, k_i, \ldots, n_i \}. Assume that level 1 is associated with the slightly heightened symptom values whereas level $n_i$ indicates the dangerous symptom intensity (Rakus-Andersson, 2009).

Universe $Y$ consists of words describing operation chance priorities. We set $Y = \text{"operation chance priorities"} = \{L_1 = \text{"none"}, L_2 = \text{"very little"}, L_3 = \text{"little"}, L_4 = \text{"moderate"}, L_5 = \text{"promising"}, L_6 = \text{"very promising"}, L_7 = \text{"totally promising"}\}$ assuming that $Y$ is experimentally restricted to seven chance priorities.

We assign $p_i^\prime$, $p_i$, $q$ and $q^\prime$ to sentences (Rakus-Andersson, 2009)

$p_i^\prime = \text{"symptom } S_i \text{ is found in patient on level } k_i, i = 1, \ldots, p\prime$,

$p_i = \text{"lower levels of } S_i \text{ are essential for a positive operation outcome"}$

$q = \text{"operation chance can be estimated on the basis of } S_i \text{ and } \ldots S_p\prime$

and

$q^\prime = \text{"patient with the } k_i \text{-level of } S_i \text{ and } \ldots \text{and the } k_p \text{-level of } S_p \text{ gets an estimated operation chance as this } L_i, \text{ which has the highest degree in } Q\prime, l = 1, \ldots, T\prime$,

Rule (2) will thus become a scheme

(IF (“symptom $S_i$ is found in patient on level $k_i$” and...“symptom $S_p$ is found in patient on level $k_p$” = $P_1 \cap \cdots \cap P_p$) AND ((IF “lower levels of $S_i$ are essential for a positive operation outcome” and...“lower levels of $S_p$ are essential for a positive operation outcome” = $P_1 \cap \cdots \cap P_p$ THEN “operation chance can be estimated on the basis of $S_i$ and...$S_p$” = $Q$ ELSE (IF it is not true that “lower levels of $S_i$ are essential for a positive operation outcome” and...“lower levels of $S_p$ are essential for a positive operation outcome” = $C(P_1 \cap \cdots \cap P_p)$ THEN “operation chance cannot be estimated on the basis of $S_i$ and...$S_p$” = $Q$)))

THEN “patient with the $k_1$-level of $S_1$ and...and the $k_p$-level of $S_p$ gets an estimated operation chance as this $L_i$, which has the highest degree in $Q\prime$, $l = 1, \ldots, T\prime$.

$C(P_1 \cap \cdots \cap P_p)$ and $CQ$ are complements of $P_1 \cap \cdots \cap P_p$ and $Q$.

### 3 DATA SETS IN $X$ AND $Y$

The decision model, sketched in Section 2, includes operations on fuzzy sets $P_1^\prime$, $P_1$, $Q$ and $Q\prime$. First we design fuzzy sets in $X_i$, $i = 1, \ldots, p$, as structures

\[
P_i = \{((1, \mu_{P_i}((1))), ((k_i, \mu_{P_i}(k_i)), \ldots, (n_i, \mu_{P_i}(n_i)))
\]

\[
= \{((k_i - 2, \frac{n_i - 2}{n_i}), (k_i - 1, \frac{n_i - 1}{n_i}), (k_i, 1),
\]

\[
(k_i + 1, \frac{n_i - 1}{n_i}), (k_i + 2, \frac{n_i - 2}{n_i}), \ldots)\}
\]

and

\[
P_i = \{((1, \mu_{P_i}((1))), ((k_i, \mu_{P_i}(k_i)), \ldots, (n_i, \mu_{P_i}(n_i))) = \{(1, \frac{n_i}{n_i}), \ldots, (k_i, \frac{n_i - 1}{n_i}), \ldots, (n_i, \frac{1}{n_i})\}
\]

referring to $S_i$ due to the definitions of $p_i$ and $p_i^\prime$.

The set $Q$ is sophisticated to be stated as a fuzzy set since its support consists of other fuzzy sets $L_i$, $l = 1, \ldots, 7$, defined in a symbolic chance reference set $Z = [z_{min}, z_{max}] = [0, 1]$. To find restrictions of $L_i$ we study the technique of Rakus-Andersson (2010).

Suppose that $L_1, \ldots, L_m$ are included in the linguistic list, where $m$ is an odd positive integer greater or equal to 5. Supports of the restrictions $\mu_{L_i}(z)$, $l = 1, \ldots, m$, will cover parts of the reference set $Z = [0, 1]$. We introduce $E$ to be the length of $Z$.

We divide all expressions $L_i$ in three groups, namely, a family of “leftmost” sets $L_1, \ldots, L_{m-1}$, the set $L_{m+1}$ “in the middle” and a collection of “rightmost” sets $X_{m+2}, \ldots, L_m$.

The “leftmost” family is given by

\[
\mu_{L_1}(z) = \begin{cases} 
1 & \text{for } z \leq z_{min} + (t - 1) \frac{E}{m-1}, \\
1 - 2 \left(\frac{z_{min} + (t - 1) \frac{E}{m-1}}{\frac{E}{m-1}}\right)^2 & \text{for } z_{min} + (t - 1) \frac{E}{m-1} \leq z \leq \frac{E}{2(m-1)} + (t - 1) \frac{E}{m-1}, \\
2 \left(\frac{z - \frac{E}{2(m-1)} - (t - 1) \frac{E}{m-1}}{\frac{E}{m-1}}\right)^2 & \text{for } \frac{E}{2(m-1)} + (t - 1) \frac{E}{m-1} \leq z \leq \frac{E}{m-1} + (t - 1) \frac{E}{m-1}, \\
0 & \text{for } z \geq \frac{E}{m-1} + (t - 1) \frac{E}{m-1}
\end{cases}
\]

for parameter $t$, $t = 1, \ldots, \frac{m-1}{2}$.

To implement the “rightmost” functions we use
\[
\mu_{L_{1\ldots m}}(z) = \begin{cases} 
0 & \text{for } z \leq E - \left( \frac{m-1}{2} + (t-1) \frac{k}{m-1} \right), \\
2 \left( \frac{\left( \frac{m-1}{2} + (t-1) \frac{k}{m-1} \right)}{m} \right)^2 & \text{for } E - \left( \frac{m-1}{2} + (t-1) \frac{k}{m-1} \right) \leq z \leq E - \left( \frac{m-3}{2(m-1)} + (t-1) \frac{k}{m-1} \right), \\
1 - 2 \left( \frac{\left( \frac{m-1}{2} + (t-1) \frac{k}{m-1} \right)}{m} \right)^2 & \text{for } E - \left( \frac{m-3}{2(m-1)} + (t-1) \frac{k}{m-1} \right) \leq z \leq E - \left( \frac{m-1}{2} + (t-1) \frac{k}{m-1} \right), \\
2 \left( \frac{\left( \frac{m-1}{2} + (t-1) \frac{k}{m-1} \right)}{m} \right)^2 & \text{for } E - \left( \frac{m-1}{2} + (t-1) \frac{k}{m-1} \right) \leq z \leq E - \left( \frac{m-1}{2} + \frac{k}{m-1} \right), \\
0 & \text{for } z \geq E - \left( \frac{m-1}{2} + \frac{k}{m-1} \right). 
\end{cases}
\]

For the list “operation chance priorities” we state \( z_{min} = 0 \), \( m = 7 \) and \( E = 1 \). Fuzzy sets \( L_i, i = 1, \ldots, 7 \), are depicted in Fig. 1.

When defuzzifying the sets \( L_i \) we consider \( z \)-coordinates of the intersection points between \( \mu_{L_i}(z) = 1 \) and \( \mu_{L_i}(z) < 1 \). We denote the \( z \)-values by \( z(L_i) = 0, z(L_2) = 0.166, z(L_3) = 0.338, z(L_4) = 0.5, z(L_5) = 0.668, z(L_6) = 0.834 \), \( z(L_7) = 1 \) and we let them represent \( L_1, \ldots, L_7 \).

![Figure 1: Fuzzy sets L1-L7](image)

Then we build the set “numerical operation chance”, which gets the constraint \( s(z, 0, 0.5, 1) \) over \([0,1]\). We compute the degrees of \( z(L_i) \) via this constraint to obtain set \( Q \) in the form

\[
Q = \{ (L_1,0),(L_2,0.055),(L_3,0.22),(L_4,0.5), (L_5,0.78),(L_6,0.945),(L_7,1) \}.
\]

Further, for all \((k_1,\ldots,k_p) \in X_1 \times \cdots \times X_p\) we determine the intersections

\[
P_1 \cap \cdots \cap P_p = \\{(1,\ldots,1),\min(\mu_{P_1}(1),\ldots,\mu_{P_p}(1))\}, \ldots, \\
((k_1,\ldots,k_p),\min(\mu_{P_1}(k_1),\ldots,\mu_{P_p}(k_p)))\}, \ldots, \\
((n_1,\ldots,n_p),\min(\mu_{P_1}(n_1),\ldots,\mu_{P_p}(n_p)))\}.
\]

In conformity with Zadeh (1973) and Zimmermann, (2002) we introduce the matrix \( R \) being a mathematical expression of the implication

(IF \( P_1 \cap \cdots \cap P_p \) THEN \( Q \))

ELSE (IF \( C(P_1 \cap \cdots \cap P_p) \) THEN \( C(Q) \)).

The membership function of \( R \) is yielded by

\[
\mu_{R}((k_1,\ldots,k_p),L_i) = \min(1, (\min_{(k_1,\ldots,k_p)} \mu_{P_1}(k_1), \ldots, \mu_{P_p}(k_p)) + \mu_{Q}(L_i)),
\]

(11)

for all \((k_1,\ldots,k_p) \in X_1 \times \cdots \times X_p\) and all \( L_i \in Y \).

Set \( Q' \) will be formed as (Zadeh, 1973)

\[
Q' = (P_1' \cap \cdots \cap P_p') \circ R .
\]

\( Q' \) is designated by the membership function

\[
\mu_{Q'}(L_i) = \max_{(k_1,\ldots,k_p) \in X_1 \times \cdots \times X_p} (\min_{(k_1,\ldots,k_p)} \mu_{P_1}(k_1), \ldots, \mu_{P_p}(k_p))
\]

(12)

(13)

By comparing magnitudes of membership degrees in set \( Q' \) with respect to all \( L_i, i = 1, \ldots, 7 \), we select this chance priority \( L_i \), which assists the largest value of \( \mu_{Q'}(L_i) \).

4 CHANCE DETERMINATION

The CRP-value and age are decisive markers of the prognosis in cancer surgery (Do-Kyong Kim et al., 2009).
The heightened values of CRP (measured in milligrams per liter) are discerned in levels:

1 = "almost normal" for CRP < 10,
2 = "heightened" if 10 ≤ CRP ≤ 20,
3 = "very heightened" if 20 ≤ CRP ≤ 25,
4 = "dangerously heightened" for CRP > 25.

The age borders are decided as:

1 = "not advanced for surgery" if "age" < 60,
2 = "advanced for surgery" if 60 ≤ "age" ≤ 80,
3 = "dangerous for surgery" if "age" > 80.

Suppose that in a seventy-year-old patient the CRP-value is measured to be 18. Due to (4) and (10) sets $P_1$, $P_2$ and their intersection are expressed as

$$P_1 = \{(1,1),(2,0.75),(3,0.5),(4,0.25)\},$$
$$P_2 = \{(1,1),(2,0.66),(3,0.34)\},$$
$$P_1 \cap P_2 = \{(1,1),(3,0.5),\ldots\},$$
$$((4,3),0.25)\}$$

while $P_1'$, $P_2'$ and their cut are computed, with respect to (3) and (9), as

$$P_1' = \{(1,0.75),(2,1),(3,0.75),(4,0.5)\},$$
$$P_2' = \{(1,0.66),(2,1),(3,0.66)\},$$
$$P_1' \cap P_2' = \{(1,1),0.66),\ldots,(3,2),0.75\},$$
$$((4,3),0.5)\}$$

provided that $X_1 = \{1,2,3,4\}$ and $X_2 = \{1,2,3\}$.

Matrix $R$, found in compliance with (11), is expanded as a two-dimensional table

$$R = \begin{bmatrix}
L_1 & \cdots & L_4 & \cdots & L_2 \\
(1,1) & 0 & \cdots & 0.5 & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
(3,2) & 0.5 & \cdots & 0.5 & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
(4,3) & 0.75 & \cdots & 0.75 & \cdots & 0.25 \\
\end{bmatrix}$$

We insert $R$ given by (16) and $P_1' \cap P_2'$ determined by (15) in (12) in order to estimate

$$Q' = \{(L_4,0.66),(L_3,0.715),(L_2,0.72),$$
$$(L_4,0.84),(L_3,0.88),(L_2,0.715),(L_7,0.66)\}.\)$$

The largest membership degree in (17) points out chance $L_5 = "promising"$ for a result of the operation on the elderly patient whose CRP-index is evaluated on the second growth level.

5 CONCLUSIONS

We have adapted approximate reasoning as a deductive algorithm to introduce the idea of evaluating the operation chance for patients with heightened values of biological indices in cancer diseases.

The formulas of membership functions in data sets have been expanded by applying a formal mathematical design invented by the author. The data sets involve parametric families of functions, which allow preparing a computer program. We have tested a large sample of patient data to get the results mostly converging to the physicians’ prognoses. This confirms reliability of the system.

ACKNOWLEDGEMENTS

The author thanks the Blekinge Research Board in Karlskrona – Sweden for the grant funding this research. The author is grateful to Medicine Professor Henrik Forssell for all helpful hints made in the subject of cancer surgery.

REFERENCES


