COMBINED BLIND EQUALIZATION AND CLASSIFICATION OF MULTIPLE SIGNALS

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Abstract: A multiuser automatic modulation classifier (MAMC) is an important component of a multiantenna cognitive radio (CR) receiver that helps the radio to better utilize the spectrum. MAMC identifies the modulation schemes of multiple users in a frequency band simultaneously. A multi-input-multi-output (MIMO) blind equalizer is another important component of a multiantenna CR receiver that improves symbol detection performance by reducing inter symbol interference (ISI) and inter user interference (IUI). In a CR scenario, it is preferable to also consider the performance of the automatic modulation classifier (AMC) while designing the blind equalizer. In this paper we propose a MIMO blind equalizer that improves the performance of both multiuser symbol detection and cumulant based MAMC.

1 INTRODUCTION

Cognitive radio (CR), introduced by Mitola (Haykin, 2005), is an emerging technology that has a wide range of military and civilian applications. For a CR operating military and public safety applications, there is no information available to the radio about signals present in the frequency band. AMC is a signal processing component that helps the CR identify the modulation format employed in the detected signal. Most of the AMC algorithms in the literature can classify only a single user present in a frequency band. The authors of this paper recently proposed a fourth order cumulant based MAMC in (Ramkumar and Bose, 2010b). The MAMC proposed in (Ramkumar and Bose, 2010b) requires multiple receiving antennas. The MAMC was developed for a more realistic multipath channel and no assumption about the transmission powers of the user was made. With multiple transmitting users and multiple receiving antennas, the overall setup can be viewed as a classical multiple input multiple output (MIMO) communication system and is depicted in Figure 1. Thus by using multiple receiving antennas apart from classifying signals from multiple users, the CR receiver can harness the benefits offered by traditional MIMO schemes. A novel blind MIMO channel estimation scheme is also proposed (Ramkumar and Bose, 2010b) which forms a integral part of the proposed multiuser AMC (refer to the block diagram of the MAMC in Figure 2).

Due to the presence of multiple signals in a frequency band, any transmitted signal is subjected to inter user interference (IUI). Also, the transmitted signals are subjected to inter symbol interference (ISI) due to multipath fading. Since there is no training sequence available in a CR scenario, MIMO blind equalizers are used to remove IUI and ISI. Both second order statistics (SOS) and higher order statistics (HOS) of the received signal are required to achieve MIMO blind equalization. Since HOS are used, MIMO blind equalizers have the potential to converge to a local minimum. Convergence of MIMO blind equalizer to local minimum not only affects symbol detection performance but also the performance of the MAMC. Typically, blind equalizers are designed to improve the symbol detection performance. In a CR, AMC is an important component and hence it is better to design a blind equalizer that improves the performance of both AMC and symbol detection. Two works in this direction are found in the literature. However, both works consider only a single user AMC and single input single output (SISO) blind equalizer. The first work is in (Wu and Wu, 2008), where a robust switching SISO blind equalizer is proposed that improves the performance of single user AMC. In the second work (Ramkumar and Bose, 2010a), the weights of the SISO blind equalizer are adapted in such a way that performance of the cumulants based single user is improved.
In this paper we propose a MIMO blind equalizer that improves the performance of both multiuser symbol detection and MAMC that was proposed in (Ramkumar and Bose, 2010b). In order to do so, we design a cost function that is related to the performance of the multiuser AMC and then choose the parameters of the blind equalizer such that the cost function is maximized. The overall block diagram of the proposed CR receiver is shown in Fig. 1. In the figure, we design the MIMO blind equalizer \( G(z^{-1}) \) by considering the performance of both symbol detection and MAMC. For designing the blind equalizer we also use the MIMO channel estimates provided by the MAMC.

The paper is organized as follows. In Section 2, we provide the channel assumptions and background theory. In Section 3, we briefly describe MAMC from (Ramkumar and Bose, 2010b). The cost function related to the performance of the MAMC is also developed in this section. In Section 4, we present the step by step procedure to design the MIMO blind equalizer. Simulation results are presented in Section 5, followed by the conclusion.

![Figure 1: Block diagram of the proposed system.](image)

## 2 BACKGROUND AND THEORY

As mentioned earlier, multiple receiving antennas are used for classifying signals from multiple users. Let \( l \) be the number of transmitting users and \( m \) be the number of receiving antennas and it is required that \( m > l \). Usually in a CR scenario, \( l \) is not known and needs to be estimated using algorithms like the one proposed in (Shi and Su, 2007). The multipath channel between the \( j^{th} \) user and \( l^{th} \) receiving antenna is denoted as \( h_{lj}(z^{-1}) \) and is given by

\[
h_{lj}(z^{-1}) = h_{lj}(0) + h_{lj}(1)z^{-1} + \ldots + h_{lj}(L)z^{-L},
\]

where \( L \) is the number of multipath components, \( z^{-1} \) is the unit delay operator and \( h_{lj}(k) \) (for \( k = 1, \ldots, L \)) is the fading coefficients of the corresponding multipaths. The overall system can now be represented by the following model

\[
y(i) = x(i) + w(i), \quad i = 0, 1, 2, \ldots
\]

\[
x(i) = H(z^{-1})s(i),
\]

where \( s(i) \) is the \( l \times 1 \) transmission vector whose elements \( s_k(i) (k = 1, 2, \ldots, l) \) denote the \( k^{th} \) transmitting user, \( y(i) \) is the \( m \times 1 \) reception vector whose elements \( y_k(i) (k = 1, 2, \ldots, m) \) denote the received signal at the \( k^{th} \) receiving antenna, \( w(i) \) denotes the \( m \times 1 \) noise vector and \( H(z^{-1}) \) is given by

\[
H(z^{-1}) = \begin{bmatrix}
    h_{11}(z^{-1}) & \cdots & h_{1l}(z^{-1}) \\
    \vdots & \ddots & \vdots \\
    h_{m1}(z^{-1}) & \cdots & h_{ml}(z^{-1})
\end{bmatrix}.
\]

Another representation of \( H(z^{-1}) \) used in this paper is

\[
H(z^{-1}) = \sum_{k=0}^{L} H_k z^{-k}
\]

where \( H_k \) (for \( k = 1, 2, \ldots, L \)) is the \( m \times l \) scalar matrix. We make the following assumptions regarding the system model (2).

**Assumption 1:** rank \( [H(z^{-1})] \) = \( l \), for all complex \( z \neq 0 \), i.e. \( H(z^{-1}) \) is irreducible.

Assumption 1 is valid for any practical wireless channel with reasonable spatial diversity. Also we assume that the signals transmitted by various users are uncorrelated and each element of the noise vector \( w(i) \) is zero mean white Gaussian with variance \( \sigma_w^2 \).

MIMO blind equalizers are used to recover the transmitted signal vector \( s(i) \) using only the received signal vector \( y(i) \) with no training sequence and knowledge of the channel transfer function \( H(z^{-1}) \). As mentioned earlier, in this paper we design a blind equalizer that takes into consideration the performance of the multiuser AMC. In order to do so, we consider the following theorem from (Tugnait, 1996).

**Theorem 1:**\( (Tugnait, 1996) \) For the system given in (2) under Assumption 1 there exists \( (l \times m) \) polynomial matrix \( G(z^{-1}) \) (not unique) such that

\[
G(z^{-1})H(z^{-1}) = I_l.
\]

Since \( G(z^{-1}) \) is not unique, we can choose \( G(z^{-1}) \) such that both symbol detection performance and MAMC performances are improved.

According to (Kailath, 1979) (Tugnait and Huang, 2000), \( G(z^{-1}) \) in (5) can be factorized as follows

\[
G(z^{-1}) = G_2(z^{-1}) \cdot G_1(z^{-1}),
\]

where \( G_2(z^{-1}) \) is a \( l \times m \) polynomial matrix and \( G_1(z^{-1}) \) is an arbitrary \( m \times m \) polynomial matrix with the condition \( \det[G_1(z^{-1})] \neq 0 \), for \( |z| \geq 1 \). Since
$G_1(z^{-1})$ is an arbitrary polynomial matrix, we design $G_1(z^{-1})$ such that the AMC performance is improved. To do so, we first construct a cost function $J_{amc}$ which is related to the performance of the multiuser AMC. We then choose the parameters of $G_1(z^{-1})$ such that $J_{amc}$ is maximized. The overall design of $G_1(z^{-1})$ can be viewed as the following constrained optimization problem

$$\max_{G_1(z^{-1})} J_{amc}$$

$$\text{s.t. } \det[G_1(z^{-1})] \neq 0, \text{ for } |z| \geq 1$$

The rest of the paper is about formulating the cost function $J_{amc}$ and solving for the polynomial matrices $G_1(z^{-1})$ and $G_2(z^{-1})$.

3 MULTIUSER AMC AND COST FUNCTION

In this section we briefly describe the multiuser AMC from (Ramkumar and Bose, 2010b). Normalized fourth order cumulants were used as a feature for multiuser AMC. For a complex random signal $v(n)$, one of the normalized fourth order cumulants is given by

$$\tilde{C}_{40v} = \frac{C_{40v}}{(C_{21v})^2},$$

where

$$C_{21v} = E(|v|^2) \text{ and } C_{40v} = E(v^4) - 3E(v^2)^2$$

For the multiuser system defined by (2), the relationship between the normalized cumulant values of each transmitting user and normalized cumulant values of the signals received at each receiving antenna is given by

$$\begin{bmatrix} \tilde{C}_{40v_1} \\ \vdots \\ \tilde{C}_{40v_m} \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \ldots & \gamma_l \\ \frac{\alpha_1}{\gamma_1} & \frac{\alpha_2}{\gamma_2} & \ldots & \frac{\alpha_l}{\gamma_l} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\gamma_1}{\gamma_m} & \frac{\gamma_2}{\gamma_m} & \ldots & \frac{\gamma_l}{\gamma_m} \end{bmatrix} \begin{bmatrix} \tilde{C}_{40y_1} \\ \vdots \\ \tilde{C}_{40y_m} \end{bmatrix}$$

(10)

or

$$\tilde{C}_{40y} = B\tilde{C}_{40v}$$

where $\tilde{C}_{40v_i}$ (for $i = 1, 2, \ldots m$) are the normalized cumulant values of each received signal. From (10) one can see that the normalized cumulant values of each received signal $\tilde{C}_{40y_i}$ (for $i = 1, 2, \ldots m$) is a weighted sum of the normalized cumulant values of all the transmitting users. The weighting coefficients are given by

$$w_{ij} = \frac{\gamma_j}{\alpha_i} \text{ (for } i = 1, 2, \ldots m, j = 1, 2, \ldots l)$$

(13)

Since the magnitude of weighting coefficients are less than one, the magnitude of the normalized cumulant values of the received signals are driven towards zero. This clustering makes it hard for
the classifier shown in Figure 2 to distinguish between the features. Thus the coefficients of the matrix polynomial $G_1(z^{-1})$ must be chosen in such a way that the features are unclustered. For this reason we propose the following cost function

$$J_{amc} = \sum_{j=1}^{m} |C_{40z_{ij}}|,$$  

where $x_2(i) = G_1(z^{-1})y(i)$ and $C_{40z_{ij}}$ is the cumulant value of the $j^{th}$ component in the vector signal $x_2(i)$. The above cost function maximizes the magnitude of the normalized cumulant values of the signals so that the classifier can distinguish between the features.

4 DESIGNING THE MATRIX POLYNOMIALS

In this section we propose the algorithm for designing the polynomials $G_1(z^{-1})$ and $G_2(z^{-1})$. We also present the overall step by step procedure for designing the blind equalizer. The cost function in (14) can be expressed as follows

$$J_{amc} = \sum_{j=1}^{m} |C_{40z_{ij}}| = J_1 + \ldots + J_m,$$  

where $J_i = |C_{40z_{ij}}|$ (for $i = 1 \ldots m$). Now we choose $G_1(z^{-1})$ to be the diagonal matrix given by

$$G_1(z^{-1}) = \text{diag} \{ C_1(z^{-1}), \ldots, C_m(z^{-1}) \},$$  

where the elements of diagonal matrix are the FIR filters given by

$$C_p(z^{-1}) = c_{p1}z^{-1} + \ldots + c_{pL1}z^{-L1} \quad (\text{for } p = 1 \ldots m)$$  

where $L1$ is the length of the filter and $c_{ij}$ (for $i = 1, \ldots, m; j = 1, \ldots, L1$) are the filter weights. Since $G_1(z^{-1})$ is chosen to be a diagonal matrix, the constraint on $G_1(z^{-1})$ (refer to (7)) implies that the FIR filter $C_p(z^{-1})$ (for $p = 1 \ldots m$) must be minimum phase. That is the filter must not have any zeros on or outside the unit circle. Let us denote the weight vector as $c_p = [c_{p1}, \ldots, c_{pL1}]$ (for $p = 1, \ldots, m$), then we use the following constrained gradient search technique for updating the weights. Due to the constraint on $G_1(z^{-1})$ we restrict the search space to the region where the weights form a minimum phase polynomial. Let $c_p(k)$ denote the coefficient vector during the iteration $k = 0, 1, 2, \ldots$

- **Step 1.** For $k = 0$ initialize $c_p(0)$ to a random value from the search space.
- **Step 2.** For $k = 1, 2, \ldots$ calculate the output of the filter

$$x_{2p}(n) = \sum_{m=0}^{L} c_p(m)y_p(n-m) \quad (\text{for } p = 1 \ldots m)$$  

- **Step 3.** Update the coefficient vector using the following equation

$$c_p(k) = c_p(k-1) - \mu \frac{\partial J_p}{\partial c_p} \quad \text{for } p = 1 \ldots m$$  

where $\mu$ is step size. The weights are updated only if the new weights lies in the search space. If not, repeat step 2.
- **Step 4.** If $J_p(c_p(k)) - J_p(c_p(k-1)) < \zeta$ terminate the iteration and go to step 5. If not, repeat step 2, where $\zeta$ is chosen to be a small number less than one.
- **Step 5.** Calculate the output $x_2(i)$ using $G_1(z^{-1})$. Now the cumulant features of the $(m \times 1)$ signal vector $x_2$ are maximized and not clustered around zero, therefore $x_2$ is given to the MAMC shown in Fig. 2 for classification. Let us denote

$$F(z^{-1}) = G_1(z^{-1})H(z^{-1}) = \sum_{k=0}^{L+L1-1} F_kz^{-k}.$$  

It can be seen from Fig. 2, that a blind MIMO channel estimator forms an integral part of the MAMC. Here we are not repeating the channel estimation algorithm (refer to (Tugnait, 1996) for a detailed explanation). Since $x_2(i)$ is fed to the MAMC, we obtain the estimate of the polynomial $F(z^{-1})$. Using the estimate of $F(z^{-1})$, we design $G_2(z^{-1})$ by solving the following equation

$$G_2(z^{-1})F(z^{-1}) = I_l,$$  

where $I_l$ is the $(l \times l)$ identity matrix. Let us denote $G_2(z^{-1})$ as

$$G_2(z^{-1}) = \sum_{k=0}^{L2-1} G_{2k}z^{-k},$$  

where $G_{2k}$ (for $k = 0, 2, \ldots, (L2-1)$) are the $l \times m$ scalar matrix. Now the solution to (21) is given by (Tugnait, 1996)

$$[G_{21} \ G_{22} \ G_{23} \ \ldots \ \ldots] = [h \ \ldots] S^l,$$  

where $S^l$ is the pseudo inverse of the $S$ matrix given by

$$S = \begin{bmatrix} F_0 & F_1 & F_2 & \ldots & \ldots \\ 0 & F_0 & F_1 & \ldots & \ldots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & F_0 \end{bmatrix}.$$  


4.1 Overall Classification and Equalization Algorithm

In this subsection we present the step by step implementation of the overall proposed system.

- **Step 1.** Given the \((m \times 1)\) received signal vector \(y(i)\) estimate the number of transmitting users \(l\) using the method proposed in (Shi and Su, 2007).

- **Step 2.** Choose the length of the matrix polynomials \(L1\) and \(L2\). Since the length of the channel impulse response is not known, choose a sufficiently large length so that the system is over modeled.

- **Step 3.** \(G_1(z^{-1})\) is chosen to be a diagonal matrix given by (16) and its coefficients are adapted using the gradient search algorithm given by (19).

- **Step 4.** The signal \(x_2(i)\) is sent to the MAMC for classification. The MAMC provides an estimate of the matrix polynomial \(F(z^{-1})\).

- **Step 5.** Using the estimated \(F(z^{-1})\), design the \((l \times m)\) matrix polynomial \(G_2(z^{-1})\) by solving (21). The output of \(G_2(z^{-1})\) is used for symbol detection.

5 PERFORMANCE ANALYSIS

In this section, we demonstrate the performance of the proposed MIMO blind equalizer using Monte Carlo simulation. Since the performance of the MAMC is also considered while designing the blind equalizer, we analyze the performance of both the MAMC and symbol detection. For the Monte Carlo simulation, 1,000 trials are considered.

5.1 MAMC Performance

In this subsection we demonstrate the performance of the MAMC using computer simulation. The performance measure considered is the probability of correct classification \(P_{cc}\). Suppose that there are \(l\) users and \(M\) modulation schemes which are denoted as \(\Omega = \{\Omega_1, \ldots, \Omega_M\}\). Then there are \(L_1 = M^l\) possible transmission scenarios denoted as \(D = \{d_1, \ldots, d_{L_1}\}\). The probability of correct classification \(P_{cc}\) is defined as

\[
P_{cc} = \sum_{i=1}^{L_1} P(d_i|d_i)P(d_i)
\]

where \(P(d_i)\) is the probability that the particular transmission scenario occurs and \(P(d_i|d_i)\) is the correct classification probability when scenario \(d_i\) has been transmitted. For the simulation we assume \(P(d_i) = \frac{1}{L_1}, \forall i\), where all scenarios are equally probable.

5.1.1 Two User Three Class Problem

In this experiment we consider \(l = 2\) transmitting users and \(m = 3\) receiving antennas. Each entry of the \(3 \times 2\) channel matrix is considered to be a three tap FIR filter whose coefficients are chosen randomly from a Gaussian distribution of unit variance. Three modulation schemes are considered for this experiment and they are \(\Omega = \{BPSK, QPSK, PSK(8)\}\). Since three modulation schemes are considered, there are nine possible scenarios. The Monte Carlo simulation results are summarized in Figure 3. In Figure 3, the curve labeled \(P_{cc1}\) shows the performance of the MAMC without the proposed blind equalizer. The curve labeled \(P_{cc2}\) illustrates the performance of the AMC using the proposed system.

Figure 3: Performance of the MAMC (Two user three class problem).

5.1.2 Two User Four Class Problem

This problem is the same as the previous one except four modulation schemes are considered. The modulation schemes considered are \(\Omega = \{BPSK, QAM(4), QAM(64), PSK(8)\}\). The Monte Carlo simulation results are summarized in Figure 4. In Figure 4 the curves labeled \(P_{cc1}\) and \(P_{cc2}\) have the same meaning as that of Figure 3.

Figure 4: Performance of the MAMC (Two user four class problem).

From Figure 3 and Figure 4 it can be seen that the proposed MIMO blind equalizer enhances the performance of the MAMC.
5.2 Symbol Detection Performance

In order to analyze the symbol detection performance, we consider the same 2-input/3-output FIR random channel considered in the previous experiment. The normalized mean square error (NMSE) and symbol error rate (SER) are taken as performance measures. The simulation results are shown in Figure 5 and Figure 6. In Figure 5 and Figure 6 the curve labeled $sd_1$ illustrates the symbol detection performance of the proposed system. The curve labeled $sd_2$ illustrates the symbol detection performance of equalizer when the channel impulse response is known (non-blind equalizer). From the figures it can be seen that the symbol detection performance of the proposed system is close to that of the non-blind MIMO equalizer.

![Figure 5](image1.png)  
**Figure 5:** Symbol detection performance of the proposed system (NMSE Vs SNR).

![Figure 6](image2.png)  
**Figure 6:** Symbol detection performance of the proposed system (SER Vs SNR).

6 CONCLUSIONS

In this paper we designed a MIMO blind equalizer that improves the performance of both cumulant based MAMC and symbol detection. The performance of proposed equalizer was analyzed using computer simulations and yielded promising results. Future work is to extend this concept to cyclic cumulant based MAMC.

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