

# MODEL AND ALGORITHM OF COMPETITION BETWEEN HIGH-SPEED RAILWAY AND AIR TRANSPORT

## *Game Theory Based*

Jianan Zhang and Peng Zhao

*School of Traffic and Transportation, Beijing Jiaotong University, Shangyuancun, Beijing, China*

**Keywords:** High-speed railway, Game theory, Competitive strategy.

**Abstract:** This paper considered the access cost of passengers, and constructed a passenger's total travel cost function. We employed the concept of "linear city" to analyze the market share between high-speed rail (HSR) and airline (AIR), and analyzed the relation between the two competitors with the non-cooperative game theory. We took the fixed fare and variable fare rate as decision variables, and established an optimizing model to calculate the fares of each mode, and then put forward a heuristic algorithm to solve the model. Taking the Wuhan-Guangzhou transportation corridor as the background, the model and algorithm were used to calculate the optimizing fares. And analyzed change of solutions with different value of time, then discussed the competitive strategies of the HSR. The result demonstrated this model reflects facts reasonably and can be used to generate better competitive strategies.

## 1 INTRODUCTION

The construction and development of high-speed rail (HSR) brings people new choice of travel and the profound impact on passenger transport market structure. In the competition with airline (AIR), HSR attracted a considerable part of AIR flow with its own advantages. This is obviously in long-distance travel. Such as Wuhan-Guangzhou HSR, which is a recent opening line, makes flights between the Wuhan to Guangzhou reduced. However, the HSR development in China is still at the initial stage, air transport, as the main mode has been developed in China for many years, it has a relatively stable market share and mature operating system, the airlines carried out fully research and policy adjustments face to the HSR opening and operation. Compared with huge investment and operating costs in HSR, AIR has flexible transport organization, experienced management, higher pure running speed, and other advantages, if HSR cannot adjust the competitive strategies for more passengers, it will affect the future development. Therefore, the study of how should HSR dealing with air transport competition is true important for its development.

The experts and scholars from home and abroad has done many researches in the competition

strategies of transportation mode, especially the passengers flow sharing and price strategy in the conditions of different transportation modes. In early researches, McFadden referenced the utility theory of economics and studied the issue of market share of transport modes (Mcfadden, 1989). Williams proposed nested Logit Model to describe the problem of flow sharing in different transport modes(Williams, 1991). Recently, Yao used Nested Structure Model to do demand forecast of various of transport modes of inter-cities(Yao, 2005), and he got the conclusion that, the amount of inter-city travel come risen with the reduction of time and cost, and the improvement of service frequency (Si, 2005).Roman, Espino had analyzed the competition of HSR and air transport in Madrid-Barcelona transport corridor in Spanish(Roman and Espino, 2007). They estimated the parameters from the survey data, and found that HSR had been more competitive in long-distance transport. Givoni explored the issue that some air companies regard the HSR network as the extension of their route and analyzed the conditions and pattern of cooperation between air transport companies and HSR (Givoni, 2007). Obviously, the analysis of the competitive strategy should be connected with the forecasts of the market sharing, and so far, the Logit Model

based on Utility Theory is the main way getting passenger volume of modes.

Actually, the main competition between different modes of transport is a game, while the use of game theory is rare. This paper according to the cost that passengers start from home to station or airport (access-cost) and the cost from station to destination (egress-cost) to constructed a cost function. Analyzed the passenger volume of HSR and AIR with the theory of “linear city” (Hsu C. W., 2009), at the same time, analysis the competition of HSR and AIR with non-cooperative game theory, on this basis, established a optimization model and gave the heuristic algorithm. Finally, we use an example to analysis the competitive strategies of HSR.

## 2 BACKGROUND

The competitive strategy about passenger service includes service planning, service level, and so on. Obviously, ticket price is the main method to adjust competitive strategy. It is also the main factor influents passenger’s choice among different transport modes. So, the main point of this paper is study how HSR pricing.

What the operators care most is passenger’s choice for mode. When passengers choose mode for their travel, cost is the key factor. In this article, the costs we talk about refer to currency expenses and time expenses. Most of the early researches only draw main attention to currency expenses which passengers pay for the travel from origination station to destination station, neglecting the access and egress cost. For passengers themselves, however, they will take access-cost and egress-cost into consideration. Usually the access-cost and egress-cost for HSR and AIR are different from each other. As a result, operators should establish pricing strategy according to passenger’s access (egress)-cost. Recently, researchers gradually realized the importance of access (egress)-cost. However, specific calculation methods of access (egress) - cost have not been mentioned yet. We brought access (egress)-cost into the function of cost, and then found that a passenger’s expenses consist of fare, access (egress)-cost and time expenses. Since there’s lots of access (egress) modes and they show little influence on the whole travel cost, the time a passenger spend on access (egress) becomes the most effective factor for cost. In order to help calculate, we only take the access (egress) time into account, and the following is function of whole travel cost:

$$C_m^k = \left\{ (gp_m + bp_m d_m^k) + t_m^k v_k \right\} + (t_{mk}^a v_k + t_{mk}^e v_k) \quad (1)$$

We summarize our notation below:

Table 1: Symbols of travel cost function.

Symbol	Meaning
$C_m^k$	The cost of passenger k choose the transport mode $m$
$K$	Passengers set
$M$	Transport modes set, $M = \{HSR, AIR\}$
$gp_m$	The fixed fare of transport mode $m$
$bp_m$	The variable fare rate of transport mode $m$
$d_m^k$	The travel distance that passenger k choose transport mode $m$
$t_m^k$	The travel time that passenger k choose transport mode $m$
$t_{mk}^a$	The access time that passenger k choose transport mode $m$
$t_{mk}^e$	The egress time that passenger k choose transport mode $m$
$v_k$	The value of time of passenger k

This function divides the travel cost into two parts. What is in the braces is the generally cost, and this part entirely depends on the mode what passengers choose. And the other part is the connection cost. It depends on access-cost and egress-cost.

## 3 THE MODEL

### 3.1 Linear City

Under the conditions of the determined fare and access (egress) modes, travel cost can be calculated depend on the passenger travel cost function. It is possible to calculate the whole amount of passengers take every transport mode when passengers choose to minimize their total travel cost. This paper will use the concept of linear city to predict passenger flow of HSR and AIR. At first we can assume that city is ribbon shape, with the ends of which are HSR station and airport (Hotelling, 1929). To simplify the problem, we assume that the sum of access cost to the railway station and the airport are the length of the linear city, and it can be marked by  $D$ . Then, set the access charges of passenger  $k$  from home to high speed rail station  $x_k$ , and set the access charges of passenger from home to airport  $D - x_k$  (Figure 1).

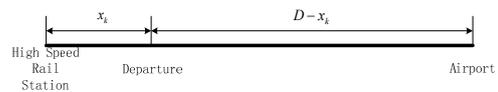


Figure 1: Schematic diagram of linear city.

Similarly, the assumption can be also used to the egress cost of passenger  $k$ , then, each line represents a linear city. Put the two lines on the same plane, we can get a two-dimensional graph of passenger  $k$  from the departure city to the destination city (Figure 2).

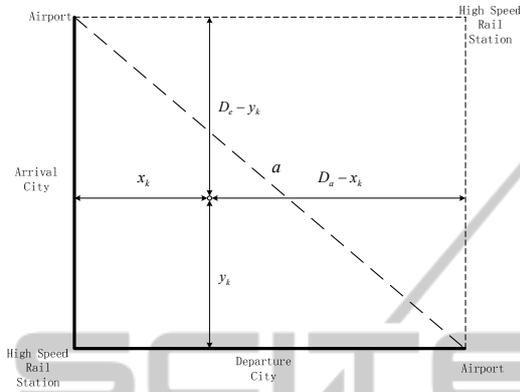


Figure 2: Schematic diagram of two-dimensional connection charges.

Each point on Figure 2 represents a combination of connection costs.  $x_k$  is the abscissa of the point and it is the access cost of passenger  $k$  from home to the HSR station, and  $D - x_k$  represents cost of passenger to the airport. We can also achieve the egress charges from the ordinate of Figure 2. The two-dimensional graph can express visual difference of connection costs among passengers. We can see, when  $D_e$  is equal to  $D_a$ , the total connection costs of passengers covered by diagonal  $a$  choosing HSR is equal to AIR. When the generally cost of HSR and AIR during operation are equal, passengers below the diagonal will preference for HSR, others will preferred to fly. At that time, diagonal of the rectangle will be the boundary of passengers to choose HSR or airline. Assuming running time of HSR and AIR is for certain, then  $t_m^k v_k$  in the formula (1) is constant. When the price of mode  $m$  change, the diagonal  $a$  will move up or down, and determines passenger flow volume of the two transport modes. For example, HSR reduce fares, and  $C_{HSR}^k$  or  $C_{AIR}^k$  represent the generally cost of passenger  $k$  takes HSR or AIR, then the minus of  $C_{HSR}^k$  and  $C_{AIR}^k$  can be expressed as:

$$\begin{aligned} \Delta C &= |C_{HSR}^k - C_{AIR}^k| \\ &= \left\{ (gp_{HSR} + bp_{HSR}d_{HSR}^k) + t_{HSR}^k v_k \right\} + (t_{HSRk}^a v_k + t_{HSRk}^e v_k) \\ &\quad - \left\{ (gp_{AIR} + bp_{AIR}d_{AIR}^k) + t_{AIR}^k v_k \right\} + (t_{AIRk}^a v_k + t_{AIRk}^e v_k) \end{aligned} \quad (2)$$

For easy calculation, assuming the population of linear cities is uniformly distribution, when the cost of passengers take two modes are different, the line  $a$  will shift up or down. Like Figure 3 shown, the reason is cost of passengers who covered by straight line  $a$  should be the same whatever mode they take. So, passenger flow volume of mode  $m$  can be calculated by the following formula:

$$q_m = \frac{Q}{D_a D_e} \left\{ \frac{1}{2} \left( D_e - \frac{D_e}{D_a} \Delta C \right) \left( D_a - \frac{D_a}{D_e} \Delta C \right) \right\} \quad (3)$$

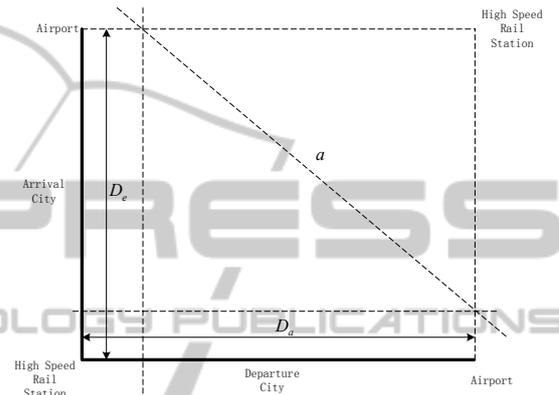


Figure 3: Schematic distribution of passenger flow.

### 3.2 Model

Price is the decision variable of the split line in the two-dimensional graph, and it also determines the passenger volume of the HSR and AIR. Goal of this problem is to maximize profits of each enterprise, so the problem is a constrained extremely problem. Because the ticket price of HSR and AIR consist of two parts, income of each mode is the product of price and the number of passengers who choose each mode, and profit for the enterprise is the difference between revenue and cost. Guide price of fixed fare and variable fare in the objective function constrained by government. So the model can be expressed as:

$$\begin{aligned} \max z_m &= gp_m q_m + \sum_{k=1}^{q_m} bp_m d_m^k - \sum_{k=1}^{q_m} c_m d_m^k q_m \\ s.t. \quad &\begin{cases} GP_m^l \leq gp_m \leq GP_m^h \\ BP_m^l \leq bp_m \leq BP_m^h \\ \left\{ (gp_m + bp_m d_m^k) + t_m^k v_k \right\} + (t_{mk}^a v_k + t_{mk}^e v_k) \\ \leq \left\{ (gp_{-m} + bp_{-m} d_{-m}^k) + t_{-m}^k v_k \right\} + (t_{-mk}^a v_k + t_{-mk}^e v_k) \end{cases} \end{aligned} \quad (4)$$

Table 2: Symbols of the model.

Symbol	Meaning
$z_m$	Profits of mode $m$
$q_m$	The number of passengers that select mode $m$
$GP_m^l$	Lower limit of the fixed fare of mode $m$
$GP_m^h$	Upper limit of the fixed fare of mode $m$
$BP_m^l$	Lower limit of variable fare rate of mode $m$
$BP_m^h$	Upper limit of variable fare rate of mode $m$

The decision variables in the model are fixed fare and variable fare rate of mode  $m$ , which is HSR or AIR. Fixed fare and variable fare rate are strategies of HSR and AIR. Actual passenger flow volume of the two sides in the game is directly related to the two kinds of fares. By solving the above model, HSR or AIR can calculate the best pricing strategies when they face to competition from the other.

#### 4 HEURISTIC ALGORITHM

According to the above model, the competition between HSR and AIR in middle distance passenger transport market aspect is non-cooperative game; non-cooperative game model's solution is Nash equilibrium. Nash equilibrium is a strategy combination, and each participant's strategy is the most superior one in the situation of other participant strategy has been determined. Specific to the problem in this paper, we can use the equation below to express Nash equilibrium (Hsu C. W., 2009):

$$\begin{aligned} & \Pi_m \{ (gp_m^*, bp_m^*), (gp_{-m}^*, bp_{-m}^*) \} \\ & \geq \Pi_m \{ (gp_m, bp_m), (gp_{-m}^*, bp_{-m}^*) \} \end{aligned} \quad (5)$$

That is to say, once the model achieved the Nash equilibrium under the condition of the other parameter has been determined, HSR and AIR cannot get more profit however they adjust their strategies. This model includes two decision variables, namely fixed fare and variable fare rate, because what this article studies is the game of HSR and AIR, it needs to determine the strategy combination which contains four variables, the best strategy can be expressed by:

$$\{ (gp_{HSR}^*, bp_{HSR}^*), (gp_{AIR}^*, bp_{AIR}^*) \} \quad (6)$$

$q_m$  is the volume of transport mode  $m$ , whose computational method has already been given in the previous section. Because the model's objective

function cannot differential everywhere, we cannot solve the model directly. In order to obtain the model's Nash equilibrium, this article gives the heuristic algorithm below.

First create an initial solution according to relatively simple rule, and this solution will be the initial ticket price of HSR and AIR. Afterward, select one solution from the two modes as the known condition, then use the (2) and (3) to calculate  $q_m$ , subsequently uses (4) to calculate ticket price of mode  $-m$  as mode  $-m$ 's second solution and this is part of the first iteration. Similarly, we can get the second solution of mode  $m$ . After times of iterations, when the solution meets the termination condition, we obtain the solution of the whole model.

This article uses the average guide ticket price of HSR and AIR as the problem's initial solution, namely:

$$\begin{aligned} & \{ (gp_m^0, bp_m^0), (gp_{-m}^0, bp_{-m}^0) \} = \\ & \left\{ \left( \frac{GP_m^l + GP_m^h}{2}, \frac{BP_m^l + BP_m^h}{2} \right), \left( \frac{GP_{-m}^l + GP_{-m}^h}{2}, \frac{BP_{-m}^l + BP_{-m}^h}{2} \right) \right\} \end{aligned} \quad (7)$$

And set the termination condition the two iterative difference of solution is smaller than a small value  $\epsilon$ .

#### 5 CASE STUDY

##### 5.1 Data

In order to confirm the model and the algorithm's validity, we take the Wuhan to Guangzhou corridor as an example, to determine the competition strategy of HSR. The result indicates how the operation cost and value of time will influence the strategy of HSR. Because only the passengers between Wuhan to Guangzhou can be fight for by HSR and AIR, this case only consider the direct passengers between Wuhan to Guangzhou. At present, Wuhan and Guangzhou's passenger already to be possible to choose the HSR, they may also choose AIR, and the relation of two transport modes meet the above game model's basic condition.

This case study involves the essential data of passenger demand between Wuhan and Guangzhou, such as the value of time, distance of each mode between Wuhan and Guangzhou, operation cost of HSR and AIR, and so on.

- Passenger Transportation Demand. At present, there are 29 pairs of EMUs between Wuhan and Guangzhou every day, each EMU can take 1200 passengers, then HSR can deliver 34800 passengers

one day in each direction; In the AIR aspect, there are 10-11 flights every day between Wuhan and Guangzhou, the plane seat capacity varies differently based on the type, this article takes the average number of one week as the capacity, and the number is 2700 passengers per day. Multiplied the above two way's delivery capacity by the average booking rate coefficient 0.7 as Wuhan and Guangzhou's passenger flow demand reference value.

- Value of Time. Residents' average income is 25000 Yuan in Wuhan and the number is 40000Yuan in Guangzhou, then the averaging value 32500 Yuan. Considering that income of passengers who take AIR or HSR is higher than the average number, therefore we multiplied by 1.5 as the passengers' income in one year, and it is 48750 Yuan per year. Supposing that everyone works 365-104-14=247 days, every day we work for 8 hours, then one year everyone works 247\*8=1976 hours. Each hour's income is 24.67 Yuan approximately is equal to 25 Yuan.
- The HSR running time is 3 hours, and AIR running time is 1.5 hours. Guangzhou and Wuhan's linear urban length is 2 hours.
- The travel distance of HSR is 1069km; with the distance of AIR is 1000km.
- The fixed fare scope of HSR is 50 Yuan to 100 Yuan, and the Variable fare rate scope is 0.3 Yuan per passenger-kilometre to 0.5 Yuan per passenger-kilometre; the fixed fare scope of AIR is 70 Yuan to 100 Yuan, and variable fare rate scope is 0.4 Yuan per passenger-kilometre to 0.8 Yuan per passenger-kilometre.
- The HSR operation cost is 0.3 Yuan per passenger-kilometre, while the cost of AIR is 0.4 Yuan per passenger-kilometre (Chang, 2004).

## 5.2 Result Analysis

After the running of computer programme, we got the results of the problem shown by Table 3:

Table 3: Result of model.

Item	HSR	AIR
Fixed fare(Yuan)	54	70
Variable fare rate(Yuan per kilometre)	0.41	0.5
Passenger volume(person)	19258	6992
Profits(Yuan)	3304480	1188640

It is easy to calculate the total ticket price of HSR is 492 Yuan, and the total ticket price of AIR is

570 Yuan. At present, the price of HSR and AIR between Wuhan to Guangzhou were 490 Yuan and 740 Yuan. Obviously, according to parameter in the article, the fare of AIR is slightly high; this is also one of the reasons that after the operation of HSR, passengers who take AIR reduce rapidly. For better show of how the value of time influences the passengers' choice, we simulated when the value of time changes between 20 Yuan per hour to 100 Yuan per hour. Result as is shown in Table 4.

Table 4: Simulation results.

Value of time	Passenger volume(person)		Profits(Yuan)	
	HSR	AIR	HSR	AIR
20	22053	4197	3699381	730320
40	15286	10964	2570327	1848410
60	12599	13651	2113980	2351950
80	11193	15057	1854550	2584850
100	10193	16057	1725674	2701130

Table 4 shown when the ticket price is stable and value of time increase, the passengers who choose HSR will reduce, and the profits of HSR and AIR will change. That is to say, although the travel time of each mode is short and almost equal, when passengers' value of time higher than the threshold, travel time will be the key factor that determine which mode to choose. Therefore, HSR operators should analyze the influence of fare to the benefit, and then create more scientific competition strategies.

## 6 CONCLUSIONS

This paper constructed a game model of competitive strategy optimization and gave the heuristic algorithm. Finally, through a case study, compared the changes of fares and revenue between the HSR and AIR, and then analyzed what HSR operators should adopt competitive strategies. In this passage, we considered the access and egress cost, and this is helpful to calculate the cost of travel more accuracy. The travel cost function can reflect the total cost of passenger travel; it is the basis to make scientific, rational and competitive strategy. In addition, we cited "linear city" theory to abstract processing the urban passenger departure and arrival, and predicting the passenger volume of various modes, experiments show that this method is easy and with rationality. Parameters in the model including access and egress time, travel distance and value of time, and so on, these parameters can be estimated according to the actual situation, and then simulate

different scenarios and analysis by changing the parameters' values. From the results of Wuhan to Guangzhou case study we can see, game model and algorithm is effectively and we can calculate the results within a reasonable time, and the solution is realistic. The results of this study can also provide a reasonable reference for HSR operators.

## ACKNOWLEDGEMENTS

We want to acknowledge the financial support from "the Fundamental Research Funds for the Central Universities".

## REFERENCES

- Chang I., Chang G. L., 2004. A Network-based Model for Estimating the Market Share of a New High-speed Rail System. *Transportation Planning and Technology*.
- Givoni, 2007. Role of the Railways in the Future of Air Transport. *Transportation Planning and Technology*.
- Hotelling, 1929. Stability in Competition. *Economic Journal*.
- McFadden, 1989. A Method of Simulated Moments for Estimation of Discrete Response Models without Numerical Integration. *Econometrical*.
- Roman, Espino, 2007. Competition of High-speed Train with Air Transport: the Case of Madrid-Barcelona. *Journal of Air Transport Management*.
- Si B. F., Gao Z. Y., 2005. Optimization model for railway passenger transportation under the condition of integrated transportation system. *Journal of the China Railway Society*.
- Williams H. C. W. L., Lam W. M., 1991. Transport Policy Appraisal with Equilibrium Models Investment Benefits in Multi-modal Systems. *Transportation Research*.
- Yao, Morikawa, 2005. A Study of on Integrated Intercity Travel Demand Model. *Transportation Research*.