A MODEL FOR DESIGNING NON COOPERATIVE SUPPLY CHAIN WHERE LOGISTICS SERVICE PROVIDERS TAKE PART

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Abstract: This paper presents a mathematical model for the problem of designing non-cooperative supply chain where the logistic service providers take part. In this problem, manufacturers are not collaborating or having any type of bargaining strategy among them, they compete for supplying products to retailers seeking to maximize their profit. Logistics service providers also compete among them for providing logistics services to manufacturers and delivering products to retailers. It is considered that manufacturers, logistics service providers and retailers collaborate to maximize services. Normally this problem can not be modeled as an optimization problem and we use a variational inequality approach to formulate it. The model determines the optimal level of production for each manufacturer, the flows of products between manufacturers and retailers, the flow of products each logistics service provider is going to move and the price retailers are willing to pay to manufacturer in a non-cooperative environment. We demonstrate and discuss theory results regarding existence and uniqueness of the solution for the model. An example is presented to illustrate some properties of the problem.

1 INTRODUCTION

As mentioned by Frankel, Bolumole, Eltantawy, Paulraj and Gundlach (2008), Giannakis and Croom (2004), Gibson, Mentzer and Cook (2005), Giunipero, Hooker, Joseph-Matthews, Yoon and Brudvig (2008), Lambert, Garcia-Dastugue, and Croxton (2005), Larson, Poist and Halldorsson (2007), Supply chain management (SCM) has become a fertile field for the application of a wide variety of disciplines, including finance, logistics, operations management, operations research, and information technology among others. For some authors, the philosophy of SCM is to combine some or all of these disciplines to produce a comprehensive strategy for improving the performance of the company (Giunipero et al., 2008). The large number of articles published till today, especially in the last twenty years, reflects the enormous interest shown in SCM by the academic and business world. Even when there is an enormous number of publications, different authors agree that despite the importance of SCM to gain competitive advantages and improve the performance of organizations (Cooper, Lambert & Pagh, 1997; Croom, Romano & Giannakis, 2000; Elmuti, 2002; Lambert, Cooper & Pagh, 1998; Gunasekaran, Patel & Tirtiroglu, 2001; Sanders, 2009), there is still no consensus on its definition, the limits for practical application and the relationship between SCM and
other disciplines or concepts (Frankel et al., 2008; Gibson et al., 2005; Lambert et al., 2005; Larson et al., 2007). For Wisner and Tan (2000) "The concept is still evolving. There is no generally accepted definition of SCM or a general understanding of how SCM affects the characteristics and organizational practices". Handfield and Bechtel (2004) noted that "what we are seeing in the field of SCM, which is that for years was defined in some way, has now become larger with different fragments of theory". Certainly the work developed after year 2000 to define SCM, has helped to close the gap of uncertainty and lack of agreement, but still remain an open question.

This article argues that a SC is a set of interacting organizations (among themselves) under a common goal and are involved in the flow of goods, services, resources and information. This is characterized by the following basic elements:

a. Organizations and / or individuals can be grouped under a common goal. The common goal does not necessarily mean that all the organizations share the same goal or objective.

b. The interactions between the organizations can take many forms, such as exchange of raw materials, exchange of goods, services, sale or purchase of various resources, information exchange, etc.:

c. The limit - or range- defining which organizations are part of the supply chain, i.e. the boundaries (scope) of the SC, is determined by the type of problem that will be addressed and the capabilities of the tools of analysis that are used. For the purposes of this article, it is used the concept of root manufacturer (provider) to describe a provider that has no other provider, i.e. for which there is no organization that provides to it products, services or resources.

In this paper it is addressed the problem of designing a supply chain involving the operation of logistics service providers (LSP) under a non-cooperative environment. In particular it is worked with triad structures for the supply chain which is composed by manufacturers, retailers and LSP layers. In each layer, members of the supply chain compete with other similar agents and work in a non-cooperative scenario. Firms belonging to a different layer of the supply chain work in a collaborative environment. There are no firms with a dominant position able to influence in the decisions of the other members of the supply chain. Each manufacturer wants to maximize its profitability and the same is valid for retailers and LSP as well. Manufacturers are located at the top layer of the supply chain and are concerned with the production of products and shipments to the retailers. The manufacturers compete among them for delivering products of equivalent quality to retailers through LSP, whom also compete to attract the manufacturers and deliver the products to retailers. In this supply chain, the LSPs are located in the middle layer. Each LSP is faced with handling and delivering the products sold by manufacturers to retailers, conducting transactions with both types of agents - whom purchase the LSP services- and retailers -whom purchase products from the manufacturers-. Retailers are located at the bottom layer of the supply chain. They demand a certain quantity of products (single commodity) and agree to purchase them from any manufacturer at a finite price. Also transactions and prices per transaction between manufacturers and LSP must be determined. Till now authors are not aware of any other paper working in this problem.

Supply chain design has been extensively studied so far. For a discussion about the design problem underlying it is recommended the book by Simchi-Levi, Kaminsky and Simchi-Levi (2003) besides the concept of supply chain assumed in this paper is a bit different from the one discussed in the book. For a detailed review of supply chain network design problems and modeling approaches it is recommended the work by Melo, Nickel and Saldanha-Da-Gama (2009). Melo et al. (2009) conducted a detailed literature review of facility location models in the context of supply chain management and particularly their applications to supply chain network design. All the literature reviewed in these works follows the traditional supply chain network design models where there is no competition among the agents and the models and solution methods are focused on facility location. For concepts and applications related to network design in a broader context it is recommend the book by Ahuja, Magnanti and Orlin (1999). For additional background on supply chain, see also the books by Bramel and Simchi-Levi (1997), Pardalos and Tsitsiringos (2002), and the volume edited by Simchi-Levi, Wu and Shen (2004).

This paper follows the work by Beckmann, McGuire and Winsten (1956) who first identified the applications of networks to conceptualize decision-making of an organization and particularly important in manufacturing processes and product flows linking also to the theory of firm. Related to this work is also the work by Nash (1950, 1951) on game theory. Nagurney Dong and Zhang (2002) addressed the problem of a three tier supply chain network design modeling where firms are located at the nodes of the network, each firm have their individual profit-maximization objective functions, and they seek to determine the optimal flows between tiers of nodes and also the prices of the product at the various tiers.
In the same line, Dong Zhang and Nagurney (2004) developed a supply chain network model where a finite-dimensional variational inequality was formulated for the behavior of various decision makers. Chiou (2008) considered a multi-tiered supply chain network which contains manufacturers, distributors and consumers and involving two-level of decision makers. He proposed a new solution scheme for the supply chain network design problem formulated as a (non-convex and non-differentiable) mathematical program with equilibrium constraints.

In some way, the problem addressed in this paper could be considered as a Strategic Planning problem involving a long-term planning horizon and the selection of some mix of manufacturer, LSP and retailers in order to satisfy customer’s demand. Eventually could also involve a facility location problem and capacity planning of manufacturing/services units to supply retailers, as well as the transportation capacity required among manufacturers and retailers. Simchi-Levi et al. (2003) stated that “the strategic level deals with decisions that have a long-lasting effect on the firm. These include decisions regarding the number, location and capacities of warehouses and manufacturing plants, or the flow of material through the logistics network”. This statement establishes a clear link between location models and strategic SCM. Problems of this type are mostly modeled as mixed integer linear programming problems, for example, see the paper by Chauhan Nagi and Proth (2004) and the annotated bibliography for a discussion. However, in the general case, the kind of problem we work on this paper cannot be modeled as an optimization problem, and then we use a variational inequality approach to formulate it. One of the central points in this type of problem is to demonstrate the existence of the solution and demonstrate its uniqueness. Normally, the complexity of this problem derives from considering that the cost functions for members of the supply chain depend on each other, then the Jacobian of the cost functions is no longer symmetric.

The paper by Meixell and Gargeya (2005) reviewed the literature on models to support the design of global supply chain and focused on the logistics aspects of the supply chain, i.e., the movement of goods from the point of origin to the point of consumption. Meixell and Gargeya (2005) studied a supply chain design problem comprising the decisions regarding the number and location of production facilities, the amount of capacity at each facility, the assignment of each market region to one or more locations, and manufacturer selection for sub-assemblies, components and materials. Global supply chain design extends this definition to include selection of facilities at international locations, and the special globalization factors this involves.

We do not make any distinction among global or domestic supply chain design. Cohen, Fisher and Jaikumar (1989) “present the main features that differentiate an international supply chain from a single-country model” (as cited in Vidal and Goetschalkx, 1997). In this paper, the selection of locations for production (manufacturers) and/or distribution facilities in global supply chains scale is modeled/considered in implicit way. The model focuses on solving the problem of how the material flows from manufacturers to retailers and the definition of how many products should be manufactured and delivered when a LSP take part.

This paper is organized as follows. In section 2 is discussed the formulation of the supply chain design problem involving the operation of logistics services providers, whom attend the demand of manufacturers for delivering products to their customers. In Section 3 is derived and analyzed the existence and uniqueness of the solution for the problem. Finally in Section 4 conclusions are presented.

2 PROBLEM FORMULATION

We address the problem of producing and delivering homogeneous products from manufacturers to retailers through logistics service providers contracted by the manufacturers. The model consists of $n$ profit-maximizing manufacturers, with a typical manufacturer denoted by $i$; $m$ profit-maximizing retailers, with retailers denoted by $j$ and with a typical demand denoted by $d_{j}$, and $q$ profit-maximizing logistics service providers (LSP), with a typical LSP denoted by $k$. The manufacturers are involved in the production of homogeneous products, which can be purchased by the retailers, who, in turn, make the product available to consumers at the demand markets. The links in the supply chain network denote transportation/transaction operations. In every layer, manufacturers, LSP and retailer compete in the sense of Nash (1950, 1951). Each firm acts in its own benefit and will determine his optimal choice given the optimal choices of the competitors, in such a way that the whole system gets an equilibrium state.
2.1 The Manufacturers Optimization Problem

Each manufacturer $i$ seeks to maximize its profit by setting the total quantity $q_i$ that he must produce and determine for each retailer $j$, the sales price $p_{ij}^s$ and the quantity of product $x_{ij}$ to deliver to retailer $j$. In this problem all the delivery services from the $n$ producers ($i=1,..,n$) to the $m$ retailers are contracted to logistics service providers (LSP) $k (k=1,..,q)$ at a price $p_{ijk}^l$. A manufacturer $i$ can contract one or more LSP $k$ to deliver the products purchased by the retailer $j$. Manufacturer $i$ have a service cost function $c_{ijk}(x_{ijk})$ to make available the products to the retailer $j$ through LSP $k$. Consider that $q_i = \sum_{j=1}^{n} x_{ij} \forall i = 1,..,n$. There are not production costs at this stage of the problem:

$$\text{Max} \; z_i = \sum_{j=1}^{m} \sum_{k=1}^{q} p_{ij}^s x_{ij} - \sum_{j=1}^{m} \sum_{k=1}^{q} p_{ijk}^l x_{ijk}$$

s.t.

$$x_{ij} \text{ and } x_{ijk} \geq 0 \; \forall \; i,j,k$$

(1)

Notice that the total production of manufacturer $i$ must satisfy that.

$$\sum_{j=1}^{m} x_{ijk} = \sum_{j=1}^{m} x_{ij} \; \forall \; i = 1,..,n$$

(3)

That is, all the products sold by manufacturer $i$ to all the retailers $j$, $j=1,..,m$ is equal to all the products delivered by manufacturer $i$ to all the retailers $j$ through LSP $k$, $k=1,..,q$.

In this problem, the service costs functions $c_{ijk}(x_{ijk})$ for each manufacturer $i$ are continuously differentiable and convex. Assuming that the manufacturers compete in a non-cooperative fashion in the sense of Nash (1950, 1951), which states, in this context, that each manufacturer will determine his optimal production quantity and shipments, given the optimal ones of the competitors, the optimality conditions for all manufacturers $i$ simultaneously are as follow: Determine $x_{ij}^*$ and $x_{ijk}^*$ $\in R$ satisfying:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{q} \left( p_{ijk}^l + \frac{\partial c_{ijk}(x_{ijk})}{\partial x_{ijk}} \right)$$

$$\times \left( x_{ijk}^* - x_{ijk} \right)$$

$$- \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij}^s \times \left( x_{ij}^* - x_{ij} \right) \geq 0,$$

$$x_{ij} \text{ and } x_{ijk} \geq 0 \; \forall \; i,j,k$$

(4)

Where $x_{ij}^*$ and $x_{ijk}^*$ are the optimal values for the corresponding variables.

2.2 The Logistics Service Providers Profit Maximizing Problem

The LSPs are also profit-maximizing agents. They seek to maximize the profit resulting from selling their services to manufacturers and the costs of servicing the retailers - the customers of the manufacturers. Remember that all the LSPs compete in the sense of Nash. For each LSP $k$, the problem is the following:

$$\text{Max} \; z_k^{SP} = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ijk}^l x_{ijk}$$

s.t.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ijk} = \sum_{j=1}^{m} x_{jk} \; \forall \; k = 1,..,q$$

(6)

LSPs incur in operating costs $c_{ijk}^l(\cdot)$ to take products from manufacturer $i$ and deliver to retailer $j$. The cost function $c_{ijk}^l(\cdot)$ is continuously differentiable and convex. The profit for LSP $k$ in (5) is given by the prices $p_{ijk}^l$ charged to manufacturer $i$ to deliver the products to retailer $j$ minus the corresponding costs $c_{ijk}^l(\cdot)$. It is reasonable consider that the LSP deliver all the products arrived from the manufacturers, what is modeled by (6).

As in the manufacturer case, the LSPs act in a non-cooperative way and there are not bargaining or any type of collaboration between them. The optimality conditions for all LSPs simultaneously are as follows, determine $x_{ijk}^* \in R$ such that:
\[ \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{q} \left( \frac{\partial c_{ijk}(x_{ijk})}{\partial x_{ijk}} - p_{ijk}^* \right) \times (x_{ijk} - x_{ijk}^*) \geq 0, \quad (7a) \]

Where \( x_{ijk}^* \) is the optimal flow of products coming from manufacturer \( i \) that LSP \( k \) will deliver to retailer \( j \).

### 2.3 The Retailers Problem

The retailers \( j \) purchase products to manufacturer \( i \) at sale price \( p_{ij}^* \), but they also consider in their decision the unit cost \( c_{ij}(x_{ij}) \) of making the transaction with this manufacturer \( i \). These costs are continuous and depend on the quantity of products purchased by other retailers, then could be interpreted as the procurement cost incurred by the retailer in a non-cooperative scenario. The retailers are willing to pay a demand price \( p_{ij}^* \) for the products. Then the equilibrium conditions for all retailers \( j = 1, \ldots, m \) are the following:

\[ p_{ij}^* + c_{ij}^d(x_{ij}) = p_{ij}^* \text{ if } x_{ij} \geq 0 \]
\[ \geq p_{ij}^* \text{ if } x_{ij} = 0 \]
\[ \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{q} (\partial c_{ijk}^d(x_{ijk})/\partial x_{ijk}) * (x_{ijk} - x_{ijk}^*) \geq 0 \]
\[ x_{ij} \text{ and } p_{ij}^* \geq 0 \forall i, j \]

### 2.4 Supply Chain Management Perspective

In a non-cooperative scenario with manufacturer, retailers and logistics service providers seeking to maximize their profit hold equilibrium conditions for the supply chain in which the total quantity of products that manufacturer produces and ships to retailers through logistics service providers must be equal to the amount of product purchased by a retailer, as well as the quantity of products the logistics service provider receives from the manufacturer to be delivered to retailers, must be equal to the shipment of the logistics service providers to the retailers. Finally the shipments and price pattern must satisfy the sum of inequalities (4), (7) and (10) described previously. Formally this is stated as follows:

#### 2.4.1 Definition 1

The equilibrium state of the given supply chain design problem in the presence of logistics service providers is one where the flows of products between manufacturers, logistics service providers and retailers coincide and the product shipments and prices satisfy the sum of the optimality conditions (4), (7), and the conditions (10).

#### 2.4.2 Theorem 1

A product shipment \( (x_{ij}^*, x_{ijk}^*) \) is an equilibrium pattern of the supply chain design model according to Definition 1 if and only if it satisfies the variational inequality problem:

\[ \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{q} \left( \frac{\partial c_{ijk}^d(x_{ijk})}{\partial x_{ijk}} - p_{ijk}^* \right) * (x_{ijk} - x_{ijk}^*) \geq 0 \]

\[ x_{ij}^* \geq 0 \forall i, j, k \]

\[ (11a) \]

**Proof**

Consider the definition 1. Sum up the inequalities (4), (7) and (10). After algebraic operations it gets the inequality (11). Now, consider the inequality (11). To the first term in bracket of inequality (11) adds \( p_{ij}^* - p_{ijk}^* \) then follows:

\[ \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{q} \left( \frac{\partial c_{ijk}^d(x_{ijk})}{\partial x_{ijk}} + \frac{\partial c_{ijk}^d(x_{ijk})}{\partial x_{ijk}} - p_{ijk}^* \right) * (x_{ijk} - x_{ijk}^*) \geq 0 \]

Then, add \( x_{ij}^* \) and \( p_{ij}^* \) then

\[ \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{q} \left( \frac{\partial c_{ijk}^d(x_{ijk})}{\partial x_{ijk}} + \frac{\partial c_{ijk}^d(x_{ijk})}{\partial x_{ijk}} - p_{ijk}^* \right) * (x_{ijk} - x_{ijk}^*) \geq 0 \]

To the second term in bracket of inequality (11) add \( p_{ij}^* - p_{ij}^* \) then

\[ \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{q} \left( \frac{\partial c_{ijk}^d(x_{ijk})}{\partial x_{ijk}} + \frac{\partial c_{ijk}^d(x_{ijk})}{\partial x_{ijk}} - p_{ijk}^* \right) * (x_{ijk} - x_{ijk}^*) \geq 0 \]
After the above additions (12) and (13), inequality (11) can be rewritten as follows:

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} \left( c_{ij}^p(x_{ij}) + p_{ij}^t - p_{ij}^r \right) * (x_{ij} - x_{ij}^*) \tag{13}
\]

The above additions (12) and (13), inequality (11) can be rewritten as follows:

\[
\begin{align*}
&\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\partial c_{ijk}^p(x_{ijk})}{\partial x_{ijk}} + p_{ij}^r * (x_{ijk} - x_{ijk}^*) \\
&- x_{ijk}^* - \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij}^r * (x_{ij} - x_{ij}^*) \\
&+ \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{q} \left( \frac{\partial c_{ijk}^p(x_{ijk})}{\partial x_{ijk}} - p_{ijk}^* \right) \\
&* (x_{ijk} - x_{ijk}^*) \\
&+ \sum_{i=1}^{n} \sum_{j=1}^{m} (p_{ij}^t + c_{ij}^q(x_{ij})) * (x_{ij} - x_{ij}^*)
\end{align*}
\geq 0 \tag{14}
\]

In the above inequality (14), the first two terms of (14) are the same of (4), the third term is equal to (7) and the last term is identical to (10). Hence inequality (11) is the sum of conditions (4), (7) and (10) according to Definition 1. So the proof is complete.

3 THEORY RESULTS

In this Section, it is presented some qualitative properties regarding inequalities (11). In particular it derived the existence and uniqueness of the solution to (11).

3.1 Theorem 2: Existence of the Solution

Assuming that the feasible set is nonempty, then variational inequality (11) admits a solution.

**Proof**

Since there is a finite demand for the products in the market, i.e., \( d \leq u \) for some \( \mu \in \mathbb{R}_+ \). Then each retailer \( j = 1, \ldots, m \) also demands a finite amount \( d_j \leq u_j \) of product from the manufacturer, for some \( \mu_j \in \mathbb{R}_+ \).

By the side of manufacturers, each of them, \( i = 1, \ldots, n \), has a finite capacity of production \( n_i \). Then for each pair \( i, j \), there is a finite capacity of shipments \( \mu_{ij} \in \mathbb{R}_+ \) for the retailers. That is \( x_{ij} \leq u_{ij} \).

Since \( x_{ij} \leq u_{ij} \), then there is \( \mu_{ij} \in \mathbb{R}_+ \) such that \( x_{ij} \leq u_{ij} \) for each \( i, j \) and \( k=1, \ldots, q \). This is, by the side of LSP, they deliver a finite amount of products to retailers already sold by manufacturers and demanded in a finite amount by the retailers.

Suppose we define \( x_{jk}^* \) as the quantity of products retailer \( j \) is receiving from LSP \( k \). Since \( x_{ij} \leq u_{ij} \) and \( x_{ijk} \leq u_{ijk} \) for each \( i, j \) and \( k \), then \( x_{jk}^* \leq u_{jk} \) for some \( j, k \).

So it can be said that the set

\[
X = \{ x_{ijk}, x_{ij}, x_{jk} \geq 0 \}
\]

is bounded, closed and convex, then \( X \) is a compact subset.

Now, let \( H \) be a real Hilbert space, whose inner product is denoted by \( \langle , \rangle \). Let \( X \) be a nonempty closed convex subset of \( H \) and \( A: X \rightarrow H \) a nonlinear map. Then (11) can be written as the problem of finding \( x^* \) in standard variational inequality format such as,

\[
\langle Ax^*, x - x^* \rangle \geq 0, \forall x \in X \tag{16}
\]

Where mapping \( A \) has a correspondence with the terms in (11).

Assuming the mapping \( A \) is continuous, from (15) and (16) there is a solution for (11).

3.2 Theorem 3: Uniqueness of the Solution

Assume the conditions in Theorem 2 and that the map \( A(X) \) is strictly monotone on \( X \), that is:

\[
\langle A(x) - A(y), x - y \rangle > 0, \forall x, y \in X \tag{17}
\]

Then the solution \( x^* \) to variational inequality (16) is unique.

**Proof**

Given that inequality (11) can be re-written as inequality (16), follows from standard theory of inequality the Theorem 3.

4 SOME NUMERICAL EXAMPLES

In this section we provide a numerical example of the model presented in previous section and also discuss the results and some interesting issues. The
example was figure out in order to illustrate the problem and be simple to solve it. We use some data from the literature (Braess, Nagurney and Wakolbinger, 2005) and adapt them to our problem. The example was solved analytically, and algorithms for solving general cases could be discussed in a future paper.

Consider a problem as represented by the graph depicted in Figure 1, with two manufacturers \((m=2)\), two logistics service providers \((q=2)\) and one retailer \((n=1)\).

![Figure 1: Example supply chain.](image)

The transaction cost functions \(c_{ijk}\) faced by manufacturers \(i\) and LSP \(k\) to meet demand of customer \(j\) are given by:

\[
\begin{align*}
c_{111}^s &= 0.5x_{111}^2 + 50x_{111}^1; \\
c_{112}^s &= 5x_{112}^2; \\
c_{211}^s &= 0.5x_{211}^2 + 100x_{211}^1; \\
c_{212}^s &= 10x_{212}^2.
\end{align*}
\]

The operating cost functions \(c_{ijk}^l\) of LSP \(k\), to deliver products from the manufacturer \(i\) to the customer \(j\) are the following:

\[
\begin{align*}
c_{111}^l &= 5x_{111}^2; \\
c_{112}^l &= 0.5x_{112}^2 + 50x_{112}^1; \\
c_{211}^l &= 10x_{211}^2; \\
c_{212}^l &= 0.5x_{212}^2 + 100x_{212}^1.
\end{align*}
\]

The transaction cost functions \(c_{ij}^d\) associated with the customer \(j\) in obtaining products from manufacturer \(i\) are given by:

\[
\begin{align*}
c_{11}^d &= 5x_{11}^1; \\
c_{21}^d &= 10x_{21}^1.
\end{align*}
\]

The demand is set to \(D=6\) units.

Analyzing the data and after some algebraic operations it is obtained the following values for the variables \(x_{ijk}\) and \(x_{ij}\):

\[
\begin{align*}
x_{111} &= x_{112} = 3; \\
x_{211} &= x_{212} = 0.
\end{align*}
\]

The demand price customer is willing to pay is 113 for the products sold by manufacturer 1. There are no products purchased to manufacturer 2. Using (8) we can obtain the price charged by manufacturer 1, that is equal to 83.

From this example and regarding the model, we can observe the following issues:

i.- The demand price retailer(s) are willing to pay to manufacturers make no difference whether the retailer is serviced by LSP 1 or LSP 2. But in practice, if LSP (anyone) has a value-added service offering to customers, and the customers perceive this difference among the LSPs then, they could be willing to pay more for the same product but making the difference by the service they receive. So this fact, proved in practice, it is considered in the model by the operating cost functions \(c_{ijk}^l\) of LSP \(k\), and also by the price \(p_{ijk}^l\) charged to manufacturer \(i\) by LSP \(k\) to service customer \(j\). Nevertheless, could be also interested to include in the model a service cost perceived by the customers depending of the LSP.

ii.- The demand price retailer(s) are willing to pay is exactly the sum of the “operating costs” of manufacturers and LSPs plus the cost of obtaining the products. This last cost in fact, could include the benefits perceived by retailers in making the purchasing, and a kind of procurement costs, including all the internal costs incurred by the retailer in making the purchasing to manufacturers.

5 CONCLUSIONS

Supply chain management sometimes confounded with logistics, is a multidisciplinary approach for effective and efficient management of the supply chain. In turn, supply chain is a set of interacting organizations (among themselves) under a common goal and they are involved in the flow of goods, services, resources and / or information. In this paper is used this approach to model a non-cooperative problem in a supply chain composed of manufacturers and retailers and where there are logistics service providers servicing the demand of retailers. The model considers that manufacturers, retailers as well as LSP act on their own advantage, seeking to maximize their profit individually. Also, the model considers that the agents located in different layers -manufacturer, retailers and LSP- collaborate to get the best available service level. In the optimal solution, the model determines the flow of products going from the manufacturers to retailers and passing across the LSP. Hence the model permits to handle the amount of products shipped from each manufacturer to each retailer and specifying the LSP servicing both the manufacturer...
and the retailer. The model also determines the price the retailer agrees to pay for the products sold by manufacturers. Some theory results are also analyzed in terms of existence and uniqueness of the solution to the problem. An example is discussed to illustrate the model.

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