A MODEL USING DATA ENVELOPMENT ANALYSIS FOR THE CROSS EVALUATION OF SUPPLIERS UNDER UNCERTAINTY

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Abstract: The paper addresses one of the key objectives of the purchasing function of a supply chain, i.e., the optimal selection of suppliers. We present a novel methodology that integrates the well-known cross-efficiency evaluation called Data Envelopment Analysis (DEA) and the Monte Carlo approach, to manage supplier selection considering uncertainty in the supply process, e.g. evaluating potential suppliers. The model allows to distinguish among several suppliers, overcoming the limitation of the traditional DEA method of not distinguishing among efficient suppliers. Moreover, the technique is able to classify suppliers with uncertain performance. The method is applied to the selection of suppliers of a Southern Italy SME.

1 INTRODUCTION

Within the purchasing management area, the process of supplier selection is currently widely investigated in the scientific literature, particularly as regards the private sector, due to its strategic role in the success of a Supply Chain (SC) (Costantino et al., 2011). Supplier evaluation techniques periodically identify and verify the best potential candidates able to provide the expected performance level to the SC. Typically, supplier selection is a multi-objective decision problem with conflicting objectives, such as, besides the obvious goal of (low) price, also quality, quantity, delivery, performance, capacity, communication, service, geographical location, etc. The wide literature of the area is a synonym of the importance of supplier choice and the interested reader may find a detailed discussion of the appeared contributions in Ho et al. (2010).

Roughly, the multi-objective approaches proposed in the related literature for the solution of the supplier selection problem may be classified into individual model and integrated techniques. One of the best-known individual methods is the so-called Data Envelopment Analysis (DEA) technique, due to its robustness and simplicity of application (Ho et al., 2010). The DEA technique is based on linear programming to determine the efficiency of several units subject to the decision (Charnes et al., 1978). However, a limitation of the classical DEA is that it distinguishes only between inefficient and efficient suppliers, without enabling a discrimination among the elements of the latter set. Such a characteristic makes it difficult to apply DEA for supplier selection, particularly in the case of a single sourcing purchasing, i.e., with one supplier only. With the aim of improving the method discriminating power, Sexton et al. (1986) proposed the so-called cross-efficiency DEA method that evaluates the decision units efficiency in a crossed way. However, the resulting technique is quite complex, since it requires the set-up and solution of a two-level optimization problem, and deterministic, so that it cannot manage uncertain data on suppliers. Nevertheless, uncertainty is one of the most relevant issues in SC management and this is particularly apparent in the supplier selection process. Indeed, such a problem is often concerned with the evaluation of potential candidates, with which the buyer has not had previous commercial relationships, so that the corresponding key performance indicators are inevitably vague.

This paper proposes an integrated model for supplier selection based on the cross-efficiency DEA and Monte Carlo simulation. The technique main advantages are two, as follows: on the one hand it enables the purchasing manager to distinguish among multiple suppliers that according to the classical DEA method are considered as equally efficient, on the other hand it is able to classify
suppliers with uncertain performance against some defined criteria for evaluation of the supply.

2 THE DEA METHOD FOR THE EVALUATION OF SUPPLIERS

2.1 The Traditional DEA Technique

The DEA method (Charnes et al., 1978) is a technique for classifying multiple Decision Making Units (DMU) in a compared way while measuring the maximum efficiency with respect to the performance of all the analyzed DMU. In particular, the units are characterized by the sharing of a set of resources used to produce goods or services. In the DEA method the efficient DMU are the vertices of a Pareto face, based on these, the other DMU efficiencies are evaluated. As regards the supplier evaluation and selection area, DEA allows determining, among a set of partners, a subset composed by the suppliers using the given (input) resources to produce the required (output) goods/services in the most efficient way. To this aim, several inputs express the contribution required to the supplier (e.g., the purchasing price, the lead time, etc.) and several outputs identify its performance in the procurement process (e.g., delivery timeliness, finite product quality).

In general, a supplier selection problem may be defined by a set of suppliers offering the requested product \( S = \{s_1, s_2, ..., s_F\} \) and by a set of conflicting objectives \( C = \{c_1, c_2, ..., c_k\} \), against which the suppliers are to be classified. We assume that the set of criteria is partitioned as \( C = C_I \cup C_O \), with \( C_I = \{c_1, c_2, ..., c_f\} \) and \( C_O = \{c_{I+1}, c_{I+2}, ..., c_{I+K}\} \), representing the set of input and output criteria respectively, with \( H+K=n \) being the overall number of criteria.

The generic supplier \( s_f \in S \) has efficiency:

\[
E_f = \frac{\sum_{h=1}^{K} u_h y_{hf}}{\sum_{h=1}^{K} v_h x_{hf}}, \quad (1)
\]

that is the ratio between the weighted sum of the outputs and the weighted sum of the inputs, where: \( y_{hf} \) is the \( k \)-th type output \( (k=1, 2, ..., K) \) referred to supplier \( s_f \in S \); \( x_{hf} \) is the \( h \)-th type input \( (h=1, 2, ..., H) \) referred to supplier \( s_f \in S \); \( u_h \) is the weight assigned to the \( k \)-th type output; \( v_h \) is the weight associated with the \( h \)-th type input.

The aim of the DEA method is associating to the outputs and inputs of supplier \( s_f \in S \), given their values, a set of weights leading to maximize efficiency, while taking into account that this cannot by definition be higher than 1. In such a way the efficiency of the \( f \)-th supplier \( (f=1, 2, ..., F) \) being the total number of analyzed suppliers) is evaluated solving the following mathematical programming problem, determining the sets of weights \( u_h \) and \( v_h \) that maximize \( E_f \):

\[
\begin{align*}
\max_{u, v} & \sum_{h=1}^{K} u_h y_{hf} \\
\text{subject to} & \sum_{h=1}^{K} v_h x_{hf} \leq 1, \quad f=1, 2, ..., F, \\
& u_h, v_h \geq 0 \quad \text{for} \quad h=1, 2, ..., H.
\end{align*}
\]

The first constraints of (2) represent an upper bound (equal to 1) in terms of absolute efficiency for all suppliers using the optimal weights for the \( f \)-th vendor, while the second constraints of (2) impose that all weights are non negative. The programming problem (2) is non-linear with its unknowns: hence, determining the solution with numerous potential suppliers and evaluation criteria is computationally intensive. However, this problem may be simplified by linearizing it, using the so-called output-oriented procedure, as follows (Charnes et al., 1978):

\[
\begin{align*}
\max_{u, v} & \sum_{h=1}^{K} u_h y_{hf} \\
\text{subject to} & \sum_{h=1}^{K} v_h x_{hf} - \sum_{h=1}^{K} v_h x_{hf} \leq 0, \quad f=1, 2, ..., F, \\
& \sum_{h=1}^{K} v_h x_{hf} = 1, \\
& u_h, v_h \geq 0 \quad \text{for} \quad h=1, 2, ..., H.
\end{align*}
\]

Solving (3) we compute the maximum efficiency of each vendor: a supplier \( s_f \in S \) is efficient if it exhibits a unitary efficiency value \( E_f \). Suppliers may accordingly be classified in a descending order of efficiency, thus leading to a ranking.
2.2 The Cross-efficiency DEA Method

The described DEA method can only distinguish between efficient and inefficient suppliers, typically providing a set of maximally efficient vendors, without discriminating among these. Such a characteristic makes it difficult its application to the supplier selection problem, particularly in the case of single sourcing purchasing. The so-called cross-efficiency DEA method (Sexton et al. 1986) improves the discriminating power of the traditional DEA technique, evaluating the efficiency of each supplier not only with respect to the set of weights that is optimal supplier itself, but also with respect to the sets of weights that are optimal for the other vendors, i.e., that maximize their efficiencies. In such a way, the f-th supplier efficiency is evaluated as the cross-efficiency CEfi given by the average of all the efficiency values that the supplier obtains by varying the considered weights. Hence the evaluation becomes a cross-evaluation rather than a self-evaluation. The resulting cross-efficiency matrix \( CE = \{CE_{fi}\} \in \mathbb{R}^{F \times H} \) is determined by evaluating, with respect to the i-th supplier, considered each time as a pivot, the related efficiency CEfi of all the other suppliers with index f. Hence we set CEi = Ei, where \( E_i \) is the optimal value of the objective function of the linear programming problem (3) solved with a pivot \( i \), i.e., with \( f=i \). Hence we set CEi = Ei, where \( E_i \) is the optimal value of the objective function of the linear programming problem (3) for the i-th supplier. As summarized earlier, for each supplier \( s_i \in S \) the values of the optimal input and output weights of its efficiency with respect to the i-th pivot supplier \( u_i^k \) and \( v_i^k \) are successively employed to determine CEfi as follows:

\[
CE_{fi} = \frac{\sum_{k=1}^{F} u_i^k \cdot y_{f_k} - \sum_{h=1}^{H} v_i^h \cdot x_{f_h}}{\sum_{k=1}^{F} u_i^k \cdot y_{f_k}} \quad \text{with} \quad f,j=1,2,\ldots,F. \tag{4}
\]

However, (4) is not univocally applicable, since there exist multiple weight combinations that maximize the i-th supplier efficiency. With them, also the efficiencies of the other suppliers CEfj with respect to \( s_i \) would vary. Hence, it is necessary to univocally choose, for each supplier, a set of weights among the combinations that maximize the efficiency. To solve the issue, Green and Doyle (1995) propose a second-level optimization procedure that is to be executed for each i-th supplier after the solution of the described linear programming problem, as follows:

\[
\begin{align*}
\min & \sum_{f,i} u_i^k \left( \sum_{j=1}^{F} y_{f_j} \right) \\
\text{subject to:} & \quad \sum_{h=1}^{H} v_i^h \left( \sum_{j=1}^{F} x_{f_j} \right) = 1, \\
& \quad \sum_{k=1}^{F} u_i^k \cdot y_{f_k} - \sum_{h=1}^{H} v_i^h \cdot x_{f_h} = 0, \\
& \quad \sum_{k=1}^{F} u_i^k \cdot y_{f_k} - \sum_{h=1}^{H} v_i^h \cdot x_{f_h} \leq 0 \quad \text{with} \quad f=1,2,\ldots,F, \\
& \quad u_i^k, v_i^h \geq 0 \quad \text{per} \quad k=1,2,\ldots,K; \quad h=1,2,\ldots,H.
\end{align*}
\]

In (5), it is imposed that each pivot supplier \( s_i \in S \) chooses, among the sets of weights maximizing its efficiency \( E_i \), the set that minimizes the overall efficiency of other vendors. The set of weights \( (u_i^k, v_i^h) \) with \( f=1,2,\ldots,K; \quad h=1,2,\ldots,H \) solving this problem is employed to determine all elements \( CE_{fi} \) with \( f=1,2,\ldots,F \) according to (4). Therefore, all suppliers \( s_j \in S \) with \( f=1,2,\ldots,F \) can be classified according to their overall cross-efficiency value that can be determined by averaging the elements of the f-th row of the cross-efficiency matrix \( CE \) as follows:

\[
CE_f = \frac{1}{F} \sum_{i=1}^{F} CE_{fi} \quad \text{with} \quad f=1,2,\ldots,F. \tag{6}
\]

3 ADDRESSING UNCERTAINTY IN SUPPLIER SELECTION BY THE DEA TECHNIQUE

The DEA technique, both in its traditional version and in its cross-efficiency extension, is deterministic. In other words, the inputs and outputs of each supplier are assumed as certain and the model does not consider any uncertainty element. On the contrary, such aspects usually characterize the supply process and should, even more importantly, be taken into account when the evaluation is referred to potential commercial partners, with whom no previous relations exist. The issue of uncertainty on DEA data was already addressed in the related literature. Dyson and Shale (2010), in particular, distinguish among four different approaches: Imprecise DEA, Bootstrapping, Chance-Constrained DEA, and Monte Carlo simulation. The Imprecise DEA model allows employing performance values that are
imprecise, i.e., expressed either as values in a range or as ordinal ranked values (that are defined by a ranking of the alternatives for the single attribute) rather than cardinal values. Bootstrapping, instead, is a technique consisting in re-sampling a sample of real observations of the uncertain variables: for each new extraction, the values of the corresponding objective function are computed, so as to obtain a sample distribution of the variable. The Chance-Constrained DEA allows employing stochastic performance values both in input and output: the probability constraints assure that the probability that the observed outputs (inputs) are higher (lower) than the best possible values overcomes a given threshold.

This work focuses on the use of Monte Carlo simulation for the application of DEA to the supplier selection problem based on stochastic data. We propose a novel method relying on the idea that the uncertain input and output values may be modelled by suitable probability distributions, based on real observations or on a simple estimate of such data. The chosen distributions (and their opportune estimated characteristic parameters) may be employed for a series of casual extractions useful to determine the efficiency of each DMU. Such a procedure was already adopted in the SC supplier choice with the traditional DEA method by Wong et al. (2008). The work by Kao and Liu (2009) is also significant, evaluating the efficiency of Taiwan banks with the DEA technique using stochastic input and output methods that are evaluated by a Beta distribution: they demonstrate that 2000 iterations are sufficient to obtain convergent results.

This paper integrates the DEA stochastic methodology proposed in Kao and Liu (2009) with the cross-efficiency DEA for application to the supplier evaluation and selection problem. In particular, let $X_{hf}$ and $Y_{kf}$ be the $h$-th stochastic input value (with $h=1,\ldots,H$) and the $k$-th stochastic output value (with $k=1,\ldots,K$), respectively, for supplier $s_j \in S$. These variables are modelled by a specific Beta probability distribution, called Beta-PERT (Vose, 2008). Such a distribution suits very well the cases in which no information is available on the values assumed in the past by the stochastic variables and it is thus impossible to define a frequency-based probability density function (Law and Kelton, 2000). Indeed, the characteristic parameters of such a distribution may be determined using a three-estimates approach that is inspired from the stochastic PERT technique for project management, while obtaining a lower standard deviation (and thus a more faithful reproduction of the expert estimate) than the well-known triangular distribution (Vose, 2008). In particular, calling $X_{hf}^{\min}$, $X_{hf}^{\mod}$, and $X_{hf}^{\max}$ respectively the lowest possible, most probable, and highest possible estimated values, respectively, that the generic input variable $X_{hf}$ may assume, the corresponding Beta (or Beta-PERT) distribution is defined by the following characteristic parameters (Vose, 2008):

$$a_{x_{hf}} = \frac{\mu_x - x_{hf}^{\min} - (2x_{hf}^{\mod} - x_{hf}^{\min} - x_{hf}^{\max})}{(x_{hf}^{\max} - \mu_x)(x_{hf}^{\max} - x_{hf}^{\min})},$$

$$a_{x_{hf}} = \frac{\mu_x - x_{hf}^{\min}}{(\mu_x - x_{hf}^{\mod})},$$

(7)

with

$$\mu_x = \frac{x_{hf}^{\min} + 4x_{hf}^{\mod} + x_{hf}^{\max}}{6}.$$  

(8)

The resulting Beta-PERT distribution is defined as follows (Vose, 2008):

$$\text{BetaPERT}(x_{hf}^{\min}, x_{hf}^{\mod}, x_{hf}^{\max}) = \text{Beta}(a_x, a_{x_{hf}})(x_{hf}^{\min} - x_{hf}^{\mod}) + x_{hf}^{\min}.$$  

(9)

Analogously, considering the stochastic variable $Y_{kf}$, given $Y_{kf}^{\min}$, $Y_{kf}^{\mod}$, and $Y_{kf}^{\max}$ respectively the lowest possible, most probable, and highest possible estimated values of the variable we set:

$$a_{y_{kf}} = \frac{\mu_y - y_{kf}^{\min} - (2y_{kf}^{\mod} - y_{kf}^{\min} - y_{kf}^{\max})}{(y_{kf}^{\max} - \mu_y)(y_{kf}^{\max} - y_{kf}^{\min})},$$

$$a_{y_{kf}} = \frac{\mu_y - y_{kf}^{\min}}{(\mu_y - y_{kf}^{\mod})},$$

(10)

with

$$\mu_y = \frac{y_{kf}^{\min} + 4y_{kf}^{\mod} + y_{kf}^{\max}}{6},$$

(11)

so that

$$\text{BetaPERT}(y_{kf}^{\min}, y_{kf}^{\mod}, y_{kf}^{\max}) = \text{Beta}(a_y, a_{y_{kf}})(y_{kf}^{\min} - y_{kf}^{\mod}) + y_{kf}^{\min}.$$  

(12)

As a consequence, all the input and output stochastic indicators $x_{hf}$ and $y_{kf}$ may be modelled as stochastic variables that are distributed according to (9) and (12), respectively. Applying the Monte Carlo method allows assigning at each iteration a casual value to all the stochastic variables according to the cited probability density functions. Such values are employed to solve at each iteration the cross-efficiency problem (3) and (5). The final cross-
efficiency values associated to the \( f \)-th supplier for \( f = 1, 2, \ldots, F \) are evaluated by averaging the overall cross-efficiency index \( CE_f \) computed for all the iterations, so that a ranking of the suppliers is established by the descending order of such values.

Summing up, the proposed method integrates both the advantages of traditional DEA (evaluating the supply efficiency and avoiding data normalization) and cross-efficiency DEA (optimizing the weight set so as to shift from self-evaluation to peer-evaluation) with respect to alternative supplier selection techniques. In addition, the technique is able to deal with uncertain data in the supply, typical of the real context.

4 USING THE STOCHASTIC DEA MODEL FOR THE SUPPLIER SELECTION OF AN SME

We apply the described model for the cross-efficiency evaluation of suppliers to the supplier selection process of a small enterprise of southern Italy, operating in the areas of sale, set-up, and maintenance of hydraulic plants. Before the subsequently reported investigation, the case study SME used the classical lowest price approach to choose suppliers. On the contrary, thanks to the proposed supplier selection tool, it was able to compare the performance of suppliers that it employed in the past with that of other potential partners, based not only on price but also considering additional uncertain data. The considered supply refers to a specific component, a cast iron mainspring check valve. While for the current suppliers the available data are deterministic, the uncertainty referring to some data of the performance of potential partners was modelled using Monte Carlo simulation and the previously introduced Beta-PERT distribution. In particular, we consider two currently existing suppliers (denoted by E1 and E2) for the product supply. Their performance is compared to that of six potential additional suppliers (P1, P2, P3, P4, P5, and P6).

The considered efficiency indicators are five. In particular, three are input indices, i.e., refer to resources that are required to the SME in the supply process: 1) component purchasing price (in €); 2) lead time for the order execution, i.e., time elapsing between order emission and product delivery (in days, d); and 3) geographical distance between supplier and SME (in Km). In addition, we consider two output indices: 1) quality of the provided component, defined as a percentage ratio between working components and overall number of delivered components (%); and 2) delivery reliability, expressed as a percentage ratio between orders that are dealt with on time and overall number of orders (%).

Table 1: Input and output data for current and potential suppliers.

<table>
<thead>
<tr>
<th>Suppl.</th>
<th>Input criteria</th>
<th>Output criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price [€]</td>
<td>Lead Time [d]</td>
</tr>
<tr>
<td>E1</td>
<td>110</td>
<td>40</td>
</tr>
<tr>
<td>E2</td>
<td>91</td>
<td>50</td>
</tr>
<tr>
<td>P1</td>
<td>95 (30,35,45)</td>
<td>970</td>
</tr>
<tr>
<td>P2</td>
<td>78 (42,45,50)</td>
<td>20</td>
</tr>
<tr>
<td>P3</td>
<td>132 (32,40,50)</td>
<td>833</td>
</tr>
<tr>
<td>P4</td>
<td>130,67 (32,40,50)</td>
<td>897</td>
</tr>
<tr>
<td>P5</td>
<td>114,50 (35,45,50)</td>
<td>813</td>
</tr>
<tr>
<td>P6</td>
<td>133,10 (32,40,50)</td>
<td>898</td>
</tr>
</tbody>
</table>

Table 2: Cross-efficiency matrix and average cross-efficiency index of suppliers.

<table>
<thead>
<tr>
<th>Suppl.</th>
<th>CE(_f) - Efficiency with respect to CE(_f)</th>
<th>Cross Eff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E1</td>
<td>E2</td>
</tr>
<tr>
<td>E1</td>
<td>0.95</td>
<td>0.87</td>
</tr>
<tr>
<td>E2</td>
<td>0.82</td>
<td>0.94</td>
</tr>
<tr>
<td>P1</td>
<td>0.99</td>
<td>0.95</td>
</tr>
<tr>
<td>P2</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>P3</td>
<td>0.94</td>
<td>0.75</td>
</tr>
<tr>
<td>P4</td>
<td>0.90</td>
<td>0.73</td>
</tr>
<tr>
<td>P5</td>
<td>0.87</td>
<td>0.79</td>
</tr>
<tr>
<td>P6</td>
<td>0.92</td>
<td>0.74</td>
</tr>
</tbody>
</table>

With reference to the data, we remark that purchasing prices and geographical distances are deterministic values. Indeed, prices may be determined using the price list of the suppliers, neglecting price variations during the year. On the contrary, the remaining indices, i.e., lead time, quality, and reliability, are deterministically defined for the current suppliers E1 and E2, using historical data of the SME, while they can be only stochastically defined for potential suppliers using the previously described the Beta-PERT approach. Table 1 summarizes the problem data. For each of the eight considered suppliers, we evaluate its cross-efficiency value according to the presented method. Due to the presence of stochastic elements among the problem data, we solve the problem using the described Monte Carlo simulation, thanks to 1000 iterations that are executed in the well-known MATLAB software environment. The results of the procedure are in Table 2. The last column of the
table show that the most efficient supplier is potential supplier P2 (that is favoured by the geographical proximity to the SME, see Table 1), followed by potential supplier P1. A key factor to this result is the optimal set of coefficients chosen for P2: it is presumable that the higher incidence falls back on the coefficients related to price and (especially) to geographical distance, two factors thanks to which supplier P2 is predominant over the remaining vendors. As a consequence, the cross-efficiency values associated to the other suppliers are two orders of magnitude lower than those characterizing P2 (ranging between 0.02 and 0.03, as the fifth column of Table 2 shows). Moreover, the last column of Table 2 remarks that the four most efficient suppliers in terms of cross-efficiency are all in the set of potential suppliers, i.e., they are P2, P1, P3, and P6, in a descending order of efficiency.

The SME purchasing manager evaluated the obtained results, underlying as major advantages of the method its ability to take into account multiple criteria, its capability to distinguish between the required resources and the overall performance, and ultimately its skill in assessing both the supply process effectiveness and efficiency.

5 CONCLUSIONS

We propose a novel method for the optimal selection of suppliers based on the well-known Data Envelopment Analysis (DEA) technique. In particular, we extend a DEA method for the cross-evaluation of efficiency, previously presented in the literature to discern among maximally efficient suppliers, using the Monte Carlo simulation method, so as to enable the treatment of uncertain data. The technique application to the supplier selection process of an Italian SME, shows its straightforwardness and its ability to discerning among suppliers, also in case of uncertain data. Future developments include further validation of the method and detailed comparison with alternative approaches presented in the scientific literature.

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