THE CLASSIFICATION OF TIME SERIES UNDER THE INFLUENCE OF SCALED NOISE

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Abstract: In this paper, we propose an improvement of a method for market time series’ classification based on fuzzy and fractal technology. Usually, the older values of time series will be cut off at a specific time point. We investigated the influence of the fractal features on the classification result. We compared a normal time series representation, a representation having a smaller box dimension (achieved by exponential smoothing), and a representation having a greater box dimension (achieved by adding scaled noise). We used different types of noises and scales to improve the classification result. Our application concerns time series of stock prices. The market performance of those approaches is analyzed, discussed, and compared with the system without the scaled noise component.

1 INTRODUCTION

There are two market hypotheses - the efficient markets and the hypothesis of inefficient markets. The hypothesis of efficient markets states that the information obtained in the time series describing the stock prices in the past is worthless to prediction because the complete information from the past is already fully evaluated, and contained in the last price value. The hypothesis of inefficient markets says that there are some influences not completely evaluated, and respected in the last price, and that these factors can be used for prediction.

Since the beginning of stock trading, traders and investors try to predict markets. One group of them uses technical analysis that developed strategies to analyse the behaviour of charts by means of technical indicators (e.g. moving average) and react accordingly, e.g. Turtle Trading (Faith, 2007). The other group uses fundamental analysis that developed strategies based on the economic data of companies and markets, e.g. Value Investing (Graham, 2006).

We do not try to predict the markets. Our research hypothesis is based on the hypothesis of inefficient markets and states that it is possible to classify securities in such classes that some of them have statistically more chances to profit in the current market situation than the others. In our previous research (Kroha and Lauschke, 2010), we implemented a system based on fuzzy and fractal technology that supported our hypothesis stated above. Fuzzy clustering and the Takagi-Sugeno inference method have been used for processing of fractal features of time series.

In this paper, we investigate the influence of time series fractal features to the classification and describe the implemented component. We analyze possibilities of this method and compare it with the properties of processes without this component by their average performance.

The paper is structured as follows. In Section 2, we discuss the related works. Section 3 explains shortly the concept of the fuzzy and fractal classifier. Sections 4 and 5 present why and how we extended our previous system by different methods of noise generation and exponential smoothing, and how we scaled them. Experiments and data used are described in Section 6. The evaluation and results are given in Section 7 where we analyze all the approaches regarding their market performance. Finally, we conclude in Section 8.

2 RELATED WORKS

This paper introduces and investigates a method of changing fractal features of time series added to ideas published in our previous paper (Kroha and Lauschke,
3 FUZZY AND FRACTAL CLASSIFICATION

In this section, we will only very briefly describe the fuzzy and fractal features that we used in our system (Kroha and Lauschke, 2010) before we introduce the improvement by changing fractal dimension described as our contribution in this paper. Our system has the following three components: fractal dimension module, c-mean clustering algorithm, and Takagi-Sugeno fuzzy inference module.

The fractal dimension module calculates the fractal features of time series. They describe how chaotic a given time series is. There are several ways to define and calculate the fractal dimension. We used box dimension and Hurst coefficient according to their definitions in (Mandelbrot, 1983). Details concerning the fractal features of market can be found in (Peters, 1994) and in (Peters, 1996).

In addition, we had to use a correlation coefficient with a line to indicate whether the time series values are going up or down.

A fuzzy classifier uses three main components: fuzzyfication, fuzzy inference and defuzzyfication. The number of classes need to be defined.

Fuzzyfication is the process of transforming sharp input data into fuzzy data using a membership function. The fuzzyfier determines what kind of cluster is more probable for a given input value describing the time series. The clustering algorithm c-mean dynamically calculates the membership function for the fuzzyfication process. As the input of the c-mean clustering of time series, we used a triple consisting of box dimension, Hurst coefficient, and correlation coefficient as mentioned above. Further, it prepares data for generation inference rules dynamically. More details can be obtained from (Castillo and Melin, 2003).

Then, using fuzzy inference according to the dynamically generated rules, a prediction value is inferred. The rules are generated dynamically on the basic of the input data. For this purpose, we used the Takagi-Sugeno inference method (Klusa, 2009). This function is valid for a certain dataset, it is, however, dynamic and individually generated for each dataset.

To interpret the output prediction value, it will be defuzzified. In our case, its definition interval is divided on three parts because we classify into three classes. The prediction value membership to a subinterval indicates whether the classified time series will be associated with the class BUY, SELL, or HOLD. Time series of the class BUY have more common features with time series that were going up in the past, and time series of the class SELL have more common features with time series that were going down in the past under similar conditions. The architecture of our time series classification system is in Fig. 1.
3.1 Why Changing Fractal Dimension Makes Sense

Supposing that a part of the future can be predicted based on the history. Then the question is how much of the history should be used to predict the future, and whether all parts of the time series should have the same weighting.

In stock market forecasting, there are many traders who believe that older data is not as important as newer data, e.g. daytraders. Other traders use some indicators based on long time intervals, e.g. 20-day or 200-day simple moving average. Even though, some features of a company are durable, many things can change during 200 days. The administration of a company may change, its product may become obsolete, new factories may be built that influence the price of a product and competition on the market.

The first version of our system was built on the idea that inside of a specified time interval (we used 30 days for our experiments) each value of the time series is as important as the other.

Classical methods of technical analysis usually state a limit of the influence, they cut the influence sharply (e.g., after 20 days, after 200 days), but all values in the moving average have the same weight, i.e., the same influence, like in the case of the simple moving average.

There are also other models known, like weighted moving average (more recent values are more heavily weighted). The commonest type of weighted moving average is exponential moving average applied as exponential smoothing. Exponential moving average (values are assigned to exponentially decreasing weights over time) reacts faster to recent prices and is used for known technical indicator MACD (moving average convergence divergence). Variable moving average is an exponential moving average that derives its smoothing percentage based on market volatility. Triangular moving average is double smoothed, i.e., it is averaged from simple moving averages. In such a case, investors using these moving averages in their trading strategies must have a special strategy for any kind of the moving average.

However, time is not the only one important factor. In (Kroha and Reichel, 2007), we classified textual market news using supervised learning, i.e., an investor decided during the training phase which of news stories is a good or bad story. When using the trained and tested classifier we stated that the ratio of good stock market news to bad stock market news build a context, e.g., investors are more enthusiastic, and has a predictive quality (Kroha and Nienhold, 2010). We experimented with time series fractal features to influence the results of this classification. To decrease the box dimension we used exponential smoothing, to increase the box dimension we used added, scaled noise as we show in the next sections.

4 INCREASING FRACTAL DIMENSION

Our original idea is to influence the fractal dimension. To increase it we use noise. The key thoughts behind this idea are:

- The addition of (random) noise to a part of time series makes the time series more chaotic, resulting in appropriate fractal dimension changes.
- When noise is added to a time series with a very distinct trend, the trend may penetrate through the noise.
- Making a decision based on a noisy time series is harder, therefore the algorithm will decide more careful when a certain part of the time series is noisy.
- If we now selectively noisify parts of the time series, e.g., the older parts, we enforce careful behavior by the algorithm. This selection we called scaling the noise. It means that the noise amplitude is controlled, scaled by a given function.
Every noise variant is generated as a sequence \( U = (u_1, u_2, \ldots, u_n) \). Every scaling variant is generated as a sequence \( S = (s_1, s_2, \ldots, s_n) \). They both have the same size \( n \), the size of the original time series \( X = (x_1, x_2, \ldots, x_n) \) for which they were generated. Now, we generate a new time series \( X' \) defined as \( (x_1 + u_1 \cdot s_1, x_2 + u_2 \cdot s_2, \ldots, x_n + u_n \cdot s_n) \). In other words, we scale the noise and add it to the time series.

There exist several different kinds of noise, which differ by Hurst coefficient. We chose to use white noise for the first version of our implementation because it is easy to generate and random. White noise is a set of uncorrelated random variables with an expected value of zero and a constant variance. The random values should be uniformly distributed. There are several methods of generating white noise. We chose to use the Central Limit Theorem (Kuo, 1996) in our implementation.

The noise is used to influence the fractal calculations on the data, i.e., we do not influence the data values. However, we do not want the whole time series to be noisy, just that parts that follow a certain (predefined) criteria, e.g., the older parts. Therefore, as already described above, we had to scale the noise. We decided to experiment with the noise scaled according to linear function, Pareto function, and good news/bad news function based on results in (Kroha et al., 2006), (Kroha et al., 2007).

The scaling can be interpreted as a model of influence probability of historical market prices on the current price, i.e., the influence of older values weakens accordingly to the scaling function.

Linear scaling takes the form of a linear function with a dynamically adjusted gradient. The idea is, that at the beginning of the time series (at time zero), the linear function intersects the y-axis at exactly 1. Precisely at the last point of the time series the linear function should reach 0. The scales are another sequence of numbers with the same amount of data points as the time series itself. The scales are defined as \( S = (s_1, s_2, \ldots, s_n) \) with \( s_i = 1 - S \cdot (1/n) \cdot i \) or \( s_i = 0 \) when the equation results in a number below 0. In the equation \( S \), there is a constant scaling parameter and is used to influence the gradient of the linear function.

As basis for the Pareto scaling method, we use the Pareto probability density function. It can be calculated as such:

\[
f(x) = \frac{\alpha}{x_0^a} x^{-a-1}, \alpha, x_0 > 0
\]

\( \alpha \) and \( x_0 \) are both parameters that can be chosen freely. The Pareto density function equals 0 for all \( x < x_0 \). Then at \( x_0 \) it has the value of \( \frac{\alpha}{x_0^a} \). Afterwards it declines rapidly and approaches 0 without ever reaching it. Pareto scales can be heavily customized by changing both the \( \alpha \) and the \( x_0 \) parameter. Ideally, the Pareto scales are fitted for each time series, since this cannot be done all the time, you need to find parameters that work for all your time series. For example, if you want to apply Pareto scales to a number of time series, each consisting of 30 points, you would need to set an \( x_0 \) in the interval \([0, 30]\). Otherwise, your Pareto scale values would be always 0 or near zero.

As already discussed above, we wanted to include not only time dependent methods of scaling but methods that use prefabricated prediction results from other systems. As an example, we used our own implemented method - a grammar-based prediction derived from good and bad market news described in (Kroha and Reichel, 2007).

5 DECREASING FRAC TAL DIMENSION

In the previous part, we described how to increase the fractal dimension. In this section, we describe how to decrease fractal dimension.

Exponential smoothing is a well known weighted moving average for time series analysis. It is used either for smoothing a time series to appear more aesthetically pleasing, or as a simple form of prediction, based on past samples.

The calculation of a smoothened time series for an input time series \( X = (x_1, \ldots, x_n) \) is done as such:

\[
y_1 = x_1
\]

\[
y_t = \alpha x_t + (1 - \alpha) y_{t-1}, t > 1
\]

\( \alpha \) is the smoothing factor. If it is 1, each point of the smoothened time series does not include information of the past. When the value is near 0, the past information is included to the fullest.

The basic idea of exponential smoothing and our concept of changing fractal features of a time series using noise is similar. The smoothing process
Table 1: System without using noise.

<table>
<thead>
<tr>
<th>variant 1</th>
<th>variant 2</th>
<th>variant 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box Dim</td>
<td>1.489</td>
<td>1.488</td>
</tr>
<tr>
<td>Hurst correlation</td>
<td>-0.418</td>
<td>0.418</td>
</tr>
<tr>
<td>correlation</td>
<td>0.288</td>
<td>0.288</td>
</tr>
<tr>
<td>result decision</td>
<td>2.687</td>
<td>-13.040</td>
</tr>
<tr>
<td>decision</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>HOLD</td>
<td>SELL</td>
<td>HOLD</td>
</tr>
</tbody>
</table>

changes fractal features of a curve or a time series. Older data points are assumed to have an exponentially lesser influence on the future than newer ones because of the member \((1 - \alpha)^n\) in the definition. To present smoothing like an addition of a noise component we can imagine that smoothing adds a function containing "complementing asperities" to the time series so that the sum builds a smoothed curve.

6 Used Data and Experimental Results

We used the same input data as used in (Kroha and Lauschke, 2010) to make it possible to compare the two systems. This data consists of 53 stocks traded in Germany. We chose to follow every stock from its beginning onto the 21.07.2009, the day to which our data extends. The trading strategy we used is from our previous work (Kroha and Lauschke, 2010). Every 30 days the program analyses this last timespan (with and without noise) and makes a decision regarding hold, buy or sell. Since our noise-based approach is experimental, we have to find out the difference to the noise-free approach.

A calculation for each specific input variable combination (box dimension, Hurst, correlation coefficient) takes around 20 minutes of computing time, regardless of adding noise or not. We used a PC with the following features - AMD Athlon Dual Core Processor 4850e, 2500 MHz (2 cores), 2 GB RAM, SAMSUNG SP2504C ATA hard disk.

6.1 Time Series without Changes

To test the existence of a difference, we used the time series of the Adobe stock from 19.2.2001 to 30.3.2001, amounting to exactly 30 different data points. There are three variants of possible input parameters. The first variant consists of box dimension and correlation coefficient. The second variant consists of Hurst Coefficient and correlation coefficient. The third variant uses box dimension, Hurst coefficient and the correlation coefficient.

Table 1 shows the input values of all three combinations obtained from the fractal method as well as the \(\text{frst}_{\text{res}}\) decision value calculated by the Takagi-Sugeno fuzzy inference for the Adobe time series. The discrete decision value is calculated using a simple border test introduced in (Kroha and Lauschke, 2010). This means that in this particular case 1 all variants suggest to hold, except the Hurst correlation variant, which suggests to sell.

6.2 Increased Fractal Dimension

Now, we have to find out what influence the addition of timescaled noise has on the parameters of the process. The following experiment was done using scaled linear noise with a gradient of -2 over the whole time series. That means at the beginning the noise is the strongest and afterwards it declines rapidly. At the middle point of the time series, the noise stops completely. The decisions were SELL (Variant 1), HOLD (Variant 2), and HOLD (Variant 3). The box dimension was shifted from 1.5 to 1.7. A value near 2 means, the time series behaves randomly. As expected, the addition of timescaled noise (which is random) makes the time series more random. The correlation coefficient however, rises from 0.3 to 0.7 and therefore indicates a positive trend. The Hurst Exponent is rather indecisive with a value near 0.5.

In the case of the news scaled white noise, the decisions were HOLD (Variant 1), BUY (Variant 2), and HOLD (Variant 3). This is the only test where the algorithm actually proposed to buy stocks.

6.3 Decreased Fractal Dimension

The next experiment used an exponentially smoothed time series, whereas \(\alpha = 0.3\). As expected, the Hurst value changed in the direction of trends. Although it is not a big change, the low smoothing factor only took out the sharpest edges of the time series. The Box Dimension changed accordingly. It decreased, which implies that the new time series contains clearer trends. Correlation became more neutral, it did not detect any positive or negative trends.

On the examples above, we just showed that the decision signals change when applying scaled noise on a time series of security prices. The next question is whether it is in a common case of a time series the same and how much performance, i.e. profit, it brings.
Table 2: Ranked results according to profit earned.

<table>
<thead>
<tr>
<th>rank</th>
<th>profit</th>
<th>noise</th>
<th>scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>175,537</td>
<td>exponential smoothing</td>
<td>linear (factor 1)</td>
</tr>
<tr>
<td>2</td>
<td>174,386</td>
<td>none</td>
<td>none - original values</td>
</tr>
<tr>
<td>3</td>
<td>167,688</td>
<td>white noise</td>
<td>linear (factor 1)</td>
</tr>
<tr>
<td>4</td>
<td>142,388</td>
<td>white noise</td>
<td>news controlled scale</td>
</tr>
<tr>
<td>5</td>
<td>112,040</td>
<td>none</td>
<td>none - 10 day’s simple moving average</td>
</tr>
</tbody>
</table>

This topic is discussed in the next section.

7 EVALUATION AND ACHIEVED RESULTS

To decide whether the change of fractal features has brought any significant changes in decision we have to test how both methods perform in trading. Our experiments and results are shown in this section.

When we try to classify the quality of a prediction we have to think about the average case. The difficulty is that in the stock market there is no average situation against which a prediction algorithm could be measured. So, we decided to process time series of each of the 53 individual stocks and compare, which approach performs best. We implemented a simulation in which 10,000 Euros are invested into each of the 53 stocks at the beginning and we evaluate the results at the end of the time interval of 30 days.

The results of the test using the process without noise show that it was profitable in sum. The variant 1 of input parameters (box dimension/correlation) was the most profitable with a gain of 174,386. We experimented also with values of 10 day’s simple moving average. It leads to practically the same box dimension in each part of the time series, but its performance is much worse probably because of the delay.

Increasing fractal dimension by a linear scaling factor of 1 for linear scaled noise, the variant 3 (box dimension/Hurst/correlation) was the most profitable one, but with a gain only 167,688. Using factor 2, the highest gain was 133,065. Using Pareto function (for $\alpha = 0.3$ and $x_0 = 15$), the highest gain was 128,395 for the variant 1. The experiment with news scaled noise produced a good result, with a gain of roughly 142,000, it ranked 5 in the result comparison (see Table 2). This shows that third party scales can be implemented and used in our system.

Decreasing fractal dimension by exponential smoothing without scaling was less promising. The method achieved an (only average) gain of roughly 40,000 in the first two methods. Using box dimension, Hurst and correlation resulted in a loss of 1500.

Exponential smoothing seemed at first sight as a good method to include time information into the fractal analysis, but performed subpar. We did another test with linear scaled exponential smoothing. This experiment showed a vast improvement over the unscaled smoothing. With a gain of roughly 175,000 using variant 3 (box dimension/Hurst/correlation), this method performed better than the original approach, if only by a small amount.

When dealing with the stock market, there were several experiments that dealt with choosing stocks randomly, e.g. using a monkey as an investor. We also tried this approach to simulate random decisions. We did several tests, each with other random number generator seeds. We found that the random decision achieved a loss of 77,975 under the same conditions. This means that it vastly underperformed the fractal analysis approaches. None of the fractal analysis methods made as big as a loss as the random method.
8 CONCLUSIONS

As we can see in Table 2, the experiment based on exponential linear scaled exponential smoothing made a greater profit than the original approach without scaled noise, i.e., decreasing fractal dimension influenced the classification positively. In all cases, the best combination of input values was used. It has to be mentioned that real life trading has some additional aspects. First, investors usually avoid some stocks that do not have a promising performance, so they are not investing in all stocks and may achieve statistically better performance. Second, we did not involve fees and taxes paid when trading. This aspect can worsen the performance very heavily.

However, there are several parameters in the generation and scaling of noise that can be tuned. For example, using a time interval of only 20 days, we obtained much less gain. So, much more experimenting in the future is necessary.

Of course, we are aware of the fact that the processes running behind the markets are sometimes more, sometimes less chaotic, and because of that the optimization of some features may fail.

Summarized, the answer to the question given in our hypothesis at the very beginning is: Yes, fractal dimension decreasing can improve the results at least in some cases as shown above.

REFERENCES


