SCHEDULING BASED UPON FREQUENCY TRANSITION
Following Agents Agreement in a NCS

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Abstract: This paper provides a strategy to schedule a kind of real-time distributed system base upon changes on frequency transmission of agents included into a distributed system. Modifications on frequency transmission (sensing periods) of system’s individual components impact on system quality performance due to limited computing resources. In this work we propose a dynamic linear time invariant model based upon frequency transmission and compute times of agent’s task which constitute a networked control system (NCS). Schedulability could be reached by controlling frequency transmission rates into a region bounded by minimum and maximum rates besides satisfy compute times. This idea is reinforced through a simulated case study based upon a helicopter simulation benchmark. It provides a good approximation of system response where main results are perform under a typical fault scenario for demonstration purposes.

1 INTRODUCTION

Nowadays distributed systems are widely used in the industrial and research. Current applications on Distributed Systems under time restrictions are Networked Control Systems (NCS) whose implementations consist of several agents which realize a part of control process and sensor/actuator activities work on a real time operating system and real time communication network. In order to achieve overall objectives of all tasks performed, it is necessary for all agents to exchange their own information through communication media properly. Therefore communication mechanisms play an important role on stability and performance (Lian, et. al., 2006). In a real-time system deterministic time requirements have to be scheduled. A task is periodic if it is time-triggered, with a regular release. The length of time between releases of successive jobs of task \( t_i \) is a constant, \( p_i \), which is called the period of the task. The deadline of each job is \( d_i \) time units after the release time. For a sporadic task there are constants \( c_i \) and \( p_i \) such that the sum of the compute times of all the jobs of \( t_i \) released in any interval of length \( p_i \) is bounded by \( c_i \). In many cases \( c_i \) is an upper bound on the compute time of each job and \( p_i \) is the minimum time between releases.

(Sha, et. al., 2004) mentions that Serling et. al. showed that a task is feasible if \( \sum_{i=1}^{n} \frac{c_i}{p_i} \leq \left( \frac{2^{2k} - 1}{2k - 1} \right) \).

Moreover, network scheduling is a priority in the design of a NCS when a group of agents are linked together through the available network resources. If there is no coordination among agents, data transmissions may occur simultaneously and someone has to back off to avoid collisions or bandwidth violations. This results in time delays or even failure to comply task’s deadlines. A good scheduling control algorithm tries to minimize this loss of system performance (Branicky, et. al., 2003), nevertheless there isn’t a global scheduler that guarantees an optimal system performance (Menéndez and Benitez, 2010). The use of a common-bus communication channel produces different forms of time delays uncertainty. (Lian et. al. 2001), (Lian et. al. 2002) have designed methodologies for networked agents to generate proper control actions and utilize communication bandwidth optimally. The effectiveness of the digital control (Figure 1) system depends on the sampling rate \( a \). A region which networked control
performance is acceptable deals with two points \( b \) and \( c \) associated to use of \( f_b \) and \( f_c \) sampling rates respectively which can be determined by characteristics an statistics of networked induced delays and device processing time delays. \( b \) implies that a certain level of control performance and it could be determined by case of study, as the sampling period gets smaller, the network traffic load becomes heavier and data loss increase in a bandwidth-limited network, time delays are longer.

\[ A \in \mathbb{R}^{n\times n} \] is the matrix of relationships between frequencies of the agents, \( B \in \mathbb{R}^{n\times n} \) is the scale frequencies matrix, \( C \in \mathbb{R}^{n\times n} \) is the matrix with frequencies ordered, \( x \in \mathbb{R}^n \) is a real frequencies vector, \( y \in \mathbb{R}^n \) is the vector of output frequencies. The input \( u = h(r-x) \in \mathbb{R}^n \) is a function of reference frequencies and real frequencies of the agents in the distributed system. It is important to note that relations between the frequencies of the \( n \) agents lead to the system (1) is schedulable with respect to the use of processors, that is, \( U = \sum_{i=1}^{n} \frac{c_i}{p_i} \).

Therefore it is possible to control the system through the input vector \( u \) such that the outputs \( y \) are in a region \( L \) non-linear where the system is schedulable. This is that during the time evolution of the system (1) the output frequencies could be stabilized by a controller within the schedulability region \( L \). This region could be unique or a set of subregions \( L_i \) in which each \( y_i \) converges. Figure 2 shows the dynamics of the frequency system and the desired effect by controlling it through a LQR controller and defining a common region \( L \) for a set of frequencies.

![Figure 1: Digital and Networked control performance.](image1.png)

![Figure 2: Frequencies controlled by a LQR controller into a schedulability region.](image2.png)

2 FREQUENCY TRANSITION MODEL

Let a distributed system with \( n \) agents that perform one task \( t_i \) with period \( p_i \) and consumption \( c_i \) each one, \( i = 1, 2, ..., n \). Network scheduling can be modeled as a linear time-invariant system whose state variables \( x_1, x_2, ..., x_n \) are the frequencies of transmission \( f_i = \frac{1}{p_i} \) of \( n \) agents involved on it. The authors assume that there is a relationship between frequencies \( f_1, f_2, f_3, ..., f_n \) and external input frequencies \( u_1, u_2, ..., u_n \) which serve as coefficients of the linear system:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

All agents of the system start with a frequency \( f_i \) and the controller modifies the period \( p_i = \frac{1}{f_i} \) of each task into a schedulable region. The real frequency \( f_i \) of the agent is modified to \( f_i' \), it means that \( p_i \) in time \( t_0 \) changes to \( p_i' \) at time \( t_1 \) to converge in a region where the system performance...
is close to optimal. The objective of controlling the frequency is to achieve coordination through the convergence of values.

### 2.1 Matrix Coefficients Proposal

Let a $a_{ij} \in A$ given by a function of minimal frequencies $f_m$ of node $i$ and $b_{ij} \in B$ given by a function of maximal frequencies $f_x$, this is, $a_{ij} = \varphi(f_{m1}^i, f_{m2}^i, ..., f_{m7}^i)$ and $b_{ij} = \psi(f_{x1}^i, f_{x2}^i, ..., f_{x7}^i)$. The control input is given by a function of the minimal frequencies and the real frequencies of agent $i$, it means $u = h(r - x) = k(f_m - f_x)$. Then, the system (1) can be written as:

$$\dot{x} = Ax + Bu$$

$$\dot{x} = Af_x + B(k(f_m - f_x))$$

(2)

### 3 CASE OF STUDY

The case of study is a prototype of a helicopter system integrated to a CanBus network with two propellers that are driven by DC motors. A description of of the helicopter can be found in (Quanser, 2006). Several Simulink models and MatLab script are used to build the helicopter dynamics model and it runs a simulation of the closed-loop response using the position controller.

Figure 3 shows closed-loop system simulation subsystem. Authors included a distributed system which performs a control close loop dynamic system based upon: sensor-controller-actuator and a centralized scheduler. Figure 3 shows the networked control system which consists of 8 processors with real-time kernel, connected by a network type CSM / AMP (CAN) with a rate of sending data of 1000000 bits / s and not likely to data loss.

These blocks of real-time kernel and network are simulated using Truetime (Ohlin, et. al., 2007). The first agent in the model, on the extreme left is the controller agent (Figure 3) that uses the values from sensors and compute control outputs. Sensor agents sample the analog signals. Two actuator agents located to the far right below (Figure 3) receives signals. Finally scheduler, main agent, above far right agent (Figure 3) organizes the activity of others 7 agents and it is responsible for periodic allocation bandwidth. 4 signals are measured to control helicopter fly: $\theta$ the pitch angle, $\psi$ the yaw angle, $\dot{\theta}$ pitch derivative, $\dot{\psi}$ yaw derivative.

(Tipsuwan and Chow, 2003) use optimal PI controller gains scheduled in real-time with respect to the monitored IP networked traffic conditions in order to maintain the best possible system performance and tries to capture changes in network traffic conditions.

In this work the authors focus on sensor agents and the objective is to control through system (1) the data frequency transmission. Each agent has a real transmission frequency and sets the minimum data frequency transmission. Each agent has a real and the objective is to control through system (1) the closed-loop response using the position controller.

Elements of the matrices for system (1) are defined as follows:

$$a_{ij} = \begin{cases} \frac{\lambda(f_m, f_m, ..., f_m)}{f_m} & i = j \\ \frac{\lambda}{f_m} & i \neq j \end{cases}$$

$$b_{ij} = \begin{cases} \frac{1}{f_m} & i = j \\ 0 & i \neq j \end{cases}$$

$$c_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$\lambda(f_m, f_m, ..., f_m)$ is the greatest common divisor of the minimum frequencies, we are going to write only $\lambda$. It is very important to consider the compute time of the task of each node $i$ as an additional state. Using (2) we can write (1) as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \lambda & f_m^1 & f_m^2 & f_m^3 \\ f_m^4 & f_m^5 & f_m^6 & f_m^7 \\ f_m^8 & f_m^9 & f_m^{10} & f_m^{11} \\ f_m^{12} & f_m^{13} & f_m^{14} & f_m^{15} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ f_m^1 \\ f_m^2 \\ f_m^3 \end{bmatrix}$$

Figure 3: Networked Control System included in helicopter model.
Table 2: Values of minimal, maximal and real frequencies and compute time.

<table>
<thead>
<tr>
<th>Agent</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Real</th>
<th>Consum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>40</td>
<td>55</td>
<td>0.001</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>30</td>
<td>50</td>
<td>0.001</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>10</td>
<td>25</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
<td>25</td>
<td>30</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The coefficient matrix A is:

\[
A = \begin{bmatrix}
0.125 & 0.750 & 0.250 & 0.625 & 0 \\
1.333 & 0.166 & 0.333 & 0.833 & 0 \\
4.000 & 3.000 & 5.000 & 2.500 & 0 \\
1.600 & 1.200 & 0.400 & 0.500 & 0 \\
0.001 & 0.001 & 0.001 & 0.001 & 1
\end{bmatrix}
\]

with eigenvalues \( \lambda_1 = 1.0000 \), \( \lambda_2 = 3.3308 \), \( \lambda_3 = -0.8556 \), \( \lambda_4 = -0.6835 \), \( \lambda_5 = -0.5000 \).

The system is unstable.

4.1 LQR Control

We chose weight matrices \( Q, R \in \mathbb{R}^{4x4} \) as follows:

\[
Q = \begin{bmatrix}
10 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 \\
0 & 0 & 10 & 0 \\
0 & 0 & 0 & 10
\end{bmatrix}
\]

\[
R = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

the gain \( K \) and \( A_c = (A - BK) \) are:

\[
K = \begin{bmatrix}
231.34 & 175.16 & 53.59 & 158.61 & -0.26 \\
189.95 & 144.00 & 52.23 & 130.28 & 0.07 \\
192.52 & 145.82 & 52.90 & 132.25 & 0.05 \\
-222.43 & -168.49 & -61.15 & 152.60 & 3.44 \\
-72.73 & -55.10 & -19.81 & -49.95 & 2.22
\end{bmatrix}
\]

\[
A_c = \begin{bmatrix}
-3.73 & -2.16 & -0.80 & -2.01 & 0.004 \\
-2.46 & -2.71 & -0.71 & -1.77 & -0.001 \\
-2.67 & -2.04 & -0.77 & -2.43 & -0.001 \\
-31865 & -241.38 & -87.76 & -21859 & -4.5374
\end{bmatrix}
\]

and eigenvalues \( \lambda_1 = -7.141 \), \( \lambda_2 = -3.330 \), \( \lambda_3 = -0.507 \), \( \lambda_4 = -0.857 \), \( \lambda_5 = -0.687 \).

Figure 4 shows the dynamics of the controlled system. The LQR controller modifies frequency transmission rate into schedulability region.

Figure 5 shows the effect of to modify transmission rate using frequency transmission.
model during 50 seconds. At the beginning sensor task periods are out of better performance interval, once passed 20 sec the model modifies dynamically frequency transmission of sensors.

Figure 5: Helicopter response using frequency transition model.

5 CONCLUSIONS

In this work, we have present a linear time invariant model of agent’s frequency transmission involved into a distributed system. The significance of control the frequencies stem from the system schedulability through information interchange between agents of distributed system. The key feature of LQR control approach is a simple design with good robustness and performance capabilities let to modify the frequencies easily. Authors have shown via numerical simulations the performance of the proposed control scheme using a helicopter prototype.

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