VEHICLE ROUTING PROBLEM WITH MULTI-DEPOT AND MULTI-TASK

Haoxiong Yang, Li Jing
School of Business, Beijing Technology and Business University, No.33 Fucheng Road, Beijing, China

Yongsheng Zhou, Mingke He
Department of Logistics Management, Beijing Technology and Business University, Beijing, China

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Abstract: This paper presents a new mathematical model of the vehicle routing problem in the context of city distribution, which considers multi-depot, multi-model vehicles and multi-task. The objective function includes three parts: transport cost, deadheading cost and time cost. To solve this mathematical model, a self-adaptive and polymorphic ant colony algorithm (APACA) has been introduced. Finally, a case study is presented to compare the results based on APACA with that under stochastic condition. Simulation results show that APACA is an effective and desirable algorithm for solving the mathematical model.

1 INTRODUCTION

Vehicle routing problem (VRP) is an important part of city distribution activities. The optimization for the vehicle routing problem contributes to improve the efficiency of goods distribution, reduce total distribution cost and increase customer satisfaction. Take Keihanshin metropolitan region in Japan for example, since the optimization strategy was taken by local government in 1989, the next surveys shown that average number of transport vehicles had been reduced by 17%, total deliver time had been reduced by 89%, and total travel distance had been reduced by 72%.

In the context of city distribution, vehicle routing problem has the following features: multi-depot, multi-model vehicles and multi-task. These features make vehicle routing problem more complicated than that under the condition of single-depot, single-model vehicle and single-task, but practical significance. In previous studies, Koehler (1999) described Kassel joint distribution system, and pointed that this system can decrease total travel time and the number of transport vehicles obviously; T Yamada et al. (2001) proposed a decisive model of co-operative vehicle routing, and analyzed the impact of co-distribution center location on deliver effectiveness; Renaud et al. proposed different branch-and-bound algorithms for symmetrical multi-depot vehicle routing problem and asymmetrical multi-depot vehicle routing problem. Yaohuang Guo (1995) and Shengce Hang et al. (1997) firstly transformed multi-depot vehicle routing problem into single-depot problem and then applied the method for solving single-depot problem into multi-depot problem; Mingshan Zhang (2002) discussed a general distributing route problem under the condition of multi-depot and full-load and proposed a heuristic algorithm for optimizing distributing route based on network flow model.

Take a broad view of previous studies, the majority of the studies have been of help to enterprises or governments; however for most of them the following aspects can be improved in the following aspects:

- More constraints such as multi-depot, multi-model vehicles, multi-task and capacity should be considered together, rather than only one or two constraints;
- The target of mathematical model is not only to minimize travel distance or cost in part distribution activities but also to minimize total travel cost in all

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distribution processes.

Since the vehicle routing problem in the context of city distribution is a complex system in which constraints including multi-depot, multi-model vehicles and multi-task are often occurred and the target to minimize total travel cost (includes transport cost, deadheading cost and time cost) is more meaningful than the target to minimize travel distance. As a result, the previous studies are not fully based on actual situations in the context of city distribution.

Based on the above studies, we first describe a practical city distribution problem whose constraints involve multi-depot, multi-model vehicles and multi-task, and establish its mathematical model whose objective function includes three parts: transport cost. Then, we state the design of an self-adaptive and polymorphic ant colony algorithm (APACA) for solving the problem. Finally the computational results of the actual instance show the effectiveness of the proposed method.

2 THE VEHICLE ROUTING PROBLEM WITH MULTI-DEPOT, MULTI-MODEL VEHICLES AND MULTI-TASK

2.1 Problem Description

In city distribution system, there are several depots. Each of them has some different model vehicles. Different model vehicle has different capacity, unit transport cost, deadheading cost and unit time cost. Each vehicle must set out from its depot to distribution center for carrying goods. Then, the vehicles deliver goods to all demand points. Each demand point has a service soft time window which means that if the arrival time at a demand point is earlier than the beginning of the time window or later than the end of the time window, the cost function will be penalized by some amount. A demand point is serviced exactly once by only one vehicle. The problem that we propose is that in order to minimize total travel cost (includes transport cost, deadheading cost and time cost) how to decide right vehicles and chose right distributing routes in the above circumstances.

2.2 Mathematical Model

Let us consider a routing network which is composed of depots, distribution center and customer points. In city distribution activities, the vehicle K sets out from the depot S to the distribution center O for loading goods, and then delivers goods to the customer point N. Finally, the vehicle K needs to return to its start point, namely depot S. The problem is defined on a complete graph $G(V,A)$ (shown in Figure 1), where $V=(v_1,v_2,\ldots,v_n,v_{n+1},v_{n+2},v_{n+3},\ldots,v_{n+m+1})$ is a vertex set and $E=\{(v_i,v_j): v_i, v_j \in V, i \neq j\}$ is the arc set. V includes three subsets: $N=\{v_1,v_2,\ldots,v_n\}$ is a customer vertex set, $O=\{v_{n+1}\}$ is a distribution center vertex set, $S=\{v_{n+2},v_{n+3},\ldots,v_{n+s+1}\}$ is a depot vertex set. For each $v_i \in N$, it includes the demand of the customer; for each $v_i \in S$, it includes the number of the vehicle model; $K=\{1,2,\ldots,k\}$ is a vehicle model vertex set; each customer v_i has a service time window $[ET_i, LT_i]$ where ET_i is the earliest time that service can begin and LT_i is the latest time that service can begin. If the arrival time at a demand point is earlier than the beginning of the time window or later than the end of the time window, the cost function will be penalized by some amount.

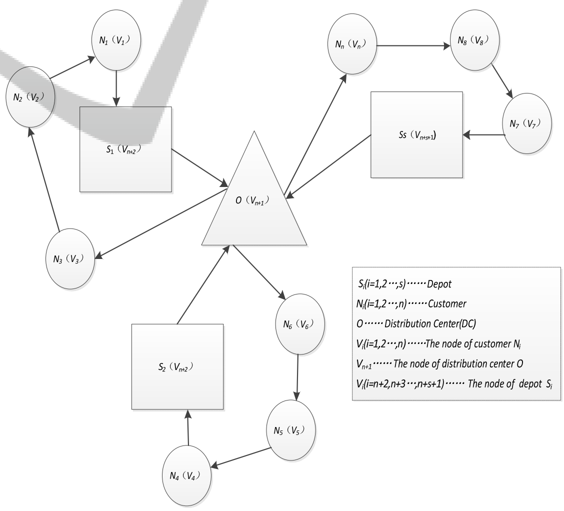

Figure 1: The illustration of the Vehicle Routing.

In order to simplify the problem, we define the sets, parameters and variables used in the mathematical model as follows:

- We denote the customers by $1,2,\ldots,N$; distribution center by $N+1$; depots by $N+2$, $N+3,\ldots,$ $N+S+1$ and the vehicle models by $1,2,\ldots,K$;
- $r_{ij}^{sk}=(1,0)$: If vehicle k in the depot s travel form vertex i to vertex j the $r_{ij}^{sk}=1$, otherwise $r_{ij}^{sk}=0$;
- D_{ij}: Distance between node i and node j;

- N: Total number of customers;
- S: Total number of depots;
- C_k: Unit transportation cost of vehicle k;
- E_k: Unit deadheading cost of vehicle k;
- V_k: Unit travel distance of vehicle k;
- Q_k: Capacity of vehicle k;
- W_i: Demand at customer i;
- T_i: Arrival time at node i;
- t_{ij}: Travel time from node i to node j;
- ET_i: Earliest arrival time at node i;
- LT_i: Latest arrival time at node i;
- A: Earliness penalty coefficient;
- B: Tardiness penalty coefficient

The target of this mathematical model is to minimize total travel cost (includes transport cost, deadheading cost and time cost) which is occurred in the process of city distribution.

The formulation of the problem is as follows:

$$\min C = \sum_{i \in O \cup N} \sum_{j \in N} \sum_{s \in S} \sum_{k \in K} r_{ij}^{sk} D_{ij} C_k + \sum_{i \in S} \sum_{j \in O} \sum_{s \in S} \sum_{k \in K} r_{ij}^{sk} D_{ij} E_k + \sum_{i \in N} \sum_{j = s} \sum_{s \in S} \sum_{k \in K} r_{ij}^{sk} D_{ij} E_k + \sum_{i \in O \cup N} \sum_{j \in N} \sum_{s \in S} \sum_{k \in K} r_{ij}^{sk} \left[a_j \max(ET_j - T_j, 0) + b_j \max(T_j - LT_j, 0) \right] \tag{1}$$

$$\sum_{i \in O \cup N} \sum_{j \in N} r_{ij}^{sk} w_j \leq Q_k, k \in K, s \in S \tag{2}$$

$$\sum_{i \in O \cup N} \sum_{s \in S} \sum_{k \in K} r_{ij}^{sk} = 1, j \in N \tag{3}$$

$$\sum_{j \in N} r_{ij}^{sk} - \sum_{j \in N} r_{ji}^{sk} = 0, k \in K, s \in S \tag{4}$$

$$\sum_{j = s} r_{ij}^{sk} = 1, i \in N, k \in K, s \in S \tag{5}$$

$$\sum_{i \in N \cup O} \sum_{s \in S} \sum_{k \in K} r_{ij}^{sk} (T_i + t_{ij}) = T_j, j \in N \tag{6}$$

$$\sum_{i \in N \cup O} \sum_{j \in N} \sum_{s \in S} \sum_{k \in K} r_{ij}^{sk} = N \tag{7}$$

$$T_s = 0, s \in S \tag{8}$$

The objectives, formula (1) is to minimize the total vehicle travel cost ((includes transport cost, deadheading cost and time cost); Formula (2) constraints the vehicle capacity; Formula (3) ensures that a customer is serviced exactly once by only one vehicle; Constraint (4) and (5) guarantee that each vehicle sets out from the depot and returns to it; Formula (6) is to compute the arrival time when the vehicle arrives at customer j; Constraint (7) ensures that each customer will be serviced; Formula (8) denotes that the initial time starts form the depot, and the initial time is equal to zero.

2.3 The Design of Self-adaptive and Polymorphic Ant Colony Algorithm (APACA) for Solving Multi-depot, Multi-model Vehicles and Multi-task Vehicle Routing Problem

2.3.1 The Overall Idea of Self-adaptive and Polymorphic ant Colony Algorithm (APACA)

The design idea of self-adaptive and polymorphic ant colony algorithm (APACA) in this paper is as follows: Based on traditional colony algorithm for the travelling salesman problem (TSP),we improve the traditional colony algorithm in information selective mechanism, information update mechanism and information collaborative mechanism, and also introduce self-adaptive migration rule into the traditional colony. Because the division of labor in ants is diverse, we allocate the tasks with many constraints to detective ants. Then, the task to find feasible solutions of the target function is completed by searching ants. We combine information self-adaptive rule and polymorphic ant colony algorithm perfectly by the collaboration between different kinds of ants. As a result, self-adaptive and polymorphic ant colony algorithm (APACA) can overcome the drawbacks such as long computing time and precocity which happen in traditional ant algorithm.

2.3.2 The Steps of Self-adaptive and Polymorphic Ant Colony Algorithm (APACA)

The heading of a subsection title should be in 12-point bold with initial letters capitalized, aligned to the left with a linespace exactly at 13-point, hanging indent of 1,0-centimeter and with According to the principle of self-adaptive and polymorphic ant colony algorithm (APACA) proposed by Meijun Chen et al. (2008), we apply this algorithm for solving multi-depot, multi-model vehicles and multi-task vehicle routing problem, Specific steps and methods are as follows:

- Step 1: Initialize Q, C and $N_{c,max}$;
- Step 2: Put n detective ants in n cities, and each detective ant revolves around city i. Then, the detective ant detects other $n\text{-}1$ cities. Calculate the detective table and assign the calculating results to S_{ij};
- Step 3: Set the information value of every route

at initial time;

- Step 4: Set that the initial value of N_c is equal to zero;
- Step 5:Randomly select the initial coordination of every ant, and put the coordination into its $tabu_k$ table;
- Step 6: Calculate the migrating coordination of every searching ant. Suppose that the migrating coordination is point j and the last coordination is point i. Then, put the coordination of point j into $tabu_k$ table of searching ant k. Until each searching ant visits all coordinations, we can get a solution;
- Step 7: Calculate all solutions of the target function $f(Z_k)$ (k=1,2,…,n), and record all the solutions;
- Step 8: Lead 2-opt local optimization to all the above solutions, if new solutions is better than initial solutions, then we replace initial solutions with new solutions; Otherwise, we have initial solutions;
- Step 9: If cycle counts comes to $N_{c,max}$, go to step12. If cycle counts does not come to $N_{c,max}$ and the solution isn't improved in recent several iterations, then we should change the value ρ, go to step 11. Otherwise, go to step 10;
- Step 10: Change the information value of every route, reset $\nabla\tau_{ij} = 0$ and clear $tabu_k$ table;
- Step 11: $N_c \leftarrow N_c+1$, go to step5;
- Step 12: Output the optimal solution.

3 SIMULATION EXAMPLE

In simulation example, in order to get the optimal solution, the algorithm is implemented under the visual studio 2005. The computational experiment assumed to have three delivery depots, one distribution center and sixteen customers. All the nodes are distributed in a 100×100 square region, each depot has two kinds of model vehicle. Customer data can be seen in Table 1; Depot and distribution center data can be seen in Table 2; Vehicle data can be seen in Table 3.

Table 1: Customer data.

Customer	1	2	3	4	5	6	7	8
Coordination	(88,55)	(39,31)	(20,76)	(65,98)	(95,23)	(50,9)	(33,65)	(79,75)
W_i	1.5	3	1	1.5	2.5	2	4	2
$[ET_i,LT_i]$	[4,6]	[4,7]	[2,5]	[1.5,4.5]	[5,8]	[1,4]	[3,6]	[5,7.5]
a	1	1	1	1	1	1	1	1
b	1	1	1	1	1	1	1	1
Customer	9	10	11	12	13	14	15	16
Coordination	(55,35)	(78,30)	(11,20)	(35,93)	(73,85)	(62,53)	(33,13)	(9,12)
W_i	5	4	1.5	2	4	2.5	45	3
$[ET_i,LT_i]$	[2,4]	[2,5]	[3,5]	[2,4]	[3,7]	[3,6]	[1,2]	[0.5,4]
a	1	1	1	1	1	1	1	1
b	1	1	1	1	1	1	1	1

Table 2: Depot and distribution center data.

Depot	18	19	20	17
Coordination	(20,20)	(75,45)	(50,8)	(40,40)
Number of model No.1	3	3	3	
Number of model No.2	3	3	3	

Table 3: Vehicle data.

Vehicle model No	1	2
Q_k	10	8
C_k	0.8	1
E_k	0.4	0.6
V_k	40	40

Table 4: Vehicle scheduling situation after optimization.

Starting depot（S）	Vehicle model No（K）	Number of Vehicles	Driving route
1	1	1	1→4→19→10→6→1
1	2	1	1→4→20→15→1
2	2	1	2→4→18→13→2
2	2	1	2→4→14→9→5→2
3	2	1	3→4→8→17→12→3
3	2	1	3→4→11→7→16→3

Table 5: Vehicle scheduling situation in in stochastic context.

Starting depot（S）	Vehicle model No（K）	Number of Vehicles	Driving route
1	1	1	1→4→19→10→1
1	2	1	1→4→15→20→6→1
2	1	1	2→4→5→9→14→2
2	2	1	2→4→13→18→2
3	1	1	3→4→8→17→12→3
3	2	1	3→4→16→7→11→3

Experimental parameters used in simulation example are as follows: α=1, β=2， ρ（t_0）=1， n=100,Q=100,C=3,Max(P_c)=10, iteration number is equal to 50. Finally, we get the total cost that is 425.6. Vehicle scheduling situation can be seen in Table 4.We also get a stochastic solution in stochastic selection context, and assume that stochastic vehicle scheduling situation can be seen in Table 5. The total cost is 440.92.Simulation results show that the total cost optimized by the self-adaptive and polymorphic ant colony algorithm (APACA) is 425.6, which is less than the total cost in stochastic selection context that is 440.92. The results also demonstrate that it is necessary to optimize the multi-depot, multi-model vehicles and multi-task vehicle routing problem and that APACA is an effective and desirable algorithm for solving the mathematical model.

4 CONCLUSIONS

In the process of city distribution, the optimal selection of the vehicle route is a key issue to improve the service quality, reduce the operation cost and increase the profits. In this paper we present a new mathematical model of the vehicle routing problem in the context of city distribution, which considers multi-depot, multi-model vehicles and multi-task. The objective function includes three parts: transport cost, deadheading cost and time cost. A self-adaptive and polymorphic ant colony algorithm (APACA) is introduced to solve this mathematical model. Finally, the effectiveness of the algorithm is demonstrated by the simulation results.

However, there are some aspects such as target system, evaluating indicator of route and the effectiveness of the algorithm that can be improved. In future studies, more factors such as customer priority and road condition should be considered when we establish mathematical models. These will make the solution more practical and perfect.

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