

TOWARDS AN INTELLIGENT SYSTEM FOR MONITORING ELECTRICAL ENERGY QUALITY

Foundations and Motivations

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Abstract: Electrical energy must be supplied in enough amount but with adequate quality. One of the components of electrical energy quality is the harmonic distortion. In this paper, we show an alternative way to measure distortion, mixing Data Envelopment Analysis (DEA) and Fourier Analysis. The technique here presented is specially useful for comparative analysis and is intended to be the basis for an intelligent system for monitoring electrical energy quality.

1 INTRODUCTION

One of the components of electrical energy quality is the proportion of harmonics in the electric signal. On the other hand, electronic equipments plugged in power lines generate harmonics. In real-life situations where a large number of such devices are plugged in power lines, the electric signal affected by harmonic distortion should be classified according to some degree of quality. Such classification helps to check whether electrical power facilities are acceptable and also to decide which devices have to be used with suitable filters.

Data Envelope Analysis (DEA) is a technique based on Linear Programming that is used to calculate the performance of operational units also known as Decision Maker Units (DMUs) in scenarios involving several inputs and outputs in a such a way that comparisons are difficult to be established. Such approach defines an index known as the relative efficiency for each DMU that results in a relative classification for each unit among all

others comprising the investigated group.

The method compares the DMU efficiencies in terms of capability of transforming inputs into outputs by means of a ratio between the output due to a particular input. In the end of the analysis, the method is able to decide which units are relatively efficient or inefficient.

In this paper, results of traditional theoretical studies in Power Electric waveforms are compared with those of DEA. The DEA modeling is carried out in a nonconventional way, once the defined inputs (Fourier coefficients) are considered the same for all DMUs (waveforms).

2 BASIC FOUNDATIONS

Mathematical tools used in this paper are shown in this section as well as the relationship between the series expansion of a periodic function and the harmonic distortion, and also the basis of Data Envelopment Analysis.

2.1 Harmonic Analysis

Harmonic analysis is based on the Fourier Series (FS), that for a function $F(x)$, defined on an interval $0 \leq x \leq 2T, \forall T > 0$, is given by equations (1), where

$$A_n = \frac{1}{L} \int_0^{2T} f(x) \cos \frac{n\pi x}{T} dx \quad B_n = \frac{1}{L} \int_0^{2T} f(x) \sin \frac{n\pi x}{T} dx .$$

$$\frac{A_0}{2} + \sum_{n=1}^{\infty} \left(A_n \cos \frac{n\pi x}{T} + B_n \sin \frac{n\pi x}{T} \right) \quad (1)$$

If $f(x)$ and $f'(x)$ are continuous and $f(x+2T) = f(x)$, then the series converges to $F(x)$. It is possible to determine the magnitude of the Harmonics (A_n and B_n) in terms of its order n (Folland, 1992).

2.2 Harmonic Distortion

Harmonic distortion is a paramount index indicating the quality of electrical energy. High harmonic distortion waveforms differ somewhat from those having a sinusoidal shape and moreover are danger to electric appliances. Usually harmonic distortion is defined in terms of the total harmonic distortion percentage (THD) that compares amplitudes of high frequency harmonics with that of the fundamental frequency that is given by the term for n equals 1 in the Fourier series expansion.

Equation (2) calculates THD where it is assumed that the distortion is evaluated for an electric waveform current (I) and the indexes in the sum are related to the harmonic number.

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1} \quad (2)$$

2.3 Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a linear programming based technique for measuring performances of predefined Decision Making Units (DMUs) in presence of multiple inputs and outputs where comparison is difficult. Such method calculates the relative efficiency of each of the DMUs among all others comprising the group in evaluation.

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The model used in (Chames, Cooper, and Rhodes, 1978) known as CCR, defines a piecewise linear, non parametric surface on the data and evaluates the DMUs efficiency over such surface. Such model is input oriented i.e. it minimizes the input number keeping the output values and assumes Constant Return to Scales (CRS) so that every input variation yields a proportional variation in the outputs. The problem is then to calculate the weights u_j and v_i such that the ratio of the outputs weighted sum over the inputs weighted sum is maximized.

The method should be repeated for each of the n DMUs so that the relative efficiency for each DMU is determined using the weights. The DMU model is shown in equation (3).

$$Max \ h_o = \frac{\sum_{j=1}^s u_j Y_{jo}}{\sum_{i=1}^r v_i X_{io}}$$

subject to

$$\frac{\sum_{j=1}^s u_j Y_{jk}}{\sum_{i=1}^r v_i X_{ik}} \leq 1, \quad k = 1, \dots, n$$

$$u_j \text{ and } v_i \geq 0 \quad \forall j, i$$

where

h_o – efficiency of DMU o

r – number of inputs

s – number of outputs

n – number of DMUs

Y_{jk} – output j produced by DMU k

X_{ik} – input i used by DMU k

u_j – weight for output j

v_i – weight for input i

The problem defined in equation (3) is known as fractional programming and can be linearized as shown in equation (4) and it is called Multiplier Problem.

$$\begin{aligned}
 & \text{Max } h_o = \sum_{j=1}^s u_j Y_{jo} \\
 & \text{subject to} \\
 & \sum_{i=1}^r v_i X_{ik} = 1 \\
 & \sum_{j=1}^s u_j Y_{jk} - \sum_{i=1}^r v_i X_{ik} \leq 0, \quad k = 1, \dots, n \\
 & u_j \text{ and } v_i \geq 0 \quad \forall j, i
 \end{aligned} \tag{4}$$

The Dual of the Multiplier Problem is known as Envelope Formulation. The solution of such problem yields, besides the values for the efficiencies, benchmarks for the inefficient DMUs. Although in management decision applications that is important information, in this paper application there is no such interest once the defined DMUs are mathematical entities that make no decision.

For the case in study, the model compares the DMUs efficiencies by their ability of transforming inputs into outputs, measuring the ratio between the produced product (output) and the input. The analysis results make possible to identify which units are relatively efficient and inefficient.

DEA has also been used to aid in defining Electric Energy Distribution targets (Pessanha et al., 2007), in evaluating the energy efficiency of cities and areas (Angulo-Meza et al., 2007 and, Soares de Mello et al., 2008) or in evaluating the performance of companies using a double perspective (Lins et al., 2007).

3 HARMONIC DISTORTION

Electrical energy have aspects that make it different from the other industrial products. It must be generated as it is used, it can not be stored by the users, nor carried through usual means of transportation and last but not least, its quality depends on both the user and the producer. Moreover, current electrical energy systems use more electronic than mechanic devices which make them easier to generate higher order harmonics due to their nonlinearity characteristics.

Currently, over 50 % of electrical energy runs on electronic devices. Although that contributed to an increase of industrial productivity and a more efficient use of electrical energy, it also changed the electrical energy quality requirements. While electromechanical systems are insensible to energy supply interruptions to the order of magnitude in

seconds, electronic systems are insensible to such interruptions to the order of magnitude in milliseconds, apart from showing sensitivity to voltage variations. Due to that larger sensitivity property, even typical procedures in electrical systems may cause interruptions in large automatic industrial unities, turning inefficient the standard evaluation indexes, as far as quality assessment is concerned.

Matters as what should be such new quality energy indexes and which actions should be taken in order to improve the quality of energy were not satisfactorily answered yet. The pressure for finding quick and efficient answers for such questions requires the use of automatic management systems and protection and filter equipments (Lima et al., 1994).

Harmonic voltage and current distortion is one of the problems usually found in electrical systems caused by highly nonlinear loads such as electronic converters, rectifiers, inverters, controllers, etc. Such high distortion indexes may cause problems such as capacitor bank faults, fuse and thyristor burn outs or even malfunctioning in electronic devices, particularly computers and electronic sensitive devices, fed by power lines degraded by the presence of harmonic distortion.

Simple solutions include the reduction of the operation time for such nonlinear loads or the use of filters. The former means production losses for the consumer whereas the latter equipment expenses. An accurate measure of the harmonic distortion allows a selective use of filters for reducing the expenses.

This paper deals only with harmonic distortion, but the quality of energy is concerned with other issues such as voltage drops and surges, frequency variations and blackouts, to mention a few.

3.1 Modeling by Harmonic Analysis

Table 1 shows the THD for typical functions used in engineering (Folland, 1992). Waveforms rank in terms of THD percentage will be later used as a basis of comparison to validate the obtained results using the DEA approach.

Table 1: THD (%) for some waveforms.

Waveform (Wav.)	% of THD
Triangular (T)	12.05
Half-wave rectification (HW)	21.75
Full-wave rectification (FW)	22.48
Square-wave (SW)	42.88
Sawtooth wave (S)	74.15
Controlled rectification (CR)	107.60

3.2 DEA based Calculations

DEA - CRS model with the ISYDS (Angulo-Meza et al., 2005) software yielded initially the results shown in Table 2. In such a model, the considered output is the RMS value for the fundamental frequency (F1) and the inputs, the RMS values for the harmonics up to order ten (H2 to H10) (Biondi Neto et al., 2003, and, Biondi Neto, 2001).

Table 2a: Initial results for DEA-CCR model.

DMUs	Inputs x 10 ⁻²					Eff. (%)
	H2	H3	H4	H5	H6	
HW	15.0	0	3.0	0	1.28	100
FW	0	0	6.0	0	2.57	100
SW	0	30.0	0	18.0	0	99.97
S	70.7	47.1	35.3	28.2	25.3	16.38
CW	16.7	11.2	6.10	3.70	3.90	11.29
T	0	10.0	0	3.6	0	100

Table 2b: Initial results for DEA-CCR model.

DMUs	Inputs x 10 ⁻²				Out. F1
	H7	H8	H9	H10	
HW	0.71	0	0	0.45	70.7
FW	0	1.42	0	0.9	30.0
SW	12.8	0	10.0	0.0	90.0
S	20.2	17.6	15.7	14.14	141.4
CW	3.70	2.80	2.20	2.20	20.90
T	1.83	0	1.11	0	90.03

Initial results show the presence of a large number of inputs for quite a few DMUs and also several null harmonics due to the properties of some waveforms, e.g. parity, which may mask the results. Moreover, there are many null inputs and so all the weights affect the units having the largest ratio output over input that is due to the fact that DEA is a benevolent model for the evaluated units.

There are several methods in the DEA literature to overcome those problems that avoid a proper waveform classification regarding their harmonic distortion, such as cross-evaluation (Sexton, 1986, and Doyle et al., 1994), inverted frontier (Yamada et al., 1994, Entani et al., 2002, Lins et al. 2005, Anderson et al., 2002, and, Soares de Mello et al., 2008), weights restrictions approach (Allen et al., 1997) and, selection of part of the set of variables to be considered in the model (Lins et al., 1999, and, Senra et al. , 2007).

A selection of variables may seem, at first sight, incoherent to traditional DEA modeling. Actually, DEA considers that all the DMU's use the same set

of inputs to produce the same set of outputs. However, one should consider that, the waveforms are not DMU's in the normal meaning of a DMU. Moreover, in this paper context, the inputs can be seen as the price one has to pay in terms of harmonics RMS values, in order to obtain the output, i.e. the fundamental RMS value. So, it makes sense consider, for each waveform, the harmonics that really cause distortion in particular those of least order. On most cases of convergent series, those harmonics show the largest values.

The results obtained using such approach are shown in Table 3. Now the results are more coherent with the application having in mind waveform distortion, since the previous detected problems are not seen making possible a plausible ranking in terms of such a distortion

Among the investigated waveforms, one can see that the triangular is the one who shows the least distortion. It should be stressed that its 100% efficiency does not mean that distortion is not present but shows that it is the best of all investigated waveforms. As a matter of fact, the sine waveform is the only one who produces no distortion. Actually, an eye inspection, indicates that the triangular waveform is the closest one to the sine waveform. The half-wave rectification shows a low distortion and all the others produce a large distortion or low efficiency particularly the controlled rectification waveform. If one of such waveforms is present in a distribution circuit, severe damages will certainly show up if filters are not used.

Several of the above methods show shortcomings. The first method results in a fixed weight model (Anderson et al., 2002) that disfigure the DEA method, the inverted frontier method is undermined by the large number of null variables, and the weights restrictions involve undesirable subjective aspects.

Table 3: Data and results for the second DEA-CCR model.

DMUs	Inputs		Output	Eff.(%)
	H1	H2		
HW	0.15	0.03	0.707	94.24
FW	0.06	0.0275	0.3	55.24
SW	0.3	0.18	0.9	33.32
S	0.707	0.471	1.414	22.21
CW	0.1670	0.12	0.309	13.9
T	0.10	0.036	0.9003	100

So the restriction of variables approach was selected. In this paper, the traditional DEA approach is substituted in a way that not all DMU's use the same input variables. The used inputs are the relevant harmonics for each investigated waveform. For each waveform, the first two non-zero harmonics (H1 and H2) are selected as inputs and the fundamental (F1) is the output.

Table 4 shows a comparison among the THD values and the DEA-CCR efficiency. Both results yield the same DMU ranking validating the approach used in this paper.

Table 4: THD and DEA-CCR Efficiency in percentage (%) for some waveforms.

Waveform	THD(%)	DEA-CCR Eff.
Triangular	12.05	100
Half-wave rectification	21.75	94,24
Full-wave rectification	22.48	55,54
Square-wave	42.88	33,32
Sawtooth wave	74.15	22,21
Controlled rectification	107.60	13,90

4 CONCLUSIONS

The use of DEA as an alternative way to measure the distortion combined with Fourier analysis techniques was shown to be very useful particularly to comparative analyses.

Although the THD and DEA methods produce nearly the same results, at least in the present theoretical study, there are reasons that justify the use of DEA instead of THD. The first reason is that DEA allows automatic calculations quite quick, particularly when they are jointly used with neural networks (Biondi, 2001, and, Biondi et al., 2004). The second regards the fact that DEA is a comparative method that can cause the decrease the number of used filters for compensating the harmonic distortion, indicating for a time instant which equipments need filters.

Finally, a mathematical comparison between the methods is needed. Although DEA can be considered a trivial linear fractional model, THD is a nonlinear model since involves a root square of a sum of squares. From a theoretical point of view, it should be stressed that the measure functions capabilities of DEA and THD methods are alike. First, both show only positive values. On the other hand, a linear version of DEA is a degree one homogeneous function. The THD numerator which comprises a square root of a homogeneous function of degree two, is also a homogeneous function of

degree one. Thus, for both ways of measure, the numerators show proportionality among variables and calculated values.

At last, it can be stressed that the fractional nonlinear version of DEA is a homogeneous function of zero degree as the final result of the THD method. Thus, the methods present a certain mathematical resemblance, although DEA is a comparative method and nondifferentiable whereas THD is an absolute method (noncomparative) and differentiable. A comparative form of THD would show several properties of a smoothed DEA frontier (Soares de Mello et al., 2002, and, Soares de Mello et al., 2004) for the investigated problem in this paper.

The results indicate that the ideas here presented could lead and inspire an intelligent system for monitoring energy quality more efficiently.

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