

# CONDITIONS FOR LONG LASTING SUSTAINABLE INNOVATION IN AN AGENT-BASED MODEL

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**Abstract:** During the last decades, innovation has become a hot topic in a variety of socio-technological contexts: in particular, a key problem is that of understanding its origins. Moreover, scientists are not able to evaluate the sustainability of innovation processes, and it is difficult to discover what sort of conditions might lead to their crisis and even collapse. In this paper we present a model where agents are able to create new artifacts and can develop and enact strategies able to sustain innovation for very long periods. We discuss some results and make observations useful for understanding the processes and the strategies that sustain the growth of diversity in social and technological organizations.

## 1 INTRODUCTION

During the last several decades, innovation has become a hot topic in a variety of social and technological contexts, including technology itself, commerce, social systems, economic development, and policy construction. There is therefore a wide range of approaches to innovation in the literature (Fagerberg et al. 2004). In this paper we focus our attention on socio-technological systems where the changes are deliberately introduced by agents, which design artifacts on the basis of specific goals. In the systems we consider innovation is typically understood as the successful introduction of something new, as for example new objects, methods, techniques, or practices or new or modified products and services, whose functionality is determined endogenously, that is, within the system itself.

At this level of abstraction, the agents could be software agents interacting in artificial environments or (groups of) human beings or organizations in the real world. What is important is that the agents have the capacity, supported by their internal sophisticated cognitive and communication structures, of creating and modifying artifacts. Aim of this work is that of identifying the minimal structures and the strategies (if any) that the agents need in order to achieve a long lasting sustainable growth of the system.

Modeling such innovation processes is a difficult challenge, involving many non-linearly interacting elements. Indeed, human societies consist of large numbers of agents (human beings or organizations composed of human beings) involved in distributed sparse interactions, mediated by the presence of artifacts (tangible, as chairs and cars, or intangible, as languages and services). These interactions give rise to macroscopic regularities such as trading relationships, protocols, widely accepted duties or technological innovations, which in turn feed back into the structure of agents interactions. The result is a complex dynamical system composed of recurrent causal chains connecting agent behaviors, interaction networks, and collective outcomes. Similar patterns of interaction may emerge also in artificial worlds, in which sophisticated software agents engage in autonomous interaction streams, through which they seek to invent new kinds of artifact.

In order to integrate a new kind of artifact into the already existing patterns of interaction there must be a certain degree of convergence of agents' attributions about the new artifact's identity (that is, about its properties and functionalities). *Several* agents have to align themselves around its use, by building or modifying other artifacts in order to combine with it and in such a way support the new functionalities. If this happens, the invention becomes an effective innovation (that is, an object

embedded in patterns of use, potentially able to foster the growth of new active zones in artifact space). The agents can use artifacts created by other agents: this fact allows the occurrence of so-called exaptation processes (Gould and Verba, 1982), very often observed in socio-technological systems (Villani et al. 2007) (Villani et al. 2009) (Villani et al. 2010).

The reciprocal feedbacks between microstructure (agents and artifacts) and macrostructure (organizations) has long been explicitly recognized as of fundamental importance for social sciences (Hayek, 1948) (Olsen, 1975) (Schelling, 1978) (Smith, 1937), but they are relatively new topics in artificial agents research area. For long time scientists have lacked the tools to quantitatively model these feedbacks, nor could they deal with their complexity. The most salient characteristic of traditional quantitative models on these topics, derived from economic or physical researches, is their top-down construction: frameworks such as fixed decision rules, common knowledge, mean field and equilibrium assumptions occupied the greatest part of the researches. Face-to-face interactions among heterogeneous economic agents typically play no role, with the only exception of the highly stylized game tournaments (Fudenberg and Tirole, 1991) (Dutta 1999).

A major advance was the introduction of agent-based models (Lane 1993a) (Lane 1993b) (Epstein and Axtell, 1996) (Gilbert and Terna, 2000) (C.Cioffi-Revilla 2002) (Ormerod et al. 2002) (Axelrod and Tesfatsion, 2006). These models deal with the topic of coordination and cooperation among heterogeneous agents, often lacking a complete knowledge of the whole system; the models aim to bridge the gap between micro-level interactions and emerging patterns at the macro-level, avoiding the misleading “representative agent” micro-macro link.

Agent-based models are very useful tools, but many of them in the innovation context underestimate the specific role of agents and, even more, the *attributions* that agents make about artifact identity, as noted above (Lane and Maxfield 1997) (Lane et al. 2005). Agents and artifacts interact in complex ways, giving rise to the so called socio-technological systems.

In fact, one of the most intriguing observations on these kinds of system is the growth of the quantity and diversity of artifacts that agents use: over time, not only the quantity and the diversity of artifacts has grown, but also the number and kinds of organizations has increased. These two phenomena

are in reciprocal relationship (van der Leeuw et al. 2009); both phenomena contribute in important ways to the system’s information coordination and processing capabilities. In particular, in the actual world, a high rate of innovation seems to be a peculiar and fundamental key to sustain the systems itself.

But can the current explosion in number of artifacts and organizational forms continue indefinitely? How can agents lacking a global vision of the whole system coordinate their actions, in order to cooperate in building a coherent system? Are there agents’ strategies that favor a sustainable growth, and others that lead to system collapse? In order to address this question, we need to understand the dynamics of innovation processes.

In this paper we make use of an agent-based model, where the relationships among agents are mediated by the presence of artifacts. Agents endowed with a suitable internal structure survey the opportunities offered by their social and material environment to create new (kinds of) artifacts, which in turn change and shape the present pattern of interactions heavily influencing the emergence of new agents-artifacts (sub)systems. This kind of approach has already provided some interesting results highlighting the importance of relationships among agents, which can influence the information flows through the system (Lane et al. 2005) (Serra et al. 2009) (Villani et al. 2007) (Villani et al. 2008). In this paper, we describe four scenarios, which taken together indicate that the conditions enabling a long lasting sustainable growth are neither simple nor widespread. The paper is organized as follows. The second section provides a detailed introduction to the basic innovation model. The third section describes results obtained by exploring four different innovation theoretic scenarios; the fourth section presents some conclusions derived from simulations based upon these scenarios.

## 2 THE MODEL

### 2.1 Agents and Artifacts

There are numerous approaches to studying innovation dynamics, but few of them attempt to construct models in which the reciprocal causality between transformations in the space of artifacts and organization in the space of agents plays an essential role. Rather, most models assume that only artifacts matter (for example, theories of technological trajectories), whereas others are agent-centric, based

on the idea that creativity or knowledge is the key factor underlying innovation dynamics (Dosi 1982) (Schumpeter 1934) (March 1991).

The model with which this paper is concerned is based on a theory of innovation developed in (Lane and Maxfield 1997) (Lane and Maxfield 2005). It represents a simplified world, inhabited by highly abstract representations of real world agents, artifacts, and attributions. The aim of the model therefore is not to describe in detail a real innovation context providing quantitative predictions: rather, its purpose is that of identifying the feedbacks and the causal connections implicit in the theory and useful in describing certain kinds of qualitative behaviours of real innovation contexts.

A claim of the theory is that agents and artifacts are both important for innovation, because artifacts mediate interactions between agents, who in turn actively produce and manipulate the knowledge needed to make effective the artifacts' functionalities.

A key point is the representation of artifacts and their combinations. For modelling purposes, we have considered different alternatives: binary coding as in classifier systems;  $\lambda$ -calculus as in the Alchemy model (Fontana, 1992) (Fontana and Buss 1994); or simply numbers, either natural or real, with functions to describe interactions. What is required is that the space has an algebraic structure, and that suitable constructors can be defined to build new artifacts by combining existing ones. We concentrated on the integer number representation and the use of arithmetic or other simple functions as operators, because it is more compact than the binary representation and simpler than the  $\lambda$ -calculus.

Despite this very simple representation, the real meaning of an artifact is not trivial, since it is determined not by the thing "in itself," but by which agents use it, and for what. For that reason, in the model the same entity is representing:

- a type of artifact, i.e., the "idea" – or archetype – of the article the producer is making (for example, the platonic idea of a chair – or of the number "12" in a particular model run): the artifact "name" in the following;
- the artifact(s) a particular producer is making (the article a particular producer is making and offering to other agents): "article" in the following;
- a single artifact token (a single chair present in the stock of an article): "item" in the following.

The entities manipulated by our algorithm are the articles, which in the model have a unique identifier

and a stock.

The intelligent part of the system is embedded in the agents, endowed with sophisticated cognitive and communication capabilities. In particular, agents

1. can explore their environment (composed of articles and other agents);
2. can manipulate articles (in order to build other articles);
3. can choose their goal (a particular name);
4. can use their knowledge in order to reach their goals.

Agents' capabilities are finite; therefore, they are not manipulating all the articles present in their world, nor know the goals of other agents. The *role* of agents is defined by what they do, and by the other agents with whom they interact. Agents have not a complete information, and this situation heavily influences their behaviour. Agents have to identify useful goals and pursue them; in so doing, they may or may not collectively build a sustainable world.

Note that at this abstract level this description applies both to living systems and to totally artificial systems. The topic with which we are dealing therefore embraces the more general theme of coordinating many different agents that can manipulate and interact with their environment, also by introducing new objects. The new objects could be tangible or intangible; the model we present here however explores worlds where these objects (the artifacts) are countable - the simplest and most common situation.

Now we can describe the agents' internal structure. In this model we aim to identify the simplest set of structures and strategies needed to assure the agents' functionality, so an agent:

1. can detect the presence of (a subset of the) already existing articles and agents;
2. can manipulate some article by means of "recipes" (ordered sequences of article identifiers and production operators), producing other articles;
3. can identify goals, derived by its world knowledge;
4. can manipulate its recipes in order to build new recipes, producing the article that match their goals.

The most complicated structure owned by an agent is the recipe, the tool it uses to process the items it obtain from other agents in order to produce the items of its output articles. In the experiments reported here, recipes employ the arithmetic operators "+" and "-".

An agent can possess more than one recipe at the same time. If the stock of a particular recipe is lower than a desired level (10 items in the following simulations) the agent repeats the production action a finite number of times (whose maximum level here is set to 10). If the stock exceeds this level, the agent decreases production; finally, in each step all the recipes owned by one agent have to be produced at least once, if inputs are available (the agents need artifacts in order to survive). A recipe that during the last 15 time steps is not produced (because the needed input names are not available in the system) or not used (because no other agents have taken items from its stock) is removed from the simulation: in such a way “useless” articles disappear.

In order to achieve their goals, the agents can create new recipes. Once the goal is set, with probability  $p_{inn}$  the agents try to build a recipe able to produce an article having a name close (in some metric) to the goal. Several optimization strategies can be applied to this aim: in the implementation used here, the agents combine their existing recipes by means of genetic algorithms, whose fitness function is the inverse of the distance between the goal and the realized name. If the genetic algorithm is not able to create a recipe building an article whose name is within a given distance from the goal, the invention process fails and the agent doesn't reach the goal (see (Serra et al. 2009) for further details).

The goal is the name the agent is trying to realize; it can be maintained until the building of an article with the same name succeeds, or it can be changed at each step with a given probability  $P_{goal}$ . The agents could use several strategies in order to set their goal: in the following simulations the agents randomly keep the name of one of the already existing articles and occasionally mutate it (by multiplying it by the value  $C_{jump}$  the 30% of the times a new goal is set – an action that correspond to a “jump” in the artifacts' space). Note that in such a way the choice of the goal is influenced by the number of articles that have the same name: replicas therefore are not negligible.

The systems' environment is very simple, and is constituted by a set of articles (the “raw materials”) whose stock is unlimited, the interesting study of systems in which raw materials have a production limit being postponed to further works.

## 2.2 Dynamics and Novelty Generation

A typical run (see also (Serra et al. 2009) and (Vil-

lani et al 2007)):

1. creates of a set of initial conditions (N agents having 2 recipes each)
2. repeats until  $n\_passi \leq \max\_passi$ 
  - a. sets  $Count=0$
  - b. repeats until  $Count \leq N$ 
    - i. the random choice of an agent (add 1 to Count)
    - ii. the determination of a new goal (with probability  $P_{inn}$ ) and its realization
    - iii. the production of the actual agent's recipes
    - iv. the increase of Count by one unit
3. final visualizations

It is possible that some stocks become empty, since very often several recipes make us of the same articles; in this case these recipes have to change provider, by finding a new one producing an article with the same name and a non-empty stock. This process has several interesting consequences:

- new articles, just built, have the possibility of being used (so allowing their inclusion on the already existing patterns of interaction);
- articles having the same name could be realized by different recipes, combining different set of articles; a frequent change of providers allows therefore the existence of a highly heterogeneous mixture of artifacts, favouring high diversity in the systems;
- cycles composed of articles and agents can be formed, and can become the source of long-lasting patterns of interaction (each article of value for the next one).

The continuous creation of new recipes making use of the already existing articles, combined with the change of providers, allows the formation of new (groups of) cycles, stabilizing in this way these new parts of the system. As a first conclusion, the change of provider seems the key feature enabling (directly or indirectly) the stabilization of the innovations, by means of the consequent formation of cycles.

## 2.3 Typical Behaviors

The model provides the basic elements for a suitable description of the creation and stabilization of innovations. Table 1 and fig.1 show the parameters and a portrait of a typical scenario, where the number of artifacts (fig.1a), the diversity (the number of different names present at a given step - fig.1b), the typical recipes' production level (fig.1c)



and the diameter (the difference between the maximum and the minimum names - fig.1d) reach a stable situation.

Table 1: The main model parameters and their values in the standard case.

Variable	Value
Number of agents	40
Initial number of recipes per agent	2
$P_{inn}$	0.20
$P_{goal}$	1.00
Names of raw material	1,2
Maximum possible name	1000000
Threshold for the success in building a new recipe	10% of the absolute value of the goals' name
$C\_jump$	[0.5,6.0]

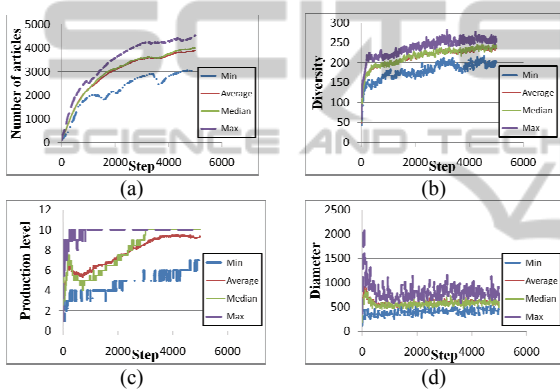


Figure 1: Behaviour of 10 systems initially composed of 40 agents, each agent having 2 recipes. Each plot reports the smallest, average, median and biggest value of each variable vs. time. (a) Number of articles; (b) diversity; (c) production levels; (d) diameter in the artifact space.

### 3 RESULTS

Fig.1 shows that the systems' diversity can undergo a considerable growth. However, there are strategies that can be adopted in order to significantly increase the number of articles and their diversity. We propose here four different scenarios, able to support different diversity levels. Besides the standard one ( $P_{stand}$  scenario, with  $p_{inn}=0.2$ ), we can significantly increase the agents' innovation probability ( $p_{inn}=1.0$ ,  $PI_0$  scenario), enable a feedback between a measure of the agents' size (as for example the number of their recipes) and the innovation probability ( $P_{chang}$  scenario); or finally we can compare these scenarios with a situation where the goal setting is random ( $p_{inn}=0.2$ ,  $P_{rand}$  scenario).

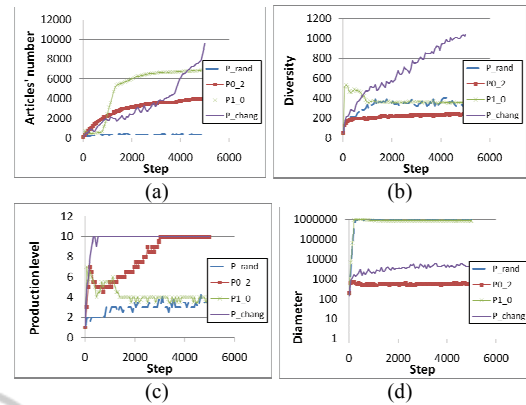


Figure 2: Median on 10 different runs of variables characterizing the 4 scenarios described in the text. (a) Number of articles; (b) diversity - the number of different names existing in the system; (c) the production levels; (d) the diameter in the artifact space (note the use of logarithmic scale in this plot).

As we can see in fig.2, the different scenarios:

- produce significantly different quantities of articles;
- support very different diversity levels;
- are able to maintain different recipes' production levels;
- explore different portion of the artifacts' space.

To these differences there correspond very different structures of the artifact space, as it is shown in fig.3 for typical runs.

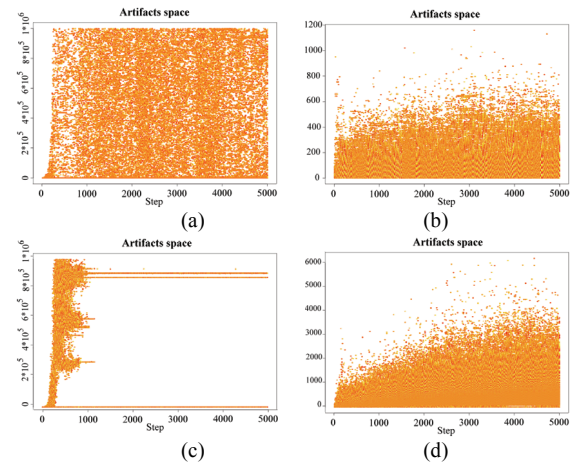


Figure 3: Artifact space of a typical run of the 4 scenarios described in the text. (a) random goals (b) standard situation; (c)  $p_{inn}=1.0$  scenario; (d)  $p_{inn}$  dependent on the number of recipes owned by each agent. Note the use of different scales in (a) and (c) cases.

From data in fig.2 and fig.3 we can draw some significant observations:

- the random system
  - has high diameter size,
  - uniformly covers the explored area,
  - support high diversity levels;
  - has low recipe production levels;
- the introduction of an imitative goal setting, with respect to the random system
  - strongly reduces the explored area;
  - reduces the diversity level;
  - allows high recipe production levels,
  - allows high number of articles;
- a very strong innovation
  - restores the previous diameter, diversity and recipes production levels
  - by maintaining a small covered area and shrinking it to some restricted “levels”;
- the feedback between agents’ size and innovation probability allows
  - long lasting periods of diversity growth;
  - long lasting periods of growth of the number of articles;
  - high numbers of articles
  - very high levels of article production.

Further clues can be inferred from table 2, which shows the efficiency of the scenarios in integrating the novelties into the already existing patterns of interactions. As we can see, in systems with high diameters, the innovation processes have high percentages of failure and produce a great number of articles that are not useful: artifacts too dispersed likely lead to the creation of other “outlier” artifacts not easily fitting with others. The systems where agents set the goal randomly have a very low article production, almost to the point that each article is a unique exemplar; the diversity is high, but so is the article turnover, because of the great difficulty in replacing inputs that have disappeared. The system wanders through artifact space. The goal setting strategy of *PI\_0* systems leads them to reduce the covered area despite the high diameter, but once the structure of artifact space reaches a kind of stationary condition, lots of agents’ innovation trials fail, because of the increased difficulty in realizing suitable recipes starting from a very sparse situation (note the emergence of levels, as partial reaction to this situation).

The combination of the diameter and the covered area in the artifact space seems therefore to play a

significant role in determining the main system behaviours. The strategies the agents adopt can influence the diameter: agents able to tune the diameter in the artifacts’ space are able to force the systems’ global behaviour, driving it toward high diversity or high production levels.

Table 2: Some results about the four scenarios presented in the text (averages on 10 runs). The “failures” column show the percentages of the unsuccessful innovation trials (there are no failures in *P\_rand* scenario, because of in absence of definite goals each single innovation is produced). The “unsuccessful successes” are articles that the agents were able to build, but that once made are not used by other agents and therefore disappear; the “successful successes” are articles that once made never disappears; the “medium successes” covers the remaining cases (the values in these last columns are percentages on the total number of articles made during the simulation). The table shows also the average final diversity of the scenarios, and the corresponding fraction on the total number of articles.

	Failures	Medium successes	Unsuccessful successes	Successful successes	Diversity	Diversity/#artifacts
<i>P_rand</i>	---	39%	60%	2%	320	0.86
<i>P_stanc</i>	26%	47%	40%	14%	240	0.06
<i>PI_0</i>	72%	27%	59%	14%	350	0.05
<i>P_chang</i>	26%	58%	34%	8%	1050	0.16

The only strategy supporting both a high number of articles and elevated production and diversity levels is the *P\_chang* scenario, where there is a feedback between agent size and probability of innovating, leading to the presence of heterogeneous agents (in fact, agents having different numbers of recipes innovate at different rates, increasing the already existing gap). We have simulated systems with each of these characteristics separately, but without obtaining any evident increase in the number of articles or the diversity. In particular, we simulated (a) worlds having from the beginning high heterogeneity in agents’ innovation probabilities (two groups of agents having  $p_{inn}=0.2$  and  $p_{inn}=1.0$ ), (b) worlds where the innovation probability increases in time and (c) worlds where the innovation probability is  $p_{inn}=1.0$  from the beginning. None of these variants sustain long lasting growth, whereas agents that develop high innovation activity after a soft growth phase are able to do so.

These remarks show the presence in the model of path dependent processes: in fact, agents having from the beginning  $p_{inn}=1.0$  are not able to sustain an endless growth, despite the high number of produced innovations, whereas agents whose innovation probability is linked to their recipes’ number (but

only during the first part of the simulation) are able to.

*P\_chang* worlds are able to efficiently recruit the new inventions, making them effective innovations (8%+58%=66% of the new articles are useful) – note that the superiority of the *P\_stand* systems in making “successful successes” (14%) is only apparent: in fact the total number of artifacts in the *P\_chang* worlds is overwhelming, as is their absolute number of completely successful innovations.

## 4 CONCLUSIONS

In this work we present an agent-based model of innovation, where agents and artifacts coevolve giving rise to a system where innovations take place and become integrated into the already existing pattern of interactions. The model captures the basic features of innovation processes, and allows the search for strategies able to support the system expansion, both in term of artifact space exploration and of the artifacts’ quantity and diversity.

Quite unexpectedly, strategies implying high innovation rates are not able to support long lasting increases in system diversity. A key aspect is that of the diameter of the explored area in the artifacts’ space: if too large, the artifacts match poorly, leading to poor worlds with very low production levels.

Systems having at the same time high diversity levels and high production levels require a kind of balance between exploration and exploitation processes. A too strong expansion in artifact space can lead to the building of artifacts that are not integrated with one another, whereas an intense propensity to build artifacts not so dissimilar to each other can limit the exploration range.

Another unexpected feature that strongly influences the integration of novelties into the already existing patterns of interaction is the change of providers, which allows:

- the existence of many agents with similar specializations;
- the simultaneous presence of several ways to build the same kind of objects (supporting in such a way high diversity levels);
- the stabilization of complete chains of integrated artifacts.

Systems without this peculiar feature cannot sustain or diffuse innovations.

A strategy able to provide a long lasting sustainable enrichment is that of varying the agents’ propensity to innovate, as for example by coupling it with a measure of the agents’ size. This strategy allows a complex interplay among each single agent and its environment (the other agents and the artifacts), bringing to growth path dependent processes.

We identify therefore several processes able to sustain diversity, and make some observations about the behaviour of the number and diversity of artifacts produced by groups of agents. Further work is needed to study the influence of artifact coding on the feedbacks discussed here and to analyze the formation of the structures in agent-artifact space that differentiate the proposed scenarios, in order to develop new strategies able to support long lasting and sustainable cascades of innovations.

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