THE RESEARCH ON STABILITY OF SUPPLY CHAIN UNDER HIGH ORDER DELAY

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Keywords: Supply chain, Stability, High order delay, Simulation, System dynamics.

Abstract: Based on system dynamics (SD), a high order delay inventory control model of supply chain is proposed. The high order delay in SD is regarded as transition between first order lag and pure time delay (PTD), it describes the more general delay mode in real systems. With simulation, the stable boundaries of inventory control model under high order delay are confirmed, and the effect of variations in delay mode on stability of inventory control system is analyzed. It is concluded that the reduction of delay order and delay time can improve the stability of inventory control model under high order delay; the sensibility of decision parameters to the change of system order is nonlinear.

1 INTRODUCTION

Bullwhip effect have triggered a series of problems including the increase of the operational risk of enterprise and the decline in supply chain management efficiency (Lee et al., 1997).In recent years, the dynamic characteristic of supply chain represented by bullwhip effect had received considerable attention. The research on stability of supply chain is helpful to reduce the bullwhip effect and improve the operation benefit of supply chain.

In the research of stability of supply chain, the delay structure is a key influencing factor of system stability. The researches based on the hypothesis of first order lag focused on revealing the influence of management practices to the fluctuant phenomenon in supply chain and proposed several management strategies to eliminate the bullwhip effect (Balakrishnan et al., 2004; Disney et al., 2004; ChengHu et al., 2005; Zhang et al., 2006). Riddalls et al. (2000) first pointed out that there was difference between first order models and actual systems. Henceforth, the studies based on pure time delay (PTD) emphasized that supply chain structure determined the convergence of inventory and order (Riddalls and Bennett, 2002; Warburton, 2004; Huixin and Hongwei, 2004). Under the influence of many factors, the delay mode of the actual inventory control system may be the high order delay which is the mixed form of first order lag and pure time delay.

As system dynamics (SD) has provided the scientific basis for describing high order delay structure, it avoids the obstacle of analytic modeling in this area. In this paper, we adopt high order delay as the transition between first-order delay and pure time delay and built the supply chain SD model. Through simulation, the dynamic characteristics and stability of the system are analyzed.

2 HIGH ORDER DELAY INVENTORY CONTROL SYSTEM MODEL

2.1 The Implication of High Order Delay

In a stable production or transportation system, there exists an average delay time and the distribution of output with delay is stable. In system dynamics, the distribution is described by high order delay.

First order delay and pure time delay are the particular cases of high order delay. System dynamics suggests that the structure of first order delay is equivalent to the structure of negative feedback with linear first-order, and the structure of high order delay is equivalent to several first-order delaying links connected in series. When the order of high order delay approaches to infinity, high order

Su-ling J., Lin W., Chang L. and Qiang W..
THE RESEARCH ON STABILITY OF SUPPLY CHAIN UNDER HIGH ORDER DELAY.
DOI: 10.5220/0003583103610367
In *Proceedings of the 13th International Conference on Enterprise Information Systems* (SSSCM-2011), pages 361-367
ISBN: 978-989-8425-54-6

delay is equivalent to pure time delay (Riddalls and Bennett , 2002).

The structure of linear first-order system with negative feedback is shown as follows:

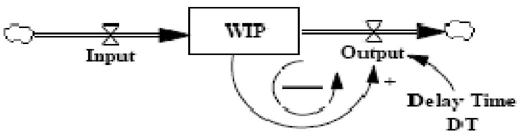

Figure 1: Structure of linear first-order system with negative feedback.

Output is described as:

$$\text{Output}(t) = \text{WIP}/\text{DT} \quad (1)$$

Connecting n first-order delaying links in series, we can obtain the n orders delay:

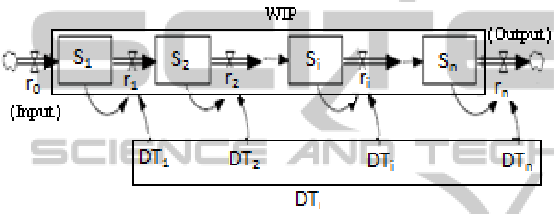

Figure 2: Structure of high order delay.

The notation for the model presented in figure 2 is as follows:

r_i: input (output) flow at the middle levels. r_0 and r_n are the input and output of the model. i = 0,1,…, n

S_i: the delayed flow at the middle levels, i = 1,2,…, n

WIP: the total flow

DT_i: the delay time at the middle levels, i = 1,2,…, n

DT: the total delay time

The relationship between the total flow and the delayed flow at the middle levels is:

$$\text{WIP} = \sum_{i=1}^{n} S_i \quad (2)$$

The total delay time and the delay time at the middle levels can be simplified (Sterman J. D, 2000):

$$\begin{cases} DT = \sum_{i=1}^{n} DT_i \\ DT_1 = DT_2 = \cdots = DT_n = DT/n \end{cases} \quad (3)$$

Refer to the structure of first order delay, the relationship between the input and output at the middle levels can be represented by differential equations:

$$\begin{cases} r_1' = \frac{n}{DT}(r_1 - r_0) \\ r_2' = \frac{n}{DT}(r_2 - r_1) \\ \quad \dots \\ r_n' = \frac{n}{DT}(r_n - r_{n-1}) \end{cases} \quad (4)$$

That is:

$$r_n - DTr_n' + \frac{DT^2}{n} r_n'' + \cdots + (-1)^n \left(\frac{DT}{n}\right)^n r_n^{(n)} = r_0 \quad (5)$$

Eq. (5) indicates that it is very difficult to calculate the output flow r_n and the delayed flow at the middle levels may be not differentiable in real system. Through the analysis of the structure of high order delay shown in figure 2, we can find that the greater the order n, the smaller the delay time at the middle levels DT_i will be. When the order n approaches infinity, DT/n is equal to DT_i and both are close to zero. That is:

$$r_0 \approx r_1 \approx r_2 \approx \dots \approx r_i \approx \dots \approx r_n \quad (6)$$

$$r_n \approx r_0(t - DT) \quad (7)$$

Therefore, pure time delay can be approximated by limited high order delay, and the smaller the DT_i, the higher the degree of approximation is. It also indicates that high order delay is the smooth transition between first order delay and pure time delay.

System dynamics simulation software, Vensim PLE, has provided the function of high order delay:

DELAY_n(Input, Delay Time, n)

Where n is the order of high order delay.

2.2 Model Structure

Figure 3 shows the high order delay inventory control model of supply chain that is built on the basis of the generic stock-management model proposed by Sterman (1989).

Considering that the validity of demand forecasting will influence the system stability, we

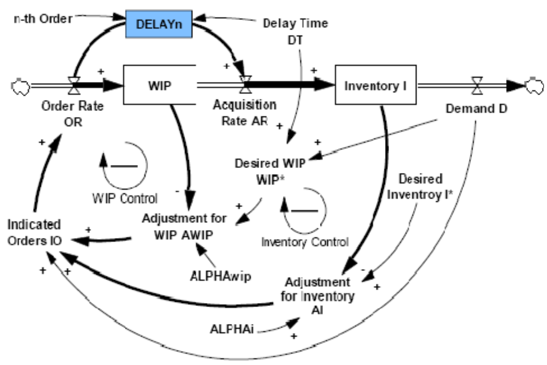

Figure 3: High order delay inventory control model.

do not make prediction on market demand. The adjustments include two aspects:

$DELAY_n$: the limited high order delay mode of WIP;

n-th Order: delay parameter, the order of high order delay.

In the model, the decision maker first determines the desired WIP based on demand and delay time, and then determines the actual adjustment for WIP. Finally, the order to upstream will be made according to demand and adjustment for inventory and WIP.

The expression of the relationship between order rate and acquisition rate is as follows:

$AR = DELAY_n(OR, DT, \text{n-th Order})$

Table 1 shows the variable settings (Sterman, 1989).

Where α_i is the rate at which the discrepancy between actual and desired inventory levels is eliminated, and α_{WIP} is the rate at which the discrepancy between actual and desired WIP levels is eliminated, $0\leq\alpha_i$, $\alpha_{WIP}\leq1$. The values of α_i and α_{WIP} represent the sensitivity of decision-maker to the gaps, that is, (I*-I) and (WIP*-WIP).

Table 1: Variable settings.

Variable	Expression
WIP	$\int_{t_0}^{t}[OR(t)-AR(t)]dt+WIP_{t_0}$
I	$\int_{t_0}^{t}[AR(t)-D(t)]dt+I_{t_0}$
AR	DELAY(OR, DT)
AI	$\alpha_i(I^* - I)$
AWIP	$\alpha_{WIP}(WIP^* - WIP)$
IO	D + AI + AWIP
OR	Max(0, IO)
ALPHAi	$0\leq\alpha_i\leq1$
ALPHAwip	$0\leq\alpha_{WIP}\leq1$

3 STABILITY ANALYSIS AND CRITERIA OF SUPPLY CHAIN

3.1 The Definition of Stability

There are different definitions of stability of supply chain. The traditional ideas of system dynamics state that only the behavior of smooth convergence is stable while the other fluctuate behaviors are unstable (Forrester, 1958). The main reason is that system dynamics methods focus on systems under first order lag. Scholars based on control theory stress the importance of pure time delay, it is commonly accepted that fluctuant convergence is a gradual process of system to be stable and oscillation with equi-amplitude is a critical state of stable system. Based on the definition of stability in control theory and the methods applied by system dynamics, we propose the following definition of stability of supply chain system:

Definition 2.1: Suppose the system is stable at the initial time, when imposing a small step disturbance on demand, if the inventory (or order rate) can get stable at a certain equilibrium level after a period of time, then the system is stable.

3.2 Stability Criterion

Combining with the above definition of stability, this paper takes inventory as research subject to obtain the stability criterion. As the underlying cause of the fluctuation of inventory is the deviation between actual inventory and desired inventory, we use the area between the two curves to describe the fluctuation in supply chain. This practice is similar to the method in cybernetics that use "noise bandwidth" to make quantitative description of bullwhip effect (Dejonckheere , 2003; Wang et al., 2006).

Figure 4 shows the general behavior pattern of inventory fluctuation.

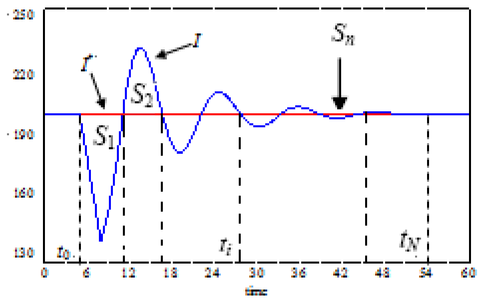

Figure 4: General behavior pattern of inventory fluctuation.

As shown in figure 4, assuming that the inventory curve begins to fluctuate at time t_0, the inventory curve and desired inventory level intersect at time t_1, t_2…t_i…t_n in succession, and the area between two curves can be divided into several parts S_1, S_2…S_i...S_n. Let the absolute value of the area between the two curves be s_n, that is:

$$S = \sum_{i=1}^{n}|S_i| = \int_{t_0}^{t_n}|I(t) - I^*|dt \quad (8)$$

We can distinguish the behavior of the system according to the form of curve S, that is, S curve can be used as the stability criterion of the system and it can be obtained by the software, Vensim PLE.

According to the definition of stability, the sufficient condition of the system to be stable is presented as following:

$$\lim_{t\to\infty} S(t) = C \text{ (Constant)} \quad (9)$$

Eq. (9) can be replaced by the following description:

Assuming t_0 is the starting time of simulation and t_F is the end time of simulation, if there exists t_s ($t_0 \le t_s \le t_F$) to make S(t) = C (Constant), then the system is stable.

The constant C can be understood as the system stable level, and the smaller the value of C, the better stability of the system. In the condition of step disturbances on demand and no prediction, C is positive.

4 STABILITY ANALYSIS OF HIGH ORDER DELAY INVENTORY CONTROL SYSTEM

4.1 Simulation Design

The initial values (unit) of variables are presented in Table 2 (Sterman, 1989; Riddalls and Bennett, 2002). The model is built using well-known system dynamics simulation software, Vensim PLE. The run length for simulation is 60 weeks.

Table 2: Initial Conditions.

WIP_{t_0}	I^*	I_{t_0}	DT	α_i	α_{WIP}	D_{t_0}	n_{MAX}	Step
300	200	200	3	1	0	100	50	0.0625

We adopt the small disturbance on demand for stability examination. This method has been widely used in the study of system stability based on control theory and general system theory. The order of delay gradually increases from 1 to n_{MAX} and the demand function is as follows:

$$D = D_{t0}(1 + STEP(h, T_s)) \quad (10)$$

Where h is the step signal of amplitude and Ts is the moment when step change happens. h = 0.2,Ts = 5.

In the presence of small disturbance, the decision parameter α_i is changed from 1 to 0 with a small decrement Δi. At the same time, α_{WIP} varies from 0 to 1 with another small increment ΔWIP, the smaller the values of Δi and ΔWIP, the higher the simulation accuracy. Meanwhile, the order of delay gradually increases from 1 to n_{MAX}.

Through simulation, we can observe the behavior patterns of the high order delay inventory control model and test the system stability in the situation of complete rationality (α_i, $\alpha_{WIP} \in [0,1]$).

4.2 Dynamics Characteristics

Figure 5 and figure 6 have shown the traverse graphs of inventory curves of tenth-order delay system and PTD system. Simulations indicate that the behavior patterns of inventory shown in figure 5 can represent the general dynamics characteristics of high order delay system. It is concluded that high order delay system can be presence of oscillation with equi-amplitude and divergent fluctuation, and the parameters α_{WIP} and α_i show entirely opposite effects on the dynamics characteristics of high order delay system. Based on the analysis, we can conclude that high order delay system has stability boundary corresponding to PTD system.

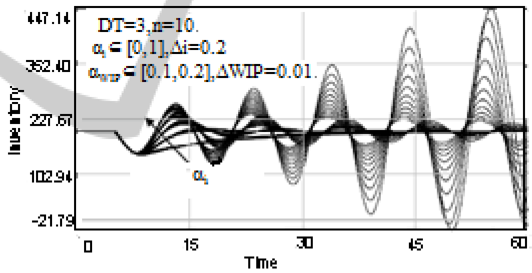

Figure 5: The traverse graph of inventory curves of tenth-order delay system.

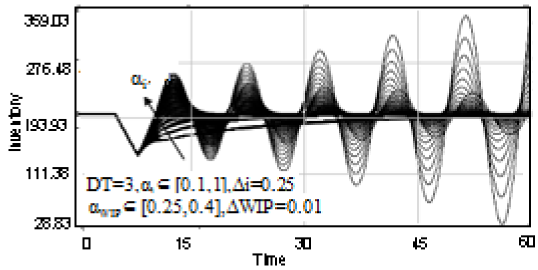

Figure 6: The traverse graph of inventory curves of PTD system.

4.3 Stability Analysis

As high order delay system covers four kinds of behavior patterns: smooth convergence, fluctuant convergence, oscillation with equi-amplitude, divergent fluctuation, it is more appropriate to PTD system from the visual point of view. The contrast between figure 5 and figure 6 shows that the range of decision parameter α_{WIP} in ninth-order delay system is smaller than that in PTD system with the same delay time and range of decision parameter α_i. That is, in high order delay system, the decision maker can consider more about inventory and consider less about WIP. Beer game has shown that most decision-makers think that the actual inventory is more important than WIP. Therefore, even if the decision maker has bias against the above conclusion, the high order delay system may still be stable. In other words, the decision strategies that lead PTD system to be unstable may make high order delay system stable. So, high order delay system is more stable than PTD system.

Through simulations, we can find the critical stable state of n-th order delay system with a given DT and obtain the critical stable points ($\alpha_{i,n}^{DT}$, $\alpha_{WIP,n}^{DT}$). Therefore, the critical stable condition of inventory control system under n-th order delay is defined as following:

Definition: Suppose the n-th order delay inventory control system is stable at the initial time. With a given DT, when imposing a small step disturbance on demand, if there exists the decision parameters ($\alpha_{i,n}^{DT}$, $\alpha_{WIP,n}^{DT}$) that can keep the inventory curve oscillate with equi-amplitude, then the state is called critical stable state, ($\alpha_{i,n}^{DT}$, $\alpha_{WIP,n}^{DT}$) is the critical stable point of the system under the given DT.

As high order delay system considers the order of delay, its critical stable points are determined by four dimensional vectors, (DT, α_i, α_{WIP}, n). Through the traversal simulation of α_i and α_{WIP} under certain DT, several critical stable points are found. After connecting these points in the plane that takes α_i as horizontal axis and α_{WIP} as vertical axis, we obtain the stability boundary of the tenth-order delay system named s curve. Figure 7 shows some s curves under different DT and the index of s represents the value of DT. For comparison, the corresponding stability boundaries of PTD system are also given.

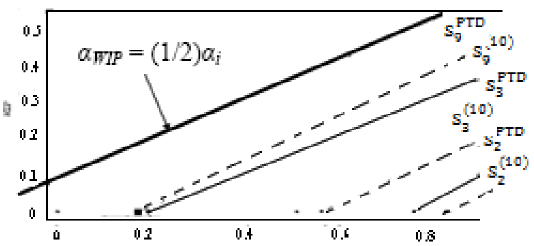

Figure 7: Comparison between the stability boundaries of tenth-order delay system and PTD system.

Furthermore, s curve of high order delay system is approximate to linear property and the lower right of s curve is the unstable region. For comparison, figure 8 shows the stability boundaries of five high order delay systems with different delay orders (n=2,4,6,10,25) under certain DT (DT=9).

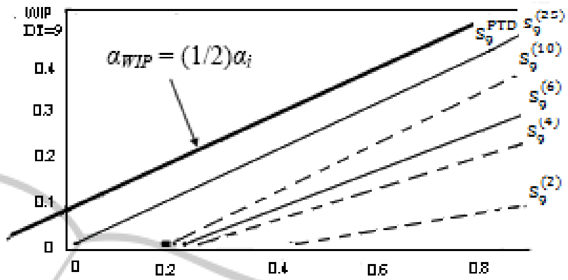

Figure 8: The effect of delay order on system stability (DT=9).

After running simulations for the high order delay inventory control model under different DT and comparing with PTD system, we can draw the following conclusions:

First, the stable region of high order delay system is larger than that of PTD system. With the same delay time, the larger order of delay, the closer the stability behavior of high order delay system to PTD system, and the oblique line (α_{WIP}=α_i/2) is the upper bound when s curve moves up to top left.

Second, as the oblique line (α_{WIP}=α_i/2) is the upper bound when s curve of n-th order delay inventory control system moves up to top left with certain DT, it can be concluded that the upper left area of oblique line (α_{WIP}=α_i/2) is the stable region which is independent of delay (IoD), and the oblique line (α_{WIP}=α_i/2) can be defined as IoD stability boundary which is only determined by systemic structure.

Third, under the same delay condition, the smaller delay time, the larger the stable region of high order delay time; and under the same delay time, the smaller the order of delay, the larger the stable region. Therefore, the stability of high order delay system with short delay time and small order of delay is closer to first-order system.

Further analysis on figure 8 reveals that when the order of delay is small, the s curves tend to be relative dispersive, and when the order of delay is large, the s curves are comparatively concentrated. This shows that the sensibility of decision parameters to the change of system order is nonlinear.

Based on figure 8, when the decision parameter α_i takes a certain value, figure 9 shows the relation curves between the decision parameter α_{WIP} and the order of delay (α_i=0.6; α_i=1).

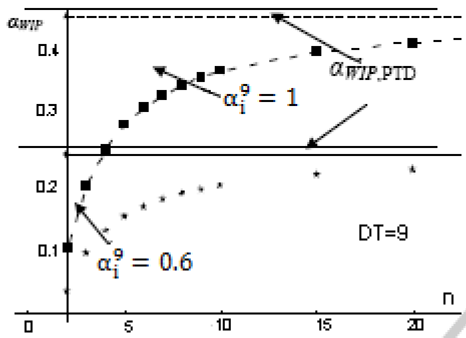

Figure 9: The delay order's sensitivity curve.

Figure 9 indicates that under the given delay time and decision parameter α_i, the value of α_{WIP} increases as the order of delay increases and it finally tends to that of PTD system in the same condition, but the increase margin of α_{WIP} drops when the order of delay increases. In other words, when the order of delay is large, the decision parameter α_{WIP} has a low sensitivity to the change of it. After simulations of delay systems under different DT, the above nonlinear characteristics still exist in the system.

The analysis above shows that in order to maintain system stability, decision maker should make clear the delay mode of WIP when focusing on the change of inventory and WIP. Overestimating the order of delay will artificially limit the stable region and restrict the implementation of certain ordering strategies. At the same time, underestimating the order of delay may enlarge the stable region and result in accidental fluctuations in the system.

4.4 Discussion

The structure of high order delay is the transition between first-order delay and pure time delay and it describes the more general delay mode in real systems. Therefore, it is more accurate to assume the delay mode in real systems to be high order delay. Researchers have shown that the delay mode of WIP is determined by the service rules of system and the order of delay is an important parameter to describe the delay mode. The order of delay can be approximately evaluated based on system data and then be used for simulation.

Simulation results show that system stability can be improved through decreasing the order of delay and shortening the delay time. Decreasing the order of delay suggests the adjustment on service rules, that is, the production or transportation system must be designed reasonablely. So decreasing the order of delay system can improve both the system stability and the overall level of service.

Finally, although the stability boundary of system exists objectively, it is definitely difficult to obtain the stability boundaries from theory or simulation. The inaccurate estimate of decision maker on the delay mode will lead to wrong decision and even the optimized rules may make the system instable. Therefore, to strengthen the control to the system and avoid the subjective error are important means of increasing system stability.

5 CONCLUSIONS

To conduct quantitative analysis on the stability of high order delay system, we first give the abstract analysis on the structure of high order delay and then built the high order delay inventory control model of supply chain. By simulation analysis, a system-dynamics-based criterion for stability judgment is proposed. With simulation, the criterion can be used to describe the nonlinearities of supply chain system and judge the influences exerted on supply chain stability by decision behavior. According to the concept of stability and stability criterion proposed in the paper, stability boundaries of high order delay inventory control system are confirmed. It is concluded that the IoD stability boundary has nothing to do with the delay time and order of delay and the stability of high order delay inventory control system is mainly decided by the features of feedback systems.

As high order delay system considers the order of delay, this paper further analyzes the effect of order of delay on system stability and finds that the sensibility of decision parameters to the change of system order is nonlinear. Synthetic analysis indicates that the subjective sensation of decision maker on the structure and behavioral pattern of system has great influence on the system stability. The paper finally points out that system stability and the overall level of service can be improved by adjusting the service rules in inventory control system, and further research needs to be done on how to adjust the service rules.

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