

AFFINE SPHARM REGISTRATION

Neural Estimation of Affine Transformation in Spherical Domain

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Abstract: In this work we propose an algorithm to perform the affine 3D surface registration using the shape modeling based on **SPHERical HARMonic**: called SPHARM. In the existing SPHARM registration algorithms the alignment is obtained using the rotation properties, that allows to perform the 3D surface rotation transforming only the spherical coefficients. The major limit is that this approach aligns the surface only by rotation. We propose a method to generalize this solution without lose the advantage to perform whole the registration process in the spherical domain. An estimation of the coefficients transformation that guarantees an affinity in the spatial domain is obtained by regression, using a set of RBF networks. The description of the 3D surface with the spherical harmonic coefficients is brief but comprehensive and provides directly a metric of the shape similarity. Therefore, the registration is obtained aligning the SPHARM model thought the minimization of the root mean square distance between the coefficients vectors. Many experiments are performed to test the affine SPHARM registration algorithm which appears efficient and effective compared with a standard registration algorithm in the spatial domain.

1 INTRODUCTION

The 3D surface registration is dealt with extensively in machine vision and computer graphics literature. In the last few years a lot of techniques were proposed (Xiao et al., 2005). Particular attention was given on a parametric surface modeling. The most widely used technique employs a description of a radial or stellar surfaces $v(\theta, \varphi)$ with the spherical harmonic decomposition (Ballard and Brown, 1982). An extension of this work allows to describe more general 3D simply connected surface using three radial functions $v(\theta, \varphi) = x(\theta, \varphi), y(\theta, \varphi), z(\theta, \varphi)^T$ called SPHARM (SPHERical HARMonic modeling): (Brechtbühler et al., 1995).

One of the most important field of application of SPHARM is medical imaging, where a good registration allows to compare shapes from the patients acquired in different moments, or different patients, or a comparison with standard atlas. In this specific field an accurate and fast registration algorithm is required. In most cases little transformations can align two different anatomical surfaces, therefore approaches that solve efficiently and effectively the registration problem in this specific domain sacrificing the generality are still well-regarded. The spherical harmonic domain lends itself well to perform the

3D surface registration. The reasons are the following: the SPHARM allows a brief representation of the 3D shapes and directly provides a shape descriptor. Moreover, the correlation of the harmonic coefficient's degree and the level of detail of the description allows a hierarchical approach to the solution. Li Shen (Shen et al., 2007) proposed a spherical surface registration algorithm based on the minimization of the root mean squared distance (RMSD) between two SPHARM models. The major disadvantage is that this approach solves the alignment problem only by rotation, that is sufficient for shape analysis but not enough for registration.

In this paper we present a novel method based on neural networks for registration SPHARM model by affine transformations. To obtain an analytical form of the "affine transformation" of the coefficients is an hard task. Indeed the orthogonality of the bases is no longer guaranteed, hence the transformation of each coefficient is function of all the other. Proceeding from these considerations a numerical approach based on neural network was chosen. The registration is obtained through the minimization of the root mean squared distance (RMSD) between two SPHARM models by the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm (Head and Zerner, 1985). The whole registration process is developed in the

spherical domain ensuring the algorithm efficiency. The experimental results show a good regression capability of the network and a good level of generalization. The dimensionality reduction obtained through the SPHARM modeling and the goodness of the shape descriptor allow an efficient and effective affine registration algorithm. The standard SPHARM alignment algorithm and our extension of this work to perform affine registration are described in the following.

2 SPHARM REGISTRATION

The aim of the SPHARM registration technique is the use of the spherical parametrization of a 3D closed surface for the description of the moving shape and static template. Consider a 3D radial object represented by a set of vertices in the cartesian space $v = (x, y, z)$. The mapping of these vertices in the spherical domain $v(\theta, \varphi) = \rho$ where $\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi]$ is performed with surface parametrization (Floater and Hormann, 2005). The spherical homogeneous sampling of the space is obtained starting with an icosahedron and iteratively subdividing each triangle into four smaller triangles. A spherical surface can be decomposed in a set of orthogonal bases through an integral transformation. The synthesis functions is the following:

$$v(\theta, \varphi) = \sum_{l=0}^L \sum_{m=-l}^l c_l^m Y_l^m(\theta, \varphi) \quad (1)$$

SPHARM surface modeling of a radial object benefits of the rotation property. The rotation of a surface, defined through the three Euler angles (α, β, γ) can be compute directly in the spherical domain. If the spherical function represents a radial object, the coefficients rotation rotates, the parametrization and also the object. The possibility to rotate a surface only by rotating the harmonic expansion coefficients makes the SPHARM alignment algorithms very efficient but restricted only to the rigid transformations. The spherical description of a surface is intrinsically a metric of the shapes similarity. The surfaces alignment is obtained by aligning the SPHARM models minimizing the root mean squared distance (RMSD) between the harmonic coefficients.

$$RMSD = \sqrt{\frac{1}{4\pi} \sum_{l=0}^{L_{max}} \sum_{m=-l}^l \|c_{1,l}^m - c_{2,l}^m\|^2} \quad (2)$$

3 AFFINE SPHARM REGISTRATION

In this section our novel method is presented, aimed to generalize the SPHARM registration algorithm for affine transformations. To exploit the good features of SPHARM modeling is necessary to perform the registration in the spherical domain. To this purpose, a transformation of the spherical coefficients that guarantees an affine transformation in a space domain is necessary. Instead of finding an analytical solution, we attempt to solve the problem through a Radial Basis Function (RBF) Neural Network. The affinity is a class of linear transformations that maps variables in new variables, it consists of a linear transformation followed by a translation.

$$x' = Ax + t \quad (3)$$

To find the affine transformation in the SPHARM domain we start by considering, at first, only the rotation: as shown by Li Shan, all the coefficients $c_l^m(\alpha, \beta, \gamma)$ of the rotated surface are a linear combination of all the coefficients of the same order and lower degree.

$$c_l^m(\alpha, \beta, \gamma) = \sum_{n=-l}^l D_{mn}^l(\alpha, \beta, \gamma) c_l^n \quad (4)$$

Observing that the affinity is a linear transformation but don't preserve the orthogonality of the basis we can suppose that all the coefficients $c_l^m(\mathbf{a})$ of the surface after affine transformation are a linear combination of all the other coefficients.

$$c_l^m(\mathbf{a}) = \sum_{k=0}^{L'} \sum_{n=-k}^k T_{mn}^{lk}(\mathbf{a}) c_k^n \quad (5)$$

The analytical definition of the function $T_{nm}^{kl}(\mathbf{a})$ is a critical aspect and is not guaranteed a closed-form expression. To asses this problem the RBF networks were introduced to regress this function. One of the easiest and effective way to model regression is that of using a finite dimensional space of function spanned by a given basis. The RBF neural network solves the regression problem by this way with a very simple structure and, differently from other types of neural network, like Multy Layer Perceptron (MLP), with a faster training (Buhmann and Buhmann, 2003). Moreover, the RBF works well if is trained with many examples, as will be shown below, in this specific application, the ground truth set can be arbitrarily large. For each $c_l^m(\mathbf{a})$ one RBF network is involved. As mentioned above the generation of the training set is easy: let be $v'(\theta, \varphi)$ the surface $v(\theta, \varphi)$ after the affine transformation $A(\mathbf{a})$

we can extract the c_l^m of the original surface and the $c_l^m(\mathbf{a})$ of the transformed surface. The input pattern of the RBF network approximates the $c_l^m(\mathbf{a})$ is $[a_1, \dots, a_{12}, c_0^0, c_{-1}^1, c_0^1, c_1^1, \dots, c_l^l, c_l^l]$. The training set is composed of a series of affine transformations of the same object, the network can generalize only in the domain of the training shape and the affine transformations of this. Depending to the training set the network can be specialized to a particular affine deformations (scaling shearing, reflection...). The alignment process is performed, like in the SPHARM registration by Li Shen, minimizing the root mean squared distance (RMSD) between the harmonic coefficients (Eq. 2).

4 EXPERIMENT RESULTS

Several experiments, to assess the capability of the proposed method and to find the best network tuning are performed. Once we have established the best network configuration in terms of number of radial basis function and training cardinality, the performance are evaluated for the two steps:

- the performance of the neural estimation of affine transformation of coefficients: that involve the network capability to regress the function and generalize.
- the performance of affine SPHARM registration: that include the goodness of the SPHARM shape descriptor and the capability of the minimization algorithm to find the affine transformation that best aligns the 3D surfaces. Our algorithm is compared with a classical registration algorithm: Demon registration (Kroon and Slump, 2009) in terms of RMSE and execution time.

The networks are trained on 750 examples of affine transformation with scaling (s_x, s_y, s_z) from 0.8 to 1.2, shearing ($s_{hxy}, s_{hxz}, s_{hyx}, s_{hyz}, s_{hzx}, s_{hzy}$) from -0.2 to 0.2 and rotation angles (α, β, γ) from $-\frac{\pi}{10}$ to $\frac{\pi}{10}$. The RMSE is computed between the surface transformed in space domain through the application of the affine matrix and performing a bicubic interpolation and the surface obtained transforming the spherical coefficients with the network and reconstruction the surface with the synthesis equation (Eq. 1). The networks performance are tested, with the K-fold technique (10 experiments). In Figure 1 the results of one of the K-fold experiment is shown for the training and test set, the mean of RMSE is 0.95 for the first and 1.16 for the second. As expected the performances are better for the training set but not so considerably, that is a symptom of a good generalization.

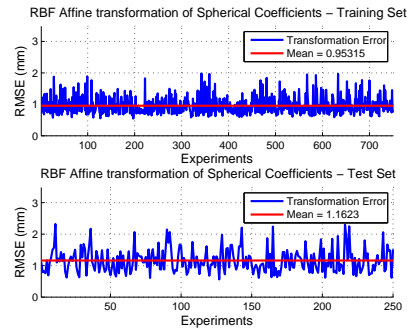


Figure 1: Neural estimation of affine coefficients transformation performance on training set and test set.

The performance of the Affine SPHARM registration are evaluated through the comparison with a standard registration algorithm the "MRI Modality transformation in Demon Registration" proposed by Kroon and Slump. The algorithm is based on a Thirlon Demon Registration (Thirion, 1998). The experiments are performed with different types of affine transformations: scaling and rotation Figure 2(a), scaling and shearing 2(b), and a general affine transformation 2(c). The performance are evaluated in term of RMSE and execution time. Note, the data used are randomly chosen in the test set.

Observing the mean of the registration error the good performance of the affine SPHARM registration is clear. The RMSE means in these experiments are always lower for our algorithm rather the demon registration. Moreover, differently to the Demon Registration the affine SPHARM performances appear invariant to the type of transformation.

In Figure 3 examples of surfaces alignment performed by the affine SPHARM registration are shown. In the three cases the RBF networks are trained with examples of the specific shape and affine transformations of these. These examples show how the performance are good also with more complex shape. Both the second and third examples belong to TOSCA data set (Toolbox for Surface Comparison and Analysis) (A. M. Bronstein, 2008): "david" and "centaur".

5 CONCLUSIONS

In this paper we propose an innovative method to solve the 3D surface registration based on the SHARM modeling. The results show good performance of the neural approach for the estimation of the coefficients transformation. The chosen RBF network allows a fast and easy training phase. Moreover the possibility of creating an arbitrary large training set

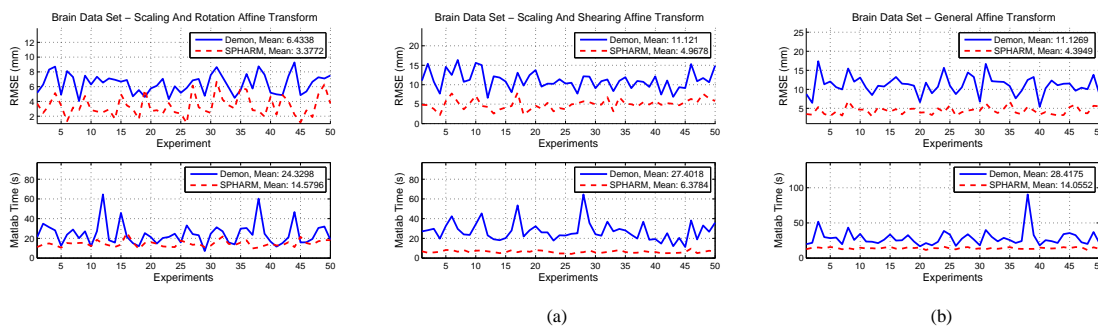


Figure 2: Performance comparison between Affine SPHARM Registration and Demon Registration in terms of registration error and execution time for different affinity class: (a) Scaling and Rotation (6 D.O.F.), Scaling and Shearing (9 D.O.F.), General Affinity (12 D.O.F.).

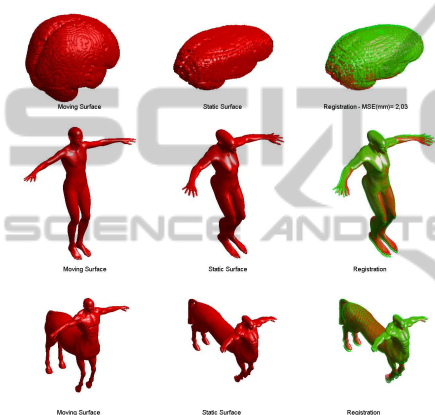


Figure 3: Examples of Surface registration: in the first column the moving surfaces, in the second the static surfaces and in the third the superposition of the static surfaces (red) and the registered surfaces (green) are shown.

or specializing it to a particular set of transformation, makes our approach very attractive. The experimental results shown good performance of the algorithm in terms of execution time and registration error for little deformation while the affine SPHARM performances worse in cases of big deformation. This problem is imputable to the loss of the basis orthogonality thought the affinity transform. In our future works we want to establish a theoretical limit to application of the affine SPHARM, and a method to solve this limitation.

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