Keywords: Particle swarm optimization (PSO), Genetic algorithms (GAs), Adaptive system, Multi-peak problems.

Abstract: To reduce a large amount of calculation cost and to improve the convergence to the optimal solution for multi-peak optimization problems with multi-dimensions, we propose a new method of Adaptive plan system with Genetic Algorithm (APGA). This is an approach for Improved Particle Swarm Optimization (PSO) using APGA. The hybrid strategy using APGA is introduced into PSO operator (H-PSOGA) to improve the convergence towards the optimal solution. The H-PSOGA is applied to some benchmark functions with 20 dimensions to evaluate its performance.

1 INTRODUCTION

The product design is becoming more and more complex for various requirements from customers and claims. As a consequence, its design problem seems to be multi-peak problem with multi-dimensions. The Genetic Algorithm (GA) (Goldberg, 1989) is the most emergent computing method has been applied to various multi-peak optimization problems. The validity of this method has been reported by many researchers. However, it requires a huge computational cost to obtain stability in the convergence to an optimal solution. To reduce the cost and to improve stability, a strategy that combines global and local search methods becomes necessary. As for this strategy, current research has proposed various methods. For instance, Memetic Algorithms (MAs) are a class of stochastic global search heuristics in which Evolutionary Algorithms-based approaches (EAs) are combined with local search techniques to improve the quality of the solutions created by evolution. MAs have proven very successful across the search ability for multi-peak functions with multiple dimensions (Smith et al., 2005). These methodologies need to choose suitably a best local search method from various local search methods for combining with a global search method within the optimization process. Furthermore, since genetic operators are employed for a global search method within these algorithms, design variable vectors which are renewed via a local search are encoded into its genes many times at its GA process. These certainly have the potential to break its improved chromosomes via gene manipulation by GA operators, even if these approaches choose a proper survival strategy.

To solve these problems and maintain the stability of the convergence to an optimal solution for multi-peak optimization problems with multiple dimensions, Hasegawa et al. proposed a new evolutionary algorithm (EA) called an Adaptive Plan system with Genetic Algorithm (APGA) (Hasegawa, 2007).

Recently, especially, meta-heuristics receive a lot of attentions. There is Particle Swarm Optimization (PSO) has been developed by Kennedy and Eberhart (2001). PSO is the method using simple iterative calculations, thus it is easy to create the program source. Therefore, PSO is applicable to wide-ranging optimization problems. However, it might be difficult to find the global optimal solution when it comes to complex objective functions which have a lot of local optimal solutions. The main problem of PSO is that it prematurely converges to stable point which is not necessarily optimum. To resolve this problem, various improvement algorithms are proposed. It is proven to be a successful in solving a variety of optimal problems.

In this paper, we purposed a Hybrid strategy for Improved PSO using APGA (H-PSOGA) to converge to the optimal solution.

This paper is organized in the following manner.
The concept of PSO is described in Section 2, concept of APGA is in Section 3. Section 4 explains the algorithm of proposed strategy (H-PSOGA), and Section 5 discussed about the convergence to the optimal solution of multi-peak benchmark functions. Finally, Section 6 includes some brief conclusions.

2 PARTICLE SWARM OPTIMIZATION

2.1 PSO Algorithms

Particle Swarm Optimization (PSO) is a population based stochastic optimization technique inspired by a social behavior of bird flocking or fish schooling (Kennedy and Eberhart, 2001).

PSO is an algorithm of a multipoint search type that is effective to solve the optimization problem. It can be said that PSO has a high versatility because it can search whether the objective function has continuous or differentiability, and using only information of the objective function value (Shi and Eberhart, 1998).

We are concerned here with gbest-model which is known to be conventional PSO. In gbest-model, each particle which makes up a swarm has information of its position and velocity at the present in the search space. And it has information of the best solution found so far of itself as \( pbest \) and of a whole swarm as \( gbest \). Each particle aims at the global optimal solution of multi-peak benchmark functions. Finally, \( OPTIMIZATION \)

\[ \begin{align*}
    v_{ij}^{k+1} &= w v_{ij}^k + c_1 \text{rand}_1 (pbest_{ij}^k - x_{ij}^k) + c_2 \text{rand}_2 (gbest_{ij}^k - x_{ij}^k) \\
    x_{ij}^{k+1} &= x_{ij}^k + v_{ij}^{k+1}
\end{align*} \]

Where \( v \) and \( x \) are velocity and position respectively; \( i \) is the particle number, and \( j \) indicates the \( j \)-th component of these vectors; \( pbest \), is the best position that \( j \)-th particle has ever found; \( gbest \) is the best position which particles has ever found; \( \text{rand}_1 \) and \( \text{rand}_2 \) are two uniformly distributed random values independently generated within \([0, 1]\) for the \( k \)-th dimension; \( w \) is the inertia weight; \( c_1 \) and \( c_2 \) are the acceleration coefficients.

The position of each particle is updated with (2) by the velocity updated in (1) as shown in Figure 1. After a number of iterations, PSO is going to get the global optimal solution as conclusive gbest.

2.2 Improved PSO

Given its simple concept and effectiveness, the PSO has become a popular optimizer and has widely been applied in practical problem solving. Meanwhile, much research on performance improvements has been reported including parameter studies, combination with auxiliary operations, and topological structures.

Linearly Decreasing Inertia Weight (LDIW) is the technique of basic PSO introduced by Shi and Eberhart (1998). The inertia weight \( w \) in (1) linearly decreasing with the iterative generation. In this method, the value of \( w \) is adopted below:

\[ w = w_{\max} - \frac{w_{\max} - w_{\min}}{iteration_{\max}} \cdot iteration \]  

Where the maximal and minimal weights \( w_{\max} \) and \( w_{\min} \) are respectively set 0.9, 0.4 known from experience.

In PSO algorithm, the concept of diversification and intensification is quite important, because it decides the characteristic of the swarm and the search performance. By using (3), the particles can be transformed from diversification to intensification by decreasing the inertia weight linearly as the search proceeds.

In addition to the inertia weight, the acceleration coefficients \( c_1 \) and \( c_2 \) are also important parameters in PSO. Both acceleration coefficients are essential to the success of PSO. Kenedy and Eberhart (2001) suggested a fixed value of 2.0, and this configuration has been adopted by many other researchers.

Another active research trend in PSO is hybrid PSO, which combines PSO with other evolutionary paradigms. All techniques have also been hybridized with PSO to enhance performance and to prevent the swarm from crowding too closely and to locate as many optimal solutions as possible. For a current review of hybrid approaches to PSO the reader is referred to, e.g. Zhan et al. (2009).
3 ADAPTIVE SYSTEM WITH GENETIC ALGORITHM

3.1 The Optimization Problem

The optimization problem is formulated in this section. Design variable, objective function and constrains condition are defined as follows:

\[
\text{Design variable}: \quad X = [x_1, \ldots, x_n] \quad (4)
\]

\[
\text{Objective function}: \quad -f(X) \rightarrow \text{Max} \quad (5)
\]

\[
\text{Constrain condition}: \quad X^{LB} \leq X \leq X^{UB} \quad (6)
\]

Where \( X^{LB} = [x_1^{LB}, \ldots, x_n^{LB}] \) and \( X^{UB} = [x_1^{UB}, \ldots, x_n^{UB}] \) and \( n \) denote the lower boundary condition vectors, the upper boundary condition vectors and the number of design variable vectors (DV) respectively. A number of DV's significant figure is defined, and DV is rounded off its places within optimization process.

3.2 APGA

The APGA concept was introduced as a new EA strategy for multi-peak optimization problems. Its concept differs in handling DVs from general EAs based on GAs. EAs generally encode DVs into the genes of a chromosome, and handle them through GA operators. However, APGA completely separates DVs of global search and local search methods. It encodes Control variable vectors (CVs) of AP into its genes on Adaptive system (AS). Moreover, this separation strategy for DVs and chromosomes can solve Memetic Algorithm (MA) problem of breaking chromosomes (Smith et al., 2005). The conceptual process of APGA is shown in Figure 2. The control variable vectors (CVs) steer the behavior of adaptive plan (AP) for a global search, and are renewed via genetic operations by estimating fitness value. For a local search, AP generates next values of DVs by using CVs, response value vectors (RVs) and current values of DVs according to the formula:

\[
X_{t+1} = X_t + NR \cdot AP(C_t, R_t) \quad (7)
\]

Where \( NR, AP(), X, C, R, t \) denote neighborhood ratio, a function of AP, DVs, CVs, RVs and generation, respectively. In addition, for a verification of APGA search process, refer to ref. (Sousuke Tooyama, 2009) (Pham Ngoc Hieu, 2010).

3.3 Adaptive Plan

It is necessary that the AP realizes a local search process by applying various heuristics rules. In this paper, the plan introduces a DV generation formula using a sensitivity analysis that is effective in the convex function problem as a heuristic rule, because a multi-peak problem is combined of convex functions. This plan uses the following equation:

\[
AP(C_t, R_t) = -\text{Scale} \cdot SP \cdot \text{sign}(\nabla R_t) \quad (8)
\]

\[
SP = 2C_t - 1 \quad (9)
\]

Where \( \text{Scale}, \nabla R \) denote the scale factor and sensitivity of RVs, respectively. A step size \( SP \) is defined by CVs for controlling a global behavior to prevent it falling into the local optimum. \( C = [c_1, \ldots, c_n] \), \((0 \leq c_i \leq 1.0)\) is used by (9) so that it can change the direction to improve or worsen the objective function, and \( C \) is encoded into a chromosome by 10 bit strings as shown in Figure 3. In addition, \( i, j \) and \( p \) are the individual number, design variable number and its size, respectively.

![Conceptual process of APGA.](image)

Figure 2: Conceptual process of APGA.  

![Encoding into genes of a chromosome.](image)

Figure 3: Encoding into genes of a chromosome.

3.4 GA Operators

3.4.1 Selection

Selection is performed using a tournament strategy to maintain the diverseness of individuals with a goal of keeping off an early convergence. A tournament size of 2 is used.
3.4.2 Elite Strategy

An elite strategy, where the best individual survives in the next generation, is adopted during each generation process.

It is necessary to assume that the best individual, i.e., as for the elite individual, generates two behaviors of AP by updating DVs with AP, not GA operators. Therefore, its strategy replicates the best individual to two elite individuals, and keeps them to next generation.

3.4.3 Crossover and Mutation

In order to pick up the best values of each CV, a single point crossover is used for the string of each CV. This can be considered to be a uniform crossover for the string of the chromosome.

Mutation are performed for each string at mutation ratio on each generation, and set to maintain the strings diverse.

3.4.4 Recombination

At following conditions, the genetic information on chromosome of individual is recombined by uniform random function.

- One fitness value occupies 80% of the fitness of all individuals.
- One chromosome occupies 80% of the population.

If this manipulation is applied to general GAs, an improved chromosome into which DVs have been encoded is broken down. However, in the APGA, the genetic information is only CVs used to make a decision for the AP behavior. Therefore, to prevent from falling into a local optimum, and to get out from the condition of being converged with a local optimum, a new AP behavior is provided by recombining the genes of the CVs into a chromosome. And the optimal search process starts to re-explore by a new one. This strategy is believed to make behavior like the re-annealing of an SA.

4 HYBRID PSO AND APGA

Many of optimization techniques called metaheuristics including PSO are designed based on the concept. In case of PSO, when a particle discovers a good solution, other particles also gather around the solution. Therefore, they cannot escape from a local optimal solution. Consequently, PSO cannot achieve the global search. However, APGA that combines the global search ability of a GA and an Adaptive Plan with excellently local search ability is superior to other EAs (MAs). With a view to global search, we propose the new algorithm based on PSO for getting hints from APGA named H-PSOGA. The communication between PSO and APGA is mentioned below.

4.1 The Concept of Proposed Method

4.1.1 Parallel Method (Type 1)

H-PSOGA Type 1 sorts all of individuals by estimating their fitness then ranks them by these sorting results. In this method, both PSO and APGA are run in parallel, with one half of individual number is adjusted to PSO operator and another half one is determined by APGA process. This type maintains the iteration of PSO and APGA for the entire run, which consists chiefly of genetic algorithm (GA), combined with PSO and the sequential steps of the algorithm are given below. Briefly, the operations are illustrated in Figure 4.

Step 1: Sort all of individuals by estimating their fitness value then rank them by results.

Step 2: The good individuals (high fitness values) are evolved with APGA and procedure offspring.

Step 3: The low individuals (low fitness values) are evolved with PSO and their best position is updated with velocity.

Step 4: Combine as the new population and calculate their fitness, choose the best position as the global.

Step 5: Repeat steps 2-4 until the termination such as a sufficiently good solution or a maximum number of generations being completed, is satisfied.

The best scoring individual in the population is taken as the global optimal solution.

4.1.2 Serial Method (Type 2)

H-PSOGA Type 2 aims at getting the direction from particle swarm to adjust into APGA. This method introduces a DV generation formula using the veloci-
ity update from PSO operator, not sensitivity analysis. AP generates next values of DVs by using CVs, velocity $v$ and current values of DVs following the equations:

$$x_{i+1} = x_i + NR \cdot AP(C_i, v_{i+1})$$  \hfill (10)$$

$$AP(C_i, v_{i+1}) = -Scale \cdot SP \cdot v_{i+1}$$  \hfill (11)

The flowchart of Type 2 is shown in Figure 5. In this approach, PSO and APGA run individually. The iteration is run by PSO operator and the velocity update following (1) is given as initial parameter of AP for APGA process.

![Flowchart of Type 2](image)

Figure 5: H-PSOGA Type 2.

4.2 The Roles of Type 1 and Type 2

As described above, the individuals which compose of Type 1 play the role of searching intensively around of $gbest$, to parallel with APGA that combines global search and local search, and the whole swarm is stabilized. On the other hand, that of Type 2 play the role of searching globally in the solution space by getting the direction from PSO operator adjusted into APGA, and the whole swarm is un-stabilized.

We can say that Type 1 achieves the intensification, and Type 2 diversification with the aim to reduce the calculation cost and improve the convergence to the optimal solution.

5 NUMERICAL EXPERIMENTS

The numerical experiments are first performed to compare among methods. To show the effects of these methods whether particles can escape from a local optimal solution and find the global optimal solution, we compare with other methodologies for the robustness of the optimization process. These experiments are performed 20 trials for every function. The initial seed number is randomly varied during every trial. In each experiment, the GA parameters used in solving benchmark functions are set as follows: selection ratio, crossover ratio and mutation ratio are 1.0, 0.8 and 0.01 respectively. The population size is 50 individuals and the terminal generation is 5000th generation. The sensitivity plan parameters in (8) and (11) for normalizing functions are listed in Table 3. The inertia weight $w$ is adopted by LDNW method with $w_{\text{max}}$, $w_{\text{min}}$ is respectively set 0.9, 0.4, and the acceleration coefficients $c_1$ and $c_2$ are set by fixed value of 2.0.

5.1 Benchmark Functions

For the H-PSOGA, we estimate the stability of the convergence to the optimal solution by using three benchmark functions with 20 dimensions Rastrigin (RA), Griewank (GR) and Rosenbrock (RO).

These functions are given as follows:

$$RA = 10n + \sum_{i=1}^{n} \left( x_i^2 - 10 \cos(2\pi x_i) \right)$$  \hfill (12)$$

$$GR = 1 + \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \cos \left( \frac{x_i}{\sqrt{i}} \right)$$  \hfill (13)$$

$$RO = \sum_{i=1}^{n} 100(x_{i+1}^2 + (x_i + 1)^2) + x_i^2$$  \hfill (14)$$

Table 1: Characteristics of the benchmark functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Epistasis</th>
<th>Multi-peak</th>
<th>Steep</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA</td>
<td>No</td>
<td>Yes</td>
<td>Average</td>
</tr>
<tr>
<td>GR</td>
<td>Yes</td>
<td>Yes</td>
<td>Small</td>
</tr>
<tr>
<td>RO</td>
<td>Yes</td>
<td>No</td>
<td>Big</td>
</tr>
</tbody>
</table>

Table 2: Design range, digits of DVs.

<table>
<thead>
<tr>
<th>Function</th>
<th>Design range</th>
<th>Number of digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>GR</td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td>RO</td>
<td>Yes</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3: Scale factor for normalizing the benchmark functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Scale factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA</td>
<td>10.0</td>
</tr>
<tr>
<td>GR</td>
<td>100.0</td>
</tr>
<tr>
<td>RO</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table 1 shows their characteristics, and the terms epistasis, multi-peak, steep denote the dependence relation of the DVs, presence of multi-peak and level of steepness, respectively. All functions are minimized to zero, when optimal DVs $X = 0$ are obtained. Moreover, it is difficult to search for their optimal solutions by applying one optimization strategy only, because each function has a different complicated characteristic. In Table 2, their design range, the digits of DVs
are summarized. If the search point attains an optimal solution or a current generation process reaches the termination generation, the search process is terminated.

5.2 Experiment Results

The experiment results are shown in Table 4. In the table, when success rate of optimal solution is not 100%, “-” is described. By H-PSOGA Type 1, the solutions of all benchmark function with 20 dimensions reach their global optimum solutions. However, H-PSOGA Type 2 could not reach the optimum solution with RO function because of the communication between PSO operator and APGA. As a result, we can confirm that Type 1 converged faster than Type 2.

H-PSOGA Type 1 converged similar to APGA process with high fitness value. APGA process could arrive at a global optimum with a high probability and PSO operator converged faster with RO function but could not reach with RA function.

From the result shown in Table 4, we employed H-PSOGA Type 1 for H-PSOGA model.

To sum up, it validity confirms that this hybrid strategy can reduce the computation cost and improve the stability of the convergence to the optimal solution.

Table 4: Num. of generations with 20 dimensions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Basic PSO</th>
<th>H-PSOGA Type 1</th>
<th>H-PSOGA Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA</td>
<td>-</td>
<td>202</td>
<td>405</td>
</tr>
<tr>
<td>GR</td>
<td>2878</td>
<td>344</td>
<td>538</td>
</tr>
<tr>
<td>RO</td>
<td>2220</td>
<td>1353</td>
<td>-</td>
</tr>
</tbody>
</table>

5.3 Comparison

H-PSOGA was compared with basic PSO. PSO algorithm converged the optimum solution with GR and RO function but did not reach with RA function. However the number of iterations is still large. The results of basic PSO are shown in Table 4.

This method was better than basic PSO in all benchmark functions, and it converged the global optimal solution with a high probability. Therefore, it is desirable to introduce APGA for improved PSO.

In particular, it was confirmed that the computational cost with these method could be reduced for benchmark functions. And it showed that the convergence to the optimal solution could be improve more significantly.

Overall, the H-PSOGA was capable of attaining robustness, high quality, low calculation cost and efficient performance on many benchmark problems.

6 CONCLUSIONS

In this paper, to overcome the weak point of PSO that particles cannot escape from a local optimal solution, and to achieve the global search for the solution space of multi-peak optimization problems with multi-dimensions, we proposed the new hybrid approach, H-PSOGA. Then, we verified the effectiveness of H-PSOGA by the numerical experiments performed three benchmark functions. The obtained points are shown below.

The search ability of H-PSOGA with the multi-dimensions optimization problems is very effective, compared with that of basic PSO or GA.

Both types of H-PSOGA achieved at the optimal solution, however they have strengths and weaknesses. The key is how to communicate between PSO and APGA, it is a future work.

Finally, this study plans to do a comparison with the sensitivity plan of AP by applying other optimization methods into AP and optimizing some kinds of benchmark functions.

REFERENCES