WHEELED MOBILE MANIPULATOR MODELING FOR TASK SPACE CONTROL

Gastón H. Salazar-Silva
UPIITA, Instituto Politécnico Nacional, Av. IPN 2580, Mexico City, D.F., Mexico

Jaime Álvarez Gallegos
Secretaría de Investigación y Posgrado, Instituto Politécnico Nacional, Mexico City, D.F., Mexico

Marco A. Moreno-Armendáriz
Centro de Investigación en Computación, Instituto Politécnico Nacional, Mexico City, D.F., Mexico

Keywords: Robot control, Mobile robots, Robots kinematics.

Abstract: Mobile manipulators have attracted a lot attention lately because they have many advantages over stationary manipulator, such as larger a work space than a stationary manipulator could have in practice. However, the proposed methods in the state of the art to obtain the kinematic model of a mobile manipulator are based in modeling separately the mobile base and the manipulator arm, and later combining both models.
This paper shows a systematic approach to obtain the kinematic model of mobile manipulators that transforms in the modeling problem of a stationary manipulator with non-holonomic kinematic constraints in the joints; it is also showed an example of the application of the method, where the kinematic and dynamic models are obtained with extensions of the same tools used in stationary robots.

1 INTRODUCTION

A mobile manipulator is a manipulator mounted on a mobile robot; an example is a manipulator arm mounted on a mobile robot with differential traction. Mobile manipulators have many advantages over stationary manipulator, such as a larger work space. Mobile manipulators can perform the tasks of locomotion and handling; those tasks have been handled as two separate problems, for example in (Wang et al., 2008) the focus is on movement of the mobile base and in (Joshi and Desrochers, 1986) the task is the motion of the manipulator arm. However, recently there are lot of attention to the performance of both tasks simultaneously, for example in (Mazur, 2010).

The problem of kinematic modeling of a mobile manipulator has been attacked by obtaining separately the kinematic models of the base and arm manipulator, and then combining both models (De Luca et al., 2006). Due to the usefulness of mobile manipulators, it is important to have methods and tools that allow an easier analysis of the mobile manipulator.

This paper shows an integrated approach to the kinematic modeling of wheeled mobile manipulators that apply the same tools used in modeling stationary robots. This method assumes that the mobile manipulator is a stationary robot which have joints with non-holonomic constraints; this approach allows the use of existing tools to obtain the kinematic and dynamic models, for example the Denavit–Hartenberg parameters and geometric Jacobians.

A kinematic modeling scheme for mobile manipulators is presented in (Bayle et al., 2003; Mazur, 2010) where the kinematic models for the mobile platform and the arm are determined separately. In (De Luca et al., 2006) a method is presented for combining the kinematic model of the mobile base with the stationary manipulator, but the mobile base and the manipulator are still modeled with different methods. Particularly interesting example is in (Mazur, 2010), where both the mobile base and the manipulator have non-holonomic constraints and yet are modeled by different methods.

The outline of this paper is as follows: the kinematic modeling of mobile robots is reviewed in Section 2. The modeling method proposed in this report, modeling the mobile manipulator simply as stationary manipulators with joints that have kinematic con-

H. Salazar-Silva G., Álvarez Gallegos J. and A. Moreno-Armendáriz M..
WHEELED MOBILE MANIPULATOR MODELING FOR TASK SPACE CONTROL.
DOI: 10.5220/0003540203130316
In *Proceedings of the 8th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2011), pages 313-316
ISBN: 978-989-8425-75-1

straints, is presented in Section 3. In Section 4 a control for task space is proposed. As an example of the method, in Section 5 the kinematic and dynamic models of a 5-degree of freedom (DOF) mobile manipulator are obtained and the implementation of the control presented in Section 4 is done.

2 KINEMATIC MODEL OF MOBILE ROBOT

A kinematic model describes the relationship between the motion of a mechanical system and the actuation velocities. The motion of a wheeled mobile robot is characterized by the constraints imposed by the wheels (Campion et al., 1996); in this section it is briefly reviewed the issues on kinematic constraints and the development of kinematics models.

A set of k kinematic constraints restricts the motion of a mechanical system and can be expressed as

$$a_i(q,\dot{q}) = 0,$$

where $q \in \mathbb{R}^n$ is the configuration variables vector, $\dot{q} \in \mathbb{R}^n$ is the configuration velocities vector and n is the size of the configuration vector, usually called degree of freedom (DOF), a_i are scalar functions on q and $\dot{q}$. If the function a_i does not depend on $\dot{q}$ then the system is called *holonomic*, otherwise it is said that the system is *nonholonomic*.

There are two kinds of kinematic models as proposed in (Campion et al., 1996). The first is the posture kinematic model and is a relationship between the motion on the task space and the motion of the actuators; for a wheeled mobile robot with differential traction it can be expressed as (Campion et al., 1996)

$$\dot{r}_b(t) = B_b(q)\eta_b \tag{1}$$

where $\dot{r}_b(t) \in \mathbb{R}^p$ are the posture velocities on a task space of dimension p, $\eta_b(t) \in \mathbb{R}^{n-k}$ is the vector which contains the velocities of the actuators, and $B_b(q) \in R^{n\times(n-k)}$ is a matrix with its columns are a base of the null space of the nonholonomic constraints.

On the other hand, the configuration kinematic model is the relationship between the velocities of the joints variables and the velocities of the actuators and it is defined as

$$\dot{q}_b(t) = S_b(q)\eta_b \tag{2}$$

where $S_b(q) \in R^{n\times(n-k)}$ is a matrix with its columns are also a base of null space of the constraints. It is also important to note that $S(q)$ is an annihilator of the kinematic constraints, such that

$$A(q)^T S_b(q) = 0; \tag{3}$$

this fact can be used to simplify the dynamic model (De Luca and Oriolo, 1995).

3 MOBILE MANIPULATOR MODELING

The kinematics of a mobile manipulator is given by the function f, defined as

$$r = f(q) \tag{4}$$

where r is the combined posture of the mobile manipulator and q are the combined generalized coordinates of the mobile base and the manipulator arm; Thus the kinematic modeling of a mobile manipulator depends on finding the Jacobian J,

$$J = \frac{\partial}{\partial q} f(q) \tag{5}$$

and which depends in turn on combining the kinematics of the manipulator and the base mobile.

A method to find the direct kinematics of the manipulator arm and mobile base, and also allows combine them, are the homogeneous transformations; specifically for the mobile manipulator it is defined as (Li and Liu, 2004):

$$T_n^0 = T_b^0 T_n^b$$

where T_b^0 is the homogeneous transformation which goes from a frame $\{b\}$ fixed on the mobile base to a frame $\{0\}$ fixed on surface on which the mobile base moves, and T_n^b is the homogeneous transformation which goes from a frame $\{n\}$ fixed on the last link of the mobile manipulator to the frame $\{b\}$; there is not a standardized method to find the transformation T_b^0. It is important to remark that T_b^0 does not take account of the nonholonomic constraints.

The proposed method is to obtain the forward kinematics of the mobile base T_b^0 by assuming that the mobile base is stationary manipulator of b DOF and considering it a unique kinematic chain, and the applying a modeling method for stationary robots, such as the Denavit–Hartenberg method. Also it is possible to obtain the Jacobian J of the whole mobile manipulator from the same geometric method used in stationary robots and then is possible to obtain the posture kinematic model of mobile manipulator

$$\dot{r} = B(q)\eta \tag{6}$$

where η is the vector of the actuation variables and is defined as

$$\eta = \begin{bmatrix} \eta_b^T & \dot{q}_m^T \end{bmatrix}^T,$$

$B(q)$ is the posture kinematic relation of the mobile manipulator, defined as

$$B(q) = J(q)S(q)$$

$S(q)$ is the configuration kinematic relation for the whole mobile manipulator

$$S(q) = \begin{bmatrix} S_b(q) & I \end{bmatrix}$$

where I is a identity matrix, showing that the configuration velocities are identical to the actuation velocities. Some advantages of the proposed method is that uses the same methods and computational tools as the stationary manipulators to obtain the kinematic models.

4 CONTROL IN TASK SPACE

The control proposed in this paper follows the classical combination of two control loops in cascade; the internal loop control uses an inverse dynamics control. The external control loop is a resolution of acceleration control over the task space. The dynamic model of a mechanical system with non-holonomic constraints is defined by a set of n second-order differential equations

$$\begin{aligned} D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) &= A(q)\lambda + S(q)\tau \\ A(q)^T\dot{q} &= 0 \end{aligned} \tag{7}$$

where $D(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix for the system, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the Coriolis and cross-velocities matrix, $g(q) \in \mathbb{R}^n$ is a vector which represents the impact of gravity on the links, $A(q) \in \mathbb{R}^{n\times k}$ is a matrix in which k kinematics constrains are expressed, $S(q) \in \mathbb{R}^{m\times n}$ in the input matrix, and $\tau \in \mathbb{R}^m$ are the generalized forces that go into system.

Taking advantage of (3), it is possible to eliminate the explicit statement of the kinematic constraint in (7) by applying (1), thus the following reduced order system is obtained (De Luca and Oriolo, 1995)

$$\begin{aligned} \dot{q} &= S(q)\eta \\ \dot{\eta} &= -M(q)^{-1}\left(m(q,\eta) + S(q)^T S(q)\tau\right) \end{aligned} \tag{8}$$

where

$$\begin{aligned} M(q) &= S(q)^T D(q) S(q) \\ m(q,\eta) &= S(q)^T \left(D(q)\dot{S}\eta + C(q,S\eta)S\eta + g(q)\right) \end{aligned} .$$

A control τ is proposed such that it cancels the dynamics in (8)

$$\tau = (S(q)^T S(q))^{-1} m(q,\eta) + (S(q)^T S(q))^{-1} Ma \tag{9}$$

where $a(t) \in \mathbb{R}^4$ is the acceleration reference for the system.

For the external control loop, the resolution of acceleration control (RAC) is used. Firstly, a measure of the error on task space is proposed, $\tilde{r}$, such that

$$\tilde{r}(t) = r^d(t) - r(t).$$

where $r^d(t) \in \mathbb{R}^n$ is the desired posture. The control is then proposed according to the following error dynamics

$$\ddot{\tilde{r}}(t) + K_1\dot{\tilde{r}}(t) + K_0\tilde{r}(t) = 0 \tag{10}$$

where $\dot{\tilde{r}}$ and $\ddot{\tilde{r}}$ are the first and second derivatives of the error with respect time. Then (10) in combination with (2) are used to obtain

$$\dot{\eta} = B(q)^\dagger \left(\ddot{r}^d - \dot{B}(q,\eta)\dot{\eta} + K_1\dot{\tilde{r}} + K_0\tilde{r}\right). \tag{11}$$

5 EXAMPLE

To test the proposed method, a mobile manipulator was modeled; it is integrated by a diferential-traction Pioneer 3DX mobile robot and a Cyton manipulator arm with 7 DOF, but only two joint were considered, thus the mobile manipulator has 5 DOF. It is assumed that the mobile base is a unicycle without slipping and the surface on which the mobile base moves is flat and horizontal. The mobile manipulator was numerically modeled with the Matlab's `robotics toolbox` (Corke, 1996).

The mobile manipulator was modeled as a stationary manipulator, as shown in Figure 1. The Denavit–Hartenberg parameters are showed on Table 1. The configuration of the mobile manipulator, $q(t)$, is defined as:

$$q = \begin{bmatrix} d_1 & d_2 & \theta_3 & \theta_4 & \theta_5 \end{bmatrix}^T$$

Table 1: The Denavit–Hartenberg parameters for the 5-DOF mobile manipulator.

i	α	a [mm]	θ	d [mm]	Kinematic pair
1	$-\pi/2$	0	0	0	prismatic
2	$\pi/2$	0	$-\pi/2$	0	prismatic
3	0	0	0	237	revolute
4	0	150	0	0	revolute
5	0	168	0	0	revolute

where d_1, d_2 are the surface coordinates (x,y) of the mobile base, $\theta_3 = \phi$ is the orientation of the mobile base, and θ_4, θ_5 are the joint variables of the manipulator arm.

On the other hand, the kinematic constraint of the 5-DOF mobile manipulator is given by the matrix $A(q) \in \mathbb{R}^{5\times 1}$ and it is defined by the expresion

$$A(q) = \begin{bmatrix} \sin q_3 & -\cos q_3 & 0 & 0 & 0 \end{bmatrix}^T. \tag{12}$$

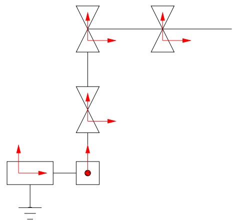

Figure 1: Kinematic representation of a mobile manipulator 5 DOF.

The actuators velocities, $\eta \in \mathbb{R}^4$, are defined as:

$$\eta = (v, \dot{q}_3, \dot{q}_4, \dot{q}_5)^T$$

where $v(t)$ is an scalar which describes the lineal velocity of the mobile robot, and configuration kinematic model $S(q) \in \mathbb{R}^{5\times 4}$ is defined by

$$S(q) = \begin{pmatrix} \cos q_3 & 0 & 0 & 0 \\ \sin q_3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (13)$$

which satisfy the property of being an annihilator for (12). The parameters of (7) are obtained according to (Spong et al., 2006) and the data that appears in Table 2.

Table 2: Link data for dynamic model from the 5-DOF mobile manipulator.

i	Length [mm]	Wide [mm]	Height [mm]	Mass [kg]
3	445	393	237	9.0
4	150	50	50	0.1
5	168	50	50	0.1

The control described in Section 4 was applied to a numerical model of the mobile manipulator. The result of the simulations are showed in Figure 2; the reference is a trajectory in task space generated by a linear interpolation between two points; it is important to note that the trajectory not necessarily satisfy the nonholonomic constraint.

6 CONCLUSIONS

This paper shows a systematic approach to modeling mobile manipulators that transforms the problem to the modeling of a stationary manipulator stationary with non-holonomic kinematic constraints on the joints. It is also presented a control that uses an estimate of the derivative of the posture kinematic model. Finally, an example is presented using this method.

In future work, it will develop a priority control in the task space for a mobile manipulator, and it will be develop a teleoperation scheme on the real system.

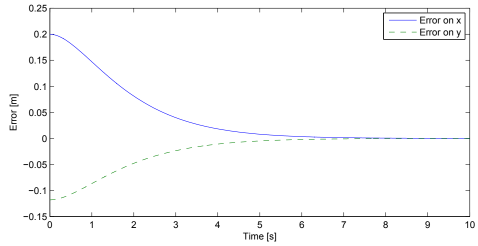

Figure 2: Posture error graph for the mobile manipulator under the control, as described in Section 4.

ACKNOWLEDGEMENTS

The authors appreciate the support of Mexican Government (SNI, SIP-IPN, COFAA-IPN, PIFI-IPN and CONACYT).

REFERENCES

Bayle, B., Fourquet, J.-Y., and Renaud, M. (2003). Kinematic modelling of wheeled mobile manipulators. In *Robotics and Automation, 2003. Proceedings. ICRA '03. IEEE International Conference on*, pages 69–74.

Campion, G., Bastin, G., and Dandrea-Novel, B. (1996). Structural properties and classification of kinematic and dynamic models of wheeled mobile robots. *IEEE Trans. Robot.*, 12(1):47–62.

Corke, P. (1996). A robotics toolbox for MATLAB. *IEEE Robotics and Automation Magazine*, 3(1):24–32.

De Luca, A. and Oriolo, G. (1995). *Kinematics and Dynamics of Multi-Body Systems*, chapter Modelling and control of nonholonomic mechanical systems, pages 277–342. Springer.

De Luca, A., Oriolo, G., and Giordano, P. R. (2006). Kinematic modeling and redundancy resolution for nonholonomic mobile manipulators. In *Proceedings of the 2006 IEEE International Conference on Robotics and Automation*, pages 1867–1873, Orlando, Florida, USA.

Joshi, J. and Desrochers, A. (1986). Modeling and control of a mobile robot subject to disturbances. In *Proceedings of the 1986 IEEE International Conference on Robotics and Automation*, pages 1508–1513.

Li, Y. and Liu, Y. (2004). Control of a mobile modular manipulator moving on a slope. In *Mechatronics, 2004. ICM '04. Proceedings of the IEEE International Conference on*, pages 135–140.

Mazur, A. (2010). Trajectory tracking control in workspace-defined tasks for nonholonomic mobile manipulators. *Robotica*, 28:1–12.

Spong, M. W., Hutchinson, S., and Vidyasagar, M. (2006). *Robot Modeling and Control*. Wiley.

Wang, Y., Lang, H., and de Silva, C. W. (2008). Visual servo control and parameter calibration for mobile multi-robot cooperative assembly tasks. In *IEEE International Conference on Automation and Logistics, 2008. ICAL 2008*, pages 635–639, Qingdao.