EVALUATING MAXIMUM TRANSMISSION UNRELIABILITY IN PERSISTENT CSMA PROTOCOL

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Abstract: The paper addresses the problem of evaluating the unreliability of transmission, undertaken by a given station, according to the persistent CSMA scheme. The unreliability of transmission is considered on the media access control level so it is defined by the probability that a given node participates in a collision. The presented results show that the maximum transmission unreliability is upper bounded by the persistence level ($p$), which is the main parameter of the protocol. The presented analysis is compared to the corresponding results for the non-persistent CSMA. As shown, both results are complementary because the maximum transmission unreliability in the non-persistent CSMA scheme is also bounded by the probability of choosing a single slot in the contention window.

1 INTRODUCTION

Although the carrier sense multiple access (CSMA) protocols have been introduced in the early 70s, due to their inherent flexibility and simplicity, they are in more advanced versions still widely used in contemporary networking, especially for wireless communication (e.g., Tay, Jamieson, Balakrishnan, 2004). In particular, the predictive CSMA protocol is employed in Local Operating Networks (LonWorks) commercial platform for sensor and control networking (Miśkowicz, Golanski, 2006). On the other hand, the non-persistent CSMA scheme with a geometric distribution has been recently proposed for sensor networking (Tay, Jamieson, Balakrishnan, 2004; Miśkowicz, 2009a; Egea-López et al., 2007).

The performance of the CSMA protocols have been investigated intensively for decades. The main criterion of performance analyses is evaluating the throughput-delay characteristics (Kleinrock, Tobagi, 1975; Lam, 1980), or the protocol energy efficiency in the context of wireless networking (Bruno, Conti, Gregori, E., 2002; Cali, Conti, Gregori, 2000; Bianchi, 1998), and especially in networked sensor and control systems (Miśkowicz, 2009b), the other class of performance analyses deal with the load scenario of finite number of active stations in which every station may produce a significant portion of network traffic. In order to model the network operation under heavy load, it is assumed that all the stations are in the asymptotic (saturation) conditions where they have always a packet ready for transmission.

The present study deals with the performance analysis of the persistent CSMA protocol that belongs to one of generic CSMA schemes introduced in (Kleinrock, Tobagi, 1975). The paper contribution is the analytical proof that the unreliability of transmission undertaken by a given station, according to the persistent CSMA scheme, is...
upper bounded by the persistence level \((p)\), which is the main parameter of the protocol.

The unreliability of transmission is considered on the media access control level so it is defined by the probability that a given node participates in a collision. The presented analysis is compared to the corresponding results for the non-persistent CSMA. As interesting, both results are complementary because the maximum transmission unreliability in the non-persistent CSMA scheme is also bounded by the probability of choosing a single slot in the contention window (Miśkowicz, Kościelnik, 2010).

The authors believe that the presented results contribute to better understanding of the persistent CSMA operation. To the best authors’ knowledge, these results have not been yet published.

The performance analysis stated in the present paper belongs to the studies of persistent CSMA scheme for the network staying in the asymptotic conditions because the evaluation of the maximum transmission unreliability needs to feed a channel with heavy load.

### 2 ANALYTICAL MODEL OF PERSISTENT CSMA

#### 2.1 Persistent CSMA Specification

The persistent CSMA scheme belongs to the slotted-CSMA protocol where the channel idle time is divided into fixed length intervals. All the stations in the network are synchronized and forced to start a transmission only at the beginning of a slot.

In the network that operates according to the persistent CSMA, when a station has a new message to transmit, it senses the channel. If the channel is detected to be idle, then it transmits a message with the probability \(p\), while with probability \(1-p\), it delays the message transmission to the next time slot. The slot duration is determined by the network propagation delay.

By a comparison, in the non-persistent CSMA, when the station senses the channel to be idle, it draws a number of a slot from a set of slots included in the contention window. The probability distribution of a random slot selection is uniform.

In the persistent CSMA protocol, the number of empty slots preceding a (successful or unsuccessful) transmission of a data packet is theoretically unbounded because the probability of starting transmission is defined by the geometric distribution where a success occurs with the probability \(p\), and a failure with the probability \((1-p)\). The mean number of trials undertaken by a given station equals \(1/p\). On the other hand, in the non-persistent CSMA protocol, the maximum number of empty slots before (successful or unsuccessful) transmission of a data packet equals \((W-1)\), and the mean number \((W-1)/2\) where \(W\) is a number of slots in the contention window.

#### 2.2 Collision Probability in Single Transmission Attempt

The probability \(p_{\text{coll}(1)}^{(k)}\) that a certain station is involved in collision in the \(k\)th transmission attempt is defined by the product of the following probabilities:

\[
P_{\text{coll}(1)}^{(k)} = P_{\text{coll}(1)}^{(k-1)}P_{\text{coll}(12)}^{(k)}
\]

where \(P_{\text{coll}(1)}^{(k)}\) is the probability that all the contending stations had not started the transmission in the previous 1,...,\(k\) transmission attempts, and \(P_{\text{coll}(12)}^{(k)}\) is the probability that at least one from the \(s=1,2,3,\ldots\) contending stations apart from a selected station starts the transmission in the \(k\)th transmission attempt.

The former probability \(P_{\text{coll}(1)}^{(k)}\) is given by the formula:

\[
P_{\text{coll}(1)}^{(k)} = \prod_{x=1}^{k-1} (1-p)^{x+1} = (1-p)^{(s+1)(k-1)}
\]

The latter probability \(P_{\text{coll}(12)}^{(k)}\) is defined as follows:

\[
P_{\text{coll}(12)}^{(k)} = p \sum_{x=1}^{s} C_s^x p^x (1-p)^{s-x}
\]

where \(C_s^x = s!/(s-x)!x!\) is the binomial coefficient, and \(s, s>1\) is an integer.

Thus, the \(P_{\text{coll}(1)}^{(k)}\) is given as:

\[
P_{\text{coll}(1)}^{(k)} = p (1-p)^{(s+1)(k-1)+s} \left[ \sum_{x=0}^{s} C_s^x \left( \frac{p}{1-p} \right)^x \right] -1
\]

The formula (4) may be transformed as follows:

\[
P_{\text{coll}(1)}^{(k)} = p (1-p)^{(s+1)(k-1)+s} \left[ \frac{1}{1-p} \right] -1
\]
As follows from (3), the probability \( p_{col}^{(k)} \) does not depend on \( k \) but only on the number of contenders \( s \). On the other hand, the probabilities \( p_{col}^{(1)} \) (see (2)), and consequently \( p_{col}^{(1)} \) also (see (5)), is a decreasing function of the number transmission attempt \( k \).

In Fig. 1, the plot of the probability \( p_{col}^{(k)} \) versus the number of transmission attempt \( k \) for selected numbers of the contending stations \( s=\{1,3,5\} \) for \( p=1/16 \) according to (5) is shown.

In Fig. 2(a,b), the plots of the probability \( p_{col}^{(k)} \) versus the population of the contending stations \( s \) for \( p=1/16=0.0625 \) and selected transmission attempts \( k=\{2,5,9\} \) (a), and \( k=\{1\} \) (b).

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As stated, the probability \( p_{col}^{(k)} \) that a certain station is involved in collision in the \( k \)th transmission attempt is defined by the formula (5).

The total probability \( p_{col}^{(1+k)} \) that a certain station participates in collision at most at the \( k \)th transmission attempt is defined as a sum:

\[
\sum_{x=0}^{s} C_x^s \left( \frac{p}{1-p} \right)^x \left( 1 - \left( \frac{p}{1-p} + 1 \right)^x \right) = \left( \frac{p}{1-p} + 1 \right)^s \tag{6}
\]

In particular, \( p_{col}^{(k)} \) reaches 0.0625 for \( p=1/16=0.0625 \) for high \( s \) as seen in Fig.2b.
The probability $P_{\text{coll}(l)}$ in the formula (9) is defined as a geometric series with the first term equal to $p[1-(1-p)^s]$, and the ratio equal to $(1-p)^{s+1}$ so it might be computed as:

$$P_{\text{coll}(l)}^{(s+1)} = p \frac{(1-(1-p)^s)(1-(1-p)^{s+1})}{1-(1-p)^{s+1}}$$

The plot of the probability $P_{\text{coll}(l)}^{(s+1)}$ versus $k$ according to (10) is shown in Fig. 3 for $p=1/16$ and $s=\{1,2,5,10\}$.  

As seen in Fig. 3, each curve approaches a horizontal asymptote with growing number of transmission attempt $k$. These asymptotes corresponding to the limits:

$$P_{\text{coll}(l)} = \lim_{k \to \infty} P_{\text{coll}(l)}^{(s+1)}$$

for various $s$ and denoted by $P_{\text{coll}(l)}$ defines the probability of collision in any attempt in a given transmission cycle.

By setting (10) to (11):

$$P_{\text{coll}(l)} = p \frac{1-(1-p)^s}{1-(1-p)^{s+1}}$$

As follows from (12), the $P_{\text{coll}(l)}$ depends both on the $p$ value and the number of contending stations $s$. The plots of $P_{\text{coll}(l)}$ versus the persistence level $p$ for various numbers of contending stations $s$ is presented in Fig. 4.

As seen in Fig. 4, the probability $P_{\text{coll}(l)}$ grows with increasing $p$ but it is at the same time smaller than $p$ for any number of contending stations $s$.

This conclusion may be also derived analytically on the basis of (12) as follows:

$$P_{\text{coll}(l)} = p \frac{1-(1-p)^s}{1-(1-p)^{s+1}} < p$$

because $\frac{1-(1-p)^s}{1-(1-p)^{s+1}} < 1$ for $s \geq 1$, and furthermore:

$$\lim_{s \to \infty} P_{\text{coll}(l)} = \lim_{s \to \infty} \frac{1-(1-p)^s}{1-(1-p)^{s+1}} = p$$

Thus, the probability $P_{\text{coll}(l)}$ of collision in any attempt in a given transmission cycle is upper bounded by the persistence level $p$ regardless of the number of contending stations $s$.
4 CONCLUSIONS

We compare the maximum transmission unreliability in the non-persistent CSMA and persistent CSMA for the same average number of contention slots in both schemes. In the persistent CSMA, the latter equals simply $1/p$. In the non-persistent CSMA, the contention window is constant in each transmission cycle and equals $W$ slots.

As proved in (Koscielnik, Miskowicz, 2010), the maximum probability of participating in a collision in the non-persistent CSMA scheme is upper bounded by $1/W$, that is, by the probability of a selection of a single slot in the transmission attempt.

On the other hand, as follows from the present paper, maximum probability of participating in a collision by a given station, according to the persistent CSMA scheme, is upper bounded by the persistence level ($p$), which a main parameter of the protocol. Thus, the complementary results defined by (14) are valid for the persistent CSMA scheme.

REFERENCES


