A SIMULATION STUDY FOR OPTIMIZING THE PERFORMANCE OF SEMI-LAYER DELTA NETWORKS

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Keywords: Delta network, Banyan networks, Performance evaluation, Buffer, Multilayer multistage interconnection networks.

Abstract: In this paper, a semi-layer multistage delta network is presented and exemplified considering various values of buffer size by using simulation. The proposed network configurations are evaluated and compared with each other. A performance evaluation was conducted via our simulator assuming uniform conditions and arrivals of Bernoulli type. Performance statistics were collected for the two most important performance indicators of the network that is throughput and packet latency. From this study emerges the appropriate configuration of single and semi-layer delta networks in terms of buffer size. The evaluation methodology can be applied to several network configurations, providing the basis for a fair comparison, and the necessary data for network engineering to optimize the performance of semi-layer delta networks.

1 INTRODUCTION

Multistage Interconnection Networks (MINs) are used for interconnecting processors in parallel systems and to ensure efficient internetworking (Suet, 2004). The advantages that they have, include their ability to route multiple communication tasks concurrently, as well as their low cost/performance ratio. Banyan MINs are MINs which have the property of the existence of one, and only one path between each source and destination. On the other hand, the non-banyan interconnection networks are more expensive and more complex to manage.

This paper is a study of the performance optimization of semi-layer multistage delta networks. Delta networks are a subclass of banyan networks. The Delta networks properties are explained in the next section.

Performance evaluation methods for delta networks (or in general banyan networks) mainly include analytical methods, Petri nets modelling and simulation. Analytical methods are considered in general to be complex. Nevertheless, they have been extensively used by some researchers. Most of the MIN analysis focuses on uniform traffic (i.e. packages) coming to a network with an equal probability of reaching any output (Hsiao and Chen, 1991), (Bouras et al., 1987). On the other hand, there are numerous non-uniform traffic patterns in real applications that require special treatment. One such non-uniform approximation can be seen in (Tutsch and Hommel, 2002). Other typical analytical studies of a MIN’s performance are exemplified by various studies (Garofalakis and Stergiou, 2008), (Bouras et al., 1987), (Garofalakis and Spirakis, 1990).

Petri nets serving as MIN modelling methods have also been employed. The (German, 2000), (Haas, 2002) and (Linderman, 1998) are examples of such approaches. Petri nets methods are also considered complex. When there is an interest in more realistic results, simulations are used. Simulations allow flexibility in network parameters, making it possible to analyze the network with different communication patterns. Examples of such approaches are (Vasiliadis at al., 2006), (Vasiliadis et al., 2007), (Vasiliadis et al., 2008). All the above cited studies involve single layer multistage interconnection networks (SiLMINs).

Dietmar Tutsch and his group (Tutsch and Hommel, 2008) introduced multilayer multistage interconnection networks (MLMINs). Firstly, they demonstrated that the single layer MINs show a high saturation when the packets population is increased.
dramatically. The MLMINs were developed mainly to meet the need for efficient handling of multicast traffic (Tutsch, 2006). MLMINs are more suitable fabrics for modern traffic as well as on-line multimedia applications, which are increasing in importance.

The main weakness of the MLMIN architecture is attributed to the exponentially growing number of layers as the stages increase, which leads to higher costs. If we try to reduce the number of layers then hardware complexity is reduced and, therefore, so is the overall cost of the fabric.

Semi-Layer MINs (SeLMINs) are special cases of the multi-layer MIN. SeLMINs are defined (Garofalakis and Stergiou, 2009), (Garofalakis and Stergiou, Oct 2010) as a multilayer MIN which consists of two segments. The second segment must keep the levels growth fixed and equal to the Switch Element (SE) size. The second segment of the MIN is an unblocked segment. Figure 2 illustrates examples of two SeLMIN cases in 2D view, which have two and four layers, respectively.

When the layers of a SeLMIN are Delta type multistage networks, we have semi-layer multistage Delta type networks, which are the kind of networks being studied here.

These multistage fabrics are devices which can be constructed using a finite buffer size. However, the main question which arises is: what is the suitable buffer size in each case of traffic? This work tries to provide an answer to this question.

Hence, the main goal of this paper is to evaluate the performance of semi-layer delta type networks assuming the offered load is of unicast type, for different buffer size constructions. Ultimately, the objective is to determine the buffer size which optimizes throughput and packet latency.

Performance evaluation was conducted through simulation, considering uniform traffic conditions. Metrics were collected for the two major important network performance factors, which is throughput and packet latency.

The remainder of this paper is organized as follows: in section II, a brief analysis of a semi-layer delta network, which is the main research subject, is presented. Subsequently, in section III, the performance criteria and parameters that are related to the above network schemes, are presented. Section IV reveals the results of our simulation-based performance analysis, examining the effect that the buffer size has on overall network performance. Finally, section V provides concluding remarks.

2 DEFINITION OF MULTILAYER DELTA NETWORKS

A typical multistage \( N \times N \) MIN is constructed by \( L = \log_c N \) parallel stages of \( (c \times c) \) Switch Elements (SEs), where \( c \) is the degree of the SEs. Each stage contains \( (N/c) \) SEs. Hence, the total number of SEs of a MIN is equal to \((N/c) \log_c N\). Thus, there are \( O(N \cdot \log N) \) interconnections between all the stages, in contrast to the crossbar network that has \( O(N^2) \) links. Also, a MIN is distinguishable from the others if we know, except of its topology, the switching techniques and the routing algorithm used.

The fabrics examined here use the store and forward switching technique and shuffle perfectly as a routing algorithm. The routing is performed in a pipeline manner, which means the routing process occurs in every stage, in parallel.

The whole network operates “synchronously”, which means that the time cycles refer to global clock ticks. The network clock consists of two phases. In the first phase, the queues are serviced and then any new packets are received. Moreover, each MIN operates under the following assumptions:

- The service time of the output queues at each switch is assumed to be fixed and equal to the network cycle time.
- The traffic feeding the first stage of the MIN switch follows a Bernoulli type distribution, so the arrivals are considered independent from each other. If \( k \) is the random variable denoting the count of arrivals of packets at the end of a network cycle on a queue of a \( c \times c \) SE at the first stage of the MIN, the formula is (Garofalakis, 2008):

\[
x^{(1)}_{k,c} = \begin{cases} 
\frac{k^c}{c^k} \left(1 - \frac{p}{k}\right)^{k-c} & \text{for } 0 \leq c \leq k \\
0 & \text{otherwise}
\end{cases}
\]  

(1)

Where \( x^{(1)}_{k,c} \) depicts the probability of \( c \) packets accepted in an arbitrary first stage queue with in general \( k \) inputs at an arbitrary time cycle. However, usually the under study systems have \( k = 2 \) inputs, hence the \( c \) can be 0, 1 or 2 at the most. Also \( p \) depicts the probability of packets arrivals in
an arbitrary input of a random first stage queue of the switch system at an arbitrary time cycle. All the packets are considered to have identical fixed sizes.

- Any arrived packet at the first stage is lost if the relevant buffer of the SE is full.
- Each queue uses the FIFO policy for all output ports.
- Any packet will be blocked at a stage, if the destination buffer at the next stage is full.
- At the last stage, output links of the MIN signify that there is no blocking. All packet conflicts are randomly resolved and the routing logic at each switch is fair.

2.1 Delta Networks Property

Delta networks were proposed by Patel (Patel, 1981). Delta networks which belong to banyan property networks, are usually used to connect a significant number of processors in a multiprocessor system. In general, delta networks are constructed by \( c_1 \times c_2 \times \ldots \times c_n \) Switch Elements (SE) (Figure 1). Let’s consider \( o_j \) an output of a random SE, where \( j = 0,1,\ldots,c_i - 1 \). If an input of a SE in \( i \) stage is connected to an output of another SE in stage \((i-1)\), then all the other inputs must be connected to outputs \( o_j \) of the same index \( j \) of SE in the previous stage.

The above described mathematical translation is deemed a delta property. All the interconnection networks which have this characteristic are said to possess the delta property. All the SEs in any delta network contains digitally controlled crossbars. Digitally controlled SEs are controlled by a sequence of bits that hold all the packets which have to traverse through the MIN. In delta networks this sequence of bits represents the packet destination. Our study case considers symmetrical SEs with \( c_1 = c_2 = c \), given that it is very common in MINs systems.

2.2 Semi-layer MINs

Semi-layer MINs are a subclass of MLMINs which consist of two distinct segments (Figure 2). The front segment (first stages) of the MIN contains only one layer which employs a backpressure blocking mechanism. Replication at the first segment is not recommended. It is a key challenge to keep the overall cost of such fabric at low levels. The second segment encompasses the rest of the construction. The second segment is the multilayer segment of a MIN (a full fan-out), which is free of blocking. If we consider the SEs of second segment to be represented by \( c \times c^e \), then the SeLMINs of the second segment keep the level growing at a fixed rate and equal to \( c \). According to (Tutsch & Hommel, 1997), the SE’s outputs in the last stage are multiplexed. In this case, if either the multiplexer or the data sink do not have enough capacity to absorb the packets, then at this point blocking can occur. However, in this study it is assumed that multiplexers (data sinks) have adequate capacity. The main drawback of MLMINs is their high cost, owing to their complexity. Semi-MLMINs were introduced as a better trade-off between cost and performance of the multistage fabric, when the traffic demands are raised to very high levels.

In a SeLMIN (Figure 2), let \( L_{SL} \) represent the number of single layer stages and let \( L_{ML} \) be the number of stages that have full layer growth which can also service multicast traffic without blocking. Hence, \( L = L_{SL} + L_{ML} \).
Semi-Layer interconnection network (4 Layers)

Semi-Layer interconnection network (2 Layers)

Single Layer interconnection network

Figure 2: Literal views of 8 layer delta networks (SiLMIN, and SeLMINs).

For a given $L_{ML}$ the total number of Layers ($NoL$) in the second segment is $NoL = c^L_{ML}$, where $c$ is the number of inputs per SE (e.g., in the case of 2x4 switches, $c$ is equal to two).

Due to their appealing performance/cost ratio, the SeLMINs are expected to play an important role in the future regarding the overall performance of internet interconnections, parallel systems and grid systems.

### 2.3 Semi-layer MINs with Delta Type Property

Semi-layer delta networks are multilayer fabrics where all the layers are maintained in a delta multistage network, keeping the same permutation pattern. Throughout this study, a performance investigation has been employed, exploring typical semi-layer delta type networks.

Our case study considers SiLMIN with $L = 8$ stages, and SeLMINs with $L = 8$ stages and $NoL = 2$ and 4, respectively. In addition, we assume that the under study semi-layer MINs use typical $2 \times 2$ SEs in the first segment and $2 \times 4$ SEs in the second segment.

### 3 METRICS & METHODOLOGY FOR PERFORMANCE EVALUATION OF SEMI-LAYER DELTA NETWORKS

This study will present results of the performance of SiLMINs and SeLMINs when they service exclusive unicast traffic. The basic performance metrics used are:

**Average throughput of a single layer delta network ($Th_{SL}$):** Average throughput of a delta interconnection network is defined as the number of packets delivered to their destination per time cycle. Formally, $Th_{SL}$ can be defined as:

$$Th_{SL} = \lim_{\tau \to \infty} \frac{\sum_{i=1}^{\tau} \omega(i)}{\tau}$$

where $\omega(i)$ denotes the number of packets that reach their destination during the $i^{th}$ time interval.

Using simulations, the throughput is calculated as the number of packets that arrived at their destinations over a certain multitude of trials.

**Average throughput of semi-layer delta network ($Th_{out}$):** If we consider the throughput of the first segment of a SeLMIN as ($Th_{SL}$), then the total SeLMIN’s throughput (at the fan-out output) can easily be calculated as follows: In the case of unicast traffic the formula is: $Th_{out} = Th_{SL}$. In the case of unicast and multicast traffic the expression is: $Th_{out} = Th_{SL} \cdot (1 + w)c_{ML}$, where $w$ is the ratio of multicast traffic (see Garofalakis and Stergiou, 2009), (Garofalakis and Stergiou, Oct 2010)) for a definition of $w$).

**Normalized throughput of single and semi-layer delta network ($Th_N$):** Normalized throughput of the delta network ($Th_N$) is the ratio of the average throughput over the network size $N$. Formally, $Th_N$ can be defined as:

$$Th_N = Th_{SL} / N : \text{in the case of a single layer MIN and}$$

$$Th_N = Th_{out} / N : \text{in the case of SeLMINs}$$

**Average packet latency of a single layer delta network ($D_{SL}$):** The packet latency of a delta network is defined as the number of time units needed for all packets of a permutation to arrive at their destinations. Formally, $D_{SL}$ can be defined as:

$$D_{SL} = \lim_{\tau \to \infty} \frac{\sum_{i=1}^{\tau} \Delta t(i)}{n}$$

where $n$ depicts the total number of packets accepted by destinations in $\tau$ time intervals and
\( \Delta t(i) \) represents the total number of network cycles that an arbitrary \( i^{th} \) packet needs in order to arrive at its destination. \( \Delta t(i) \), includes the total number of network cycles for a packet waiting at any stage and the total number of network cycles the same \( i^{th} \) packet needs to remain in active transmission mode until it reaches its destination. The network latency is directly related to the maximum multitude of time cycles needed to route a certain number of packets to their destinations via permutations.

**Average packet latency of a SeLMIN delta network** (\( D_{out} \)): Assuming that the packet latency of the first segment is \( D_{SL} \), then the packet delay of a SeLMIN \( D_{out} \) can be expressed as:

\[
D_{out} = D_{SL} + L_{ML}.
\]

This occurs because in the second segment the packets don’t suffer from contentions, so the delay to traverse the second (\( L_{ML} \)) stage of the fan-out is exactly equal to (\( L_{ML} \)) time cycles.

**Normalized latency of a single and semi-layer delta network** (\( D_N \)): Normalized packet latency \( D_N \) of a delta network is the ratio of the average packet latency \( D_{SL} \) over the minimum packet delay which is considered as equal to \( L \) number of time cycles. Formally, \( D_N \) can be expressed by:

- In case of single layer MIN: \( D_N = D_{SL} / L \)
- In case of SeLMIN: \( D_N = D_{out} / L \)

A unique indicator for performance evaluation of multilayer networks

From the initial experiments it became apparent that the values of MIN's throughput and the values of packet latency are inversely proportional to each other for various values of buffer size.

Nevertheless, the optimal solution is to have high throughput rates and low values of packet latency. Hence, it is interesting to have a general evaluation using only one factor. The factor must reveal the better overall performance, that is, the first factor maximized and the second factor minimized simultaneously. So, this demanding overall performance factor is defined based on the correlation of the two individual performance factors. Because the individual factors have different measurement units and ranges, it is necessary to normalize them to obtain a common reference value domain. We call this factor the **Combined Performance Factor (CPF)** which is expressed by the following formula (Garofalakis and Stergiou, March 2010):

\[
CPF = \sqrt{Th_N^2 + \left( \frac{1}{D_N} \right)^2} \quad (4)
\]

In any multi-criteria decision-making problem, however, the importance of each criterion is a design problem. Therefore, when it is of interest to assign a weight (in terms of its importance in the network) to each separate metric, then the above formula can be replaced by:

\[
CPF (w_{Th}, w_{D}) = \sqrt{w_{Th}Th_N^2 + w_{D}\left( \frac{1}{D_N} \right)^2} \quad (5)
\]

where \( w_{Th} \), \( w_{D} \) are the corresponding weights of the normalized system’s parameters: normalized throughput and normalized packet latency. According to this equation, when the \( Th_N \) metrics become larger and/or the \( D_N \) metrics become smaller, the \( CPF \) becomes larger. The reference value domain of \( CPF \) ranges from 0 to 1.

The main condition which must be satisfied when the \( CPF \) factor is applied, is the assumption that \( D_N \neq 0 \). Besides this, all the measured factors must be calculated and manipulated as inter-individual metrics.

Hence, as the \( CPF \) becomes higher, the performance of the MIN is considered to have been improved.

Here we limit our study to two performance evaluation factors knowing that the proposed methodology is general, and that it is available to add additional factors chosen to evaluate the performance of a MIN.

Consequently, the following parameters affect the above performance aspects of multistage delta networks.

- **Network size** \( L \), where \( L = \log_2 N \), is the number of stages in a \( (N \times N) \) multistage delta network. In our study it is assumed that \( N = 256 \), thus \( L = \log_2 256 \, \approx \, 8 \).
- **Offered load** (\( p \)) is the steady-state fixed probability of packet arrivals at each queue on inputs. In our study, \( p \) is assumed to be \( p = 0.10, 0.20 \ldots 0.50, 0.60 \ldots 1 \).
- **Buffer size** (\( b \)) is the maximum number of
packets that an input buffer of a SE has the ability to hold. In our study, \( b \) is assumed to be \( b = 1, 2, 3 \) and 4. In addition to those values of buffer size, we chose constructions with higher values of buffer size that are considered to be extremely expensive fabrics yet not as good in performance. This happens because the cost of multilayer delta type fabrics is an exponential function of the buffer size.

4 SIMULATION AND PERFORMANCE RESULTS

4.1 Simulation

Here we estimate the performance of multilayer delta networks using simulations. We are interested in \((N \times N)\) multilayer delta networks that consist of \((2 \times 2)\) and \((2 \times 4)\) SEs, using internal queues. We developed a general simulator for SeLMINs that was capable of handling several switch types and load conditions which work at the packet level. The simulator was programmed in C++ and is capable of running various configuration schemas. In building the simulator, every \((2 \times 2)\) and \((2 \times 4)\) SE was modelled by two buffered queues. Each buffer operates according to FCFS principle. Each packet is forwarded by the store and forward mechanism and in each time slot, they are forwarded by at most one stage. Cases of packet contention, are solved randomly with equal probability.

We use as input parameters, the probability of packet arrivals, the buffer length, the number of inputs/outputs ports, the number of stages and the number of layers.

Metrics such as throughput and packet latency are gathered at the output of the system. The simulation needs at least \(10^4\) iterations (clock cycles) in order to ensure that the system operates in steady-state operating condition.

4.2 Results

Figure 3 shows the normalized throughput of an 8-stage SiLMIN and SeLMIN versus the probability of packet arrivals for MIN’s buffer size \( b = 1, 2, 3 \) and 4 when the offered load is exclusively of unicast type. The dot-dashed curves depict results for SiLMINs, while the solid curves illustrate results for SeLMIN with 4 layers for buffer size \( b = 1, 2, 3 \) and 4, respectively.

From Figure 3 it becomes apparent that the larger the values of the buffer size in MINs, the greater the value of the MIN’s throughput.

Also, we can notice that the throughput of SeLMINS – here 4 layer constructions - have higher values of throughput compared with the corresponding, in terms of number of stages and buffer size, single layer MINs. In addition, for offered load \( p \geq 0.7 \), throughput stabilization can be observed in the system due to the high value of blockings that takes place in the system.

Figure 4 represents the values of normalized packet latency of 8-stage SiLMINs and SeLMINs versus the probability of packets arrivals on the inputs for MINs with buffer size \( b = 1, 2, 3 \) and 4 when the offered load is exclusively of unicast type. The dot-dashed and solid curves depict results for SiLMINs and SeLMINs (NoL=4), respectively, when buffer ranges from 1 to 4.

From Figure 4 it can be seen that the Semi-layer MINs with 4 delta type layers, and with a single buffer size, achieve the best values (lower) of packet latency in comparison to the corresponding SeLMINS with higher values of buffer size. In addition, in the single layer MINs, the packets delay increases sharply, especially for high values of offered load, as compared to the corresponding SeLMINS in terms of buffer size. So, it is obvious that as the buffer size is increased, the packet delay also deteriorates (values become higher).

The SiLMINs maintain low values \((D_N \leq 1.5)\) of packet delay when the offered load is \(p \leq 0.5\). On the other hand, the same packet delay values are achieved when the offered load is \(p \leq 0.6\). This gain which the SeLMINs fabrics have over the SiLMINs, is owing to the exploitation of the
additional layers at the last stages, which on one hand provide routes to packets, and on the other eliminates the phenomenon of packet collisions, thus improving the packets’ speed, as they move to the outputs.

By observing Figures 3 and 4 it is obvious that the two performance indicators (throughput and packet delay) are contrary to each other. For a given MIN’s configuration, when the buffer size is increased, the throughput follows incrementally while the packets delay deteriorates. Hence, to evaluate the system by one general performance indicator we use the CPF factor which has been defined above.

### 4.3 Simulator Validation

To validate our simulator, a single-layer, single buffer and 6-stage MIN is modelled assuming the offered load on inputs is of unicast type. The results that are obtained by our simulations are compared with the corresponding results reported in other works of the literature. So, in the case of unicast traffic, Figure 5 depicts the normalized throughput versus the probability of packet arrivals on 64x64 MIN inputs for buffer sizes 1 and 2.

The results of this simulation which include Figure 5 curves: ‘BS=1 Our Simulation’ and ‘BS=2 Our Simulation’, are almost identical with the results reported in Mun’s model (Mun and Yoon, 1994) (curves: BS=1 and BS=2 from Mun’s model), while the Yoon’s model (Yoon et al., 1990) (BS=2 from Yoon’s model) deviates significantly. All the foregoing validates the results from our simulations.

### 4.4 Throughput and Latency CPF

Figure 6 shows the Combined Performance Factor (CPF) for 256x256 Semi-layer MINs with 4 layers versus the probability of packet arrivals when the total offered load is of unicast type. Figure 6 illustrated the CPF indicator for fabrics with buffer sizes equal to 1, 2, 3 and 4, respectively.

For buffer size \( b = 4 \), the value of CPF is low owing to the high packet delay values. It shows better behaviour but very near the fabrics with \( b = 2, 3 \). By looking at Figure 6 it is obvious that the best performance is achieved when the 4 layer SelMIN has a buffer size equal to 1. This happens because the delay of packets is significantly reduced.

Moreover, the performance of a MIN can be applied and tailored to the needs that a specific type of load demands.
Figure 7 illustrates the Combined Performance Factor (CPF) in 8-stage and 4 layer MINs for cases of applications traffic in which it is necessary to have extra low prices of packet delay. Therefore, the calculation of a general CPF indicator considers the packets delay factor with a weight of 2.

Figure 7 shows that the 4 layer Delta network with buffer size equal to 1 provides the best performance. On the other hand, the general performance indicator (CPF) deteriorates as buffer sizes increases.

![Figure 7: CPF of an 8 stage semi-layer delta MIN vs. probability of packets arrivals.](image)

Finally, the main finding of this study remains that the single buffered SeLMINs constructions present optimum performance behavior in terms of throughput and latency, compared to the corresponding SiLMINs with higher values of buffer size. This performance behavior of SeLMINs is strengthened when it comes to service applications that require small values of latency or jitter.

Also, the single buffered SeLMINs present as better performance as many number of layers they have for a given network size N. Also, they have an earlier point in starting the layer replication and thus eliminating the backpressure phenomenon.

Figure 6 and 7 reveal that the single buffered SeLMINs are more suitable devices for applications which demand low values of packets latency and jitter when considering jitter as a variation of packets latency. Hence, e.g. applications like streaming media of voice tracking devices present better attributes when they are constructed by single buffers. Contrary to this, cases which require high throughput rates and are indifferent to the information’s time transmission, are rather rare.

5 CONCLUSIONS

In this paper we studied Delta networks of SeLMINs, which is a possible performance improving strategy for Delta MINs. We present also, an evaluation and comparison methodology of MINs. This approach was applied on Delta type SeLMINs and Delta type SiLMINs.

It is obvious that the delta type SeLMINs seem to be more powerful but this is due to a higher complexity, relatively speaking, than delta type SiLMINs. However, in the literature there is a lack of studies relevant to multi or semi layer MINs.

It is noteworthy that the predictions of the simulations are validated in marginal cases by existing related works in the literature.

The findings of this study can be utilized by MINs designers to optimally configure their networks.

The methodology presented herein is to be used in future work in order to estimate the improvement in performance of Delta networks when servicing unicast and multicast traffic. Future work will also focus on studying other load patterns where there is hotspot and burst type of traffic. Additional work will also examine the MIN’s performance under different selection algorithms.

REFERENCES


Garofalakis J., Stergiou El., 2009. Performance evaluation for multistage interconnection networks servicing multicast traffic. The First International Conference on Advances in Future Internet. AFIN


