ANT COLONY OPTIMIZATION FOR THE UNEQUAL-AREA FACILITY LAYOUT PROBLEM

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Abstract: In this paper, an ant colony optimization (ACO) approach is proposed to solve the Facility Layout Problem (FLP) with unequal area departments. The flexible bay structure (FBS) is relaxed by allowing empty spaces in bays, which results in more flexibility while assigning departments in bays. The comparative results show that the ACO approach is very promising.

1 INTRODUCTION

The Facility Layout Problem (FLP) is generally defined as locating $N$ departments in an area of size $W \times H$. The inputs of the problem include department area $a_i$ and minimum side length $h_{\text{min}}$ requirements for each department $i$ as well as material flow $f_{ij}$ and material handling cost $c_{ij}$ (per unit flow per unit distance travelled) between each department pair $i$ and $j$. The goal is to minimize the total material handling cost, which is generally expressed as follows:

$$F(s) = \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} f_{ij} d_{ij}(s)$$

where $d_{ij}(s)$ is the distance between the centre points of departments $i$ and $j$ for a given layout $s$. The decision variables of the FLP include determining department centres $(x_i, y_i)$ and department shapes for each department $i$. Satisfying the area requirements of the departments, the boundaries of the layout and restrictions on the departments’ shapes are the problem constraints. The output of the FLP is called block layout, which specifies relative location and shape of each department in the area.

In this paper, an ant colony inspired algorithm is proposed to solve the FLP with unequal area departments in the flexible bay structure (FBS). In the FBS, departments are located only in parallel-bays with varying width, bays are bounded by straight aisles on both sides, and departments are restricted to be located only in one bay. Recently, Komarudin and Wong (2010), Wong and Komarudin (2010), and Kulturel-Konak and Konak (2011a) have proposed ACO approaches to solve the unequal area FLP. In this paper, an ACO approach for the relaxed FBS, called ACO-RFBS, is developed. The relaxed FBS (RFBS) concept was originally proposed by Kulturel-Konak and Konak (2011b) to remedy the drawbacks of the FBS. The RFBS allows empty spaces in bays, which results in more flexibility while assigning departments in bays. Being different from the Particle Swarm Optimization (PSO) by Kulturel-Konak and Konak (2011b), the ACO-RFBS uses a different encoding scheme, a dynamic penalty handling method, and a two-phase diversification scheme. Moreover, in this paper, facility areas are expanded, and the proposed approach is used to solve the problems with expanded areas to demonstrate the advantages of the RFBS.

2 THE ACO-RFBS

2.1 Solution Construction Definition

In the ACO-RFBS, first a layout sketch is constructed by filling bays one department at a time, from bottom to top. A layout sketch defines the relative locations of the departments within bays. After creating a layout sketch, the actual locations and shapes of the departments are calculated according to the RFBS as described by Kulturel-Konak and Konak (2011b). Figure 1 demonstrates a
step-by-step example of constructing a layout sketch with five departments.

![Layout Sketch Example](image_url)

Figure 1: An example of layout construction where (x) represents admissible cells to assign departments.

As demonstrated in the example in Figure 1, three types of department assignments are admissible while adding an unassigned department \( i \) to a partial layout sketch as follows:

- \((i, j, 1)\): department \( j \) is the first department in the leftmost bay of a partial layout sketch. As a result of this assignment, department \( i \) is located immediately to the left of department \( j \).
- \((i, j, 2)\): department \( j \) is the last department in a bay. As a result of this assignment, department \( i \) is located immediately above department \( j \).
- \((i, j, 3)\): department \( j \) is the first department in the rightmost bay of a partial layout sketch. As a result of this assignment, department \( i \) is located immediately to the right of department \( j \).

Pheromone \( \tau(i, j, k) \) is defined as the favourability of assignment \((i, j, k)\). Let \( A \) be the set of all admissible assignments. While constructing a layout sketch, an admissible assignment \((i, j, k)\) is randomly selected from \( A \), and department \( i \) is added to the sketch according to the assignment rules defined above. The probability of selecting an admissible assignment \((i, j, k)\) from \( A \) is given as follows:

\[
p(i, j, k) = \frac{\tau(i, j, k)^{1-\beta} \eta(i, j)^{\beta}}{\sum_{(x, y, z) \in A} \tau(x, y, z)^{1-\beta} \eta(x, y)^{\beta}}
\]  

where \( \eta(i, j) \) is the problem specific heuristic information, which is defined as a function of the normalized flows between departments \( i \) and \( j \) as follows:

\[
\eta(i, j) = 1 + \frac{N \times f_{ij}}{\max_{j \neq i} \sum_{j \neq i} f_{ij}}
\]

Unlike the standard ACO (Dorigo et al., 1996; Dorigo and Gambardella, 1997), only a single parameter, \( 0 < \beta < 1 \), is used in the ACO-RFBS to scale the relative importance of the pheromone and the problem specific heuristic information. To do so, the pheromone and heuristic information values are normalized in the same range. Layout construction initially starts with an empty sketch. While assigning the first department, however, equation (2) cannot be used because \( A \) is an empty set. The first department is randomly selected with the following probability,

\[
p(j) = \frac{\sum_{i=1}^{N} (\tau(i, j, 1) + \tau(i, j, 3))}{\sum_{i=1}^{N} \sum_{j=1}^{N} (\tau(i, j, 1) + \tau(i, j, 3))}
\]

where \( p(j) \) denotes the probability of selecting department \( j \) as the first department. In equation (4), only pheromones \( \tau(i, j, 1) \) and \( \tau(i, j, 3) \) are considered while calculating probability of selecting department \( j \). Therefore, the layout sketch is likely to start with a department that might yield good solutions if it is located as the first department in a bay.

**Procedure Solution Construction()**

**Step 1.** Set \( A=\{(i, \alpha, k) : i \in U, k \in \{1, 2, 3\}\} \) and calculate \( p(i) \) for \( i=1,\ldots,N \).

**Step 2.** Randomly select a department \( i \) with probability \( p(i) \) to assign to the layout sketch. Let \( i^* \) denote the selected department.

**Step 3.** Set \( U=U\setminus\{i^*\} \) and \( A=A\setminus\{(i^*, \alpha, k) : i^* \in U, k \in \{1, 2, 3\}\} \).

**Step 4.** Calculate \( p(i, j, k) \) for all \((i, j, k)\in A\), and randomly select an assignment. Let \((i^*, j^*, k^*)\) denote the selected assignment.

**Step 5.** Set \( U=U\setminus\{i^*, j^*, k^*\}, A=A\setminus\{(i^*, j^*, k^*) : i^* \in U, j^* \in U, k^* \in \{1, 2, 3\}\} \).

**Step 6.** If \( U=\{\} \), then go to Step 4.

**Step 7.** Create the actual layout from the sketch.

### 2.2 Solution Evaluation

Although the FBS representation is relaxed in this paper, some solutions may still have departments with impractical shapes, such as a very narrow/long rectangular department. In addition, the width of the layout may exceed the maximum allowed width of the area because adjusted bay widths are wider than regular bay widths. The ACO-RFBS uses the maximum aspect ratio, which is defined as the ratio of a department’s longer side to its shorter side, to quantify the infeasibility of solutions with respect to department shapes. Therefore, a small-sized and a large-sized department can be penalized in the same scale. Let \( a_i(s) \) represent the aspect ratio of department \( i \) for solution \( s \) and let \( a_i \) be the given
maximum aspect ratio of department $i$. Let $W(s)$ be
the width of the layout for solution $s$. Solution $s$ is
said to be feasible if and only if $\alpha_i(s) \leq \alpha_i$ for
each department $i$ and $W(s) > W$. Infeasible solutions are
dynamically penalized using the near feasibility
threshold (NFT) concept (Kulturel-Konak et al.,
2004).

2.3 Local Search

After evaluating the fitness of the solutions in an
iteration, a local search attempts to improve the best
solution of the iteration, $s^*$, where new solutions are
created from $s^*$ by swap and insert operators.
Operator swap($i$, $j$) swaps the positions of
departments $i$ and $j$. Operator insert($i$, $b$, $k$) inserts
department $i$ into the $k^{th}$ position of bay $b$. The insert
operator changes the relative locations of the bay
breaks in a layout. The swap and insert operators are
randomly selected in each loop of the local search
and performed for all possible combinations. If a
better solution is found, $s^*$ is updated, and the local
search continues until no improvements possible.

2.4 Pheromone Update, Diversification,
and Overall Algorithm Evaluation

In each iteration, $\mu$ solutions are generated as
described in the previous section, pheromone values
are updated based on the best feasible solution $s^*$ or the
best solution of the iteration $s^*$ as follows:

$$\tau(i,j,k) = \begin{cases} 
\rho \times \tau(i,j,k) + \\
\alpha(i,j,k | s^*) & \text{if } s^* \text{ is available,} \\
\rho \times \tau(i,j,k) + \\
\alpha(i,j,k | s^*) & \text{otherwise.} 
\end{cases}$$

where $\rho < 1$ is the evaporation parameter and
$\alpha(i,j,k | s) = 1$ if assignment $(i, j, k)$ is used to construct solution $s$ and $\alpha(i,j,k | s) = 0$, otherwise.

During the search, if $s^*$ has not been updated for
a certain number of iterations, new solutions cannot
be generated. The ACO-RFBS uses a two-phase
diversification schema when the search stagnates in
such cases as follows:

$$\tau(i,j,k) = \begin{cases} 
U(1,N) & \text{if } g_0 > g^* \\
\max\{0,N - \tau(i,j,k)\} & \text{if } g_0 > g^* \text{ and } g_0 > g^{**}, 
\end{cases}$$

where $g_0$ is the number of consecutive iterations
such that $s^*$ has not been updated, $g_0$ is the number
of the consecutive iterations in which the same $s^*$
is obtained, and $g^*$ and $g^{**}$ are diversification
parameters. Observing the same $s^*$ in the last $g^*$
iterations indicates stagnation of the search.
Therefore, all pheromone values are randomly reset
between one and $N$ to restart the search at a different
location in the search space. If the search is
stagnated without improving $s^*$ in the last
consecutive $g^{**}$ iterations, the pheromone values are
reversed in the second case of equation (6). The
search is terminated after performing $g_d$
diversifications.

Procedure ACO-RFBS ($\mu, \rho$, $\beta$, $\beta$; $g_0$, $g^*$, $g^{**}$)

Step 1. $g=0$, $g_0=0$, $g^* = 0$, $g^{**} = 0$ for all $i$ and $j$, and $k=1, 2, 3$.

Step 2. Generate $\mu$ solutions using Procedure
Create_Solution() and calculate the fitness of
each solution.

Step 3. Identify $s^*$ and if $s^*$ is different than the best
solution of the previous iteration, set $g=0$ and apply
the local search on $s^*$ if it is not equal to $s^{**}$ or $s^*$
of the previous iteration. If a better solution is
found, replace $s^*$. Update $s^{**}$ and $F^*$
if necessary. Set $g_0=0$ if $s^*$ is updated.

Step 4. Update pheromone values using (5).

Step 5. If one of the conditions in equation (6) is
satisfied, apply diversification and set $g=0$.

Step 6. Set $g=g+1$, $g^* = g^* + 1$, $g^{**} = g^{**} + 1$. If $g_0$ number of
phase diversifications have been
performed, then stop and return $s^{**}$; else, go to
Step 2.

3 COMPUTATIONAL
EXPERIMENTS

To compare the performance of the ACO-RFBS,
seven test problems ranging from twelve to 35
departments are used as given in Table 1. All these
problems have been previously solved in the
literature using the FBS. Additional information
about these problems as well as their best RFBS
solutions can be found in (Kulturel-Konak and
Konak, 2011b). Herein this paper, these problems
were first solved with their original dimensions
given in the literature, and then, they were solved
again with their relaxed dimensions in which the
layout widths were increased about 10% allowing
empty spaces in bays. In addition, the problems were
solved using horizontal and vertical running bays. In
problems Tam20, Tam30, SC30, and SC35,
Table 1: Properties of the test problems.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Relaxed Area</th>
<th>Best Known</th>
<th>Reference</th>
<th>Best Known RFBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nug12</td>
<td>5×3</td>
<td>262.00</td>
<td>(Kulturel-Konak and Konak, 2011a)</td>
<td>257.50</td>
</tr>
<tr>
<td>Nug15</td>
<td>4×5</td>
<td>524.75</td>
<td></td>
<td>524.75</td>
</tr>
<tr>
<td>AB20(4)</td>
<td>35×20</td>
<td>5073.82</td>
<td>(Liu and Meller, 2007)</td>
<td>5336.36</td>
</tr>
<tr>
<td>Tam20</td>
<td>44×35</td>
<td>9003.82</td>
<td>(Kulturel-Konak and Konak, 2011a)</td>
<td>8753.57</td>
</tr>
<tr>
<td>Tam30</td>
<td>50×40</td>
<td>19667.45</td>
<td></td>
<td>19462.41</td>
</tr>
<tr>
<td>SC30</td>
<td>18×12</td>
<td>3679.85</td>
<td>(Wong and Komarudin, 2010)</td>
<td>3443.34</td>
</tr>
<tr>
<td>SC35</td>
<td>20×15</td>
<td>3604.00</td>
<td>(Liu and Meller, 2007)</td>
<td>3700.75</td>
</tr>
</tbody>
</table>

Table 2: Solutions found by the ACO-RFBS for the test problems.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Original Area</th>
<th>Relaxed Area</th>
<th>Best Imp (%) over Best-known</th>
<th>Average CPU Sec</th>
<th>Best Imp (%) over Best-known RFBS</th>
<th>CPU Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nug12</td>
<td>257.50</td>
<td>1.75</td>
<td>257.50</td>
<td>57</td>
<td>253.00</td>
<td>53</td>
</tr>
<tr>
<td>Nug15</td>
<td>524.75</td>
<td>0.00</td>
<td>524.75</td>
<td>120</td>
<td>511.50</td>
<td>121</td>
</tr>
<tr>
<td>AB20(4)</td>
<td>5336.36</td>
<td>-5.17</td>
<td>5336.36</td>
<td>1940</td>
<td>5023.23</td>
<td>288</td>
</tr>
<tr>
<td>Tam20</td>
<td>8753.57</td>
<td>2.86</td>
<td>8778.15</td>
<td>311</td>
<td>8727.45</td>
<td>327</td>
</tr>
<tr>
<td>Tam30</td>
<td>19462.41</td>
<td>1.05</td>
<td>19528.96</td>
<td>1881</td>
<td>19462.41</td>
<td>1669</td>
</tr>
<tr>
<td>SC30</td>
<td>3443.34</td>
<td>6.87</td>
<td>3499.20</td>
<td>1655</td>
<td>3259.61</td>
<td>1797</td>
</tr>
<tr>
<td>SC35</td>
<td>3700.75</td>
<td>-2.68</td>
<td>3971.76</td>
<td>2393</td>
<td>3607.60</td>
<td>3102</td>
</tr>
</tbody>
</table>

Imp (%) = Percent improvement from the previously reported best-solution.

In Table 2, ACO-RFBS results are compared to their best-known RFBS solutions as well as their best-known solutions. It should be noted that the best-known solutions of several FLP test problems (i.e., Nug12, Tam20, Tam30, and SC30) were improved by the ACO-RFBS in this paper despite the limitations of the FBS as stated in the introduction section. These improvements indicate that the proposed ACO-RFBS is effective. Moreover, when the department widths were relaxed, the proposed ACO-RFBS was able to improve the best solutions for all problems excluding Tam30. Note that such improvements may not be achieved using the original FBS representation. Therefore, these results demonstrate an advantage of the RFBS over the original FBS.

4 CONCLUSIONS

In this paper, an ACO algorithm is proposed to solve FLP with relaxed FBS and compared with the existing methods in the literature with promising results. With the ability of incorporating problem specific heuristic information into the search process, the ACO approach is well suited to effectively solve various facility layout problems. In this paper, it is demonstrated that the relaxed FBS may result in a block layout with a lower material handling cost by expanding the width of a facility.

REFERENCES


