COMBINING TEMPORAL AND FREQUENCY BASED PREDICTION FOR EEG SIGNALS

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Abstract: This paper presents a novel approach for electroencephalogram (EEG) signal prediction. It combines temporal and frequency based prediction to achieve a good final prediction result. Artificial neural networks are used as the predictive model for signals both in the temporal and frequency domain. In frequency based prediction, the amplitude and the phase of the frequency response are predicted separately. Experiments were conducted on the prediction of EEG data recorded from 13 subjects. Eight performance measures were used to evaluate the performance of our proposed method. Experiment results show that the proposed combined prediction method gives the overall best performance compared with the temporal based prediction alone and the frequency based prediction alone.

1 INTRODUCTION

Time series prediction problem has wide range of research interest due to its diverse potential applications such as electroencephalogram (EEG) signal analysis, financial data prediction, and environmental monitoring. To measure brain activity, non-invasive EEG is one of the most important bio-signals and many researchers are working on EEG signal prediction.

Researchers have used time series prediction methods to check the linearity of EEG signals. They found that nonlinear properties are present in EEG signals and that some data are not predictable using linear stochastic system (Robert A. Stêpieñ, 2002). It was found that EEG recordings from subjects with schizophrenia contain some degree of determinism (low order chaotic), but are not completely deterministic and contain properties of nonlinearity (Ying-Jie Li, 2005). The linear EEG model cannot perfectly describe the spontaneous EEG that displays nonlinear phenomena (Ou Bai, 2000).

Time series prediction methods were also applied to find the occurrence of seizures from the EEG of epilepsy patients (Florian Mormann, 2007). EEG time series prediction also has been used to extract features for motor imagery task classification in Brain Computer Interfaces (Stefan Cososchi, 2006). EEG time series prediction pre-processing shows better performance compared with Common Spatial Pattern (Damien Coyle, 2008). From previous research, it is clear that EEG time series prediction has a high impact on medical and engineering applications.

Different algorithms for EEG signal prediction have been proposed to enhance the predictive model's convergence performance in the time domain, such as Least Square Support Vector Machine (LS-SVM), Support Vector Regression (SVR), Neuro-Fuzzy System, recurrent or time delay network, and feature selection methods such as mutual information based feature selection. (Nicholas I., 2009) Researchers also combine Principal Component Analysis (PCA) (Paul Cristea, 2008), Kernel PCA and SVM (Qisong Chen, 2008), Independent Component Analysis (ICA) (Juan M. Gorriz, 2003), for feature selection purpose in the time domain. Future EEG signal prediction is
It is necessary to predict the future brain activity in which users may have different stages of intention. In this work, the EEG to be predicted is recorded during a time in which tasks conditions are changing. At some points in time, subjects are responding to rewards or making decisions, making movements, or doing none of these things. To predict the EEG would be related to what task conditions the subject was performing at particular points in the prediction interval.

For nonlinear time series prediction, the future to some extent may be predicted, but the accuracy of the non-linear forecast falls off with increasing intervals of prediction time for uncorrelated noise (K.J. Blinowska, 1991). On the other hand, the EEG reflects thousands of simultaneous ongoing brain processes. The brain’s response to a single stimulus or event of interest is not usually visible in the EEG recording of a single trial. To see the brain response to the stimulus, many trials are typically averaged (Coles, 1996).

We propose a method for EEG signal prediction that combines temporal and frequency based prediction. In our problem, a segment of future EEG signals is predicted given some known values of the EEG signal in the past. Using only time domain data, prediction causes high prediction error for noise and for the model error. On the other hand, brain activities such as internal and external cognitive processing have great impact on particular frequency bands. This provides good motivation to perform the prediction in the frequency domain. The prediction from the temporal domain and the one from the frequency domain are combined by considering their performance in the training data.

The remainder of the paper is organized as follows. The proposed method is described in Section 2. Experiments and results are provided in Section 3. The conclusions and future work are stated in Section 4.

2 PROPOSED METHOD

2.1 Input Data Representation

Time series prediction is a well known problem for forecasting future value. One-step-ahead time series prediction can be presented by Equation (1):

\[ y_t = f(y_{t-1}, y_{t-2}, \ldots, y_{t-L}) \]  

where \( y_t \) is predicted based on the past \( L \) values in the time series.

For future brain activity or event prediction segments of multiple time samples are predicted. As illustrated in Figure 1, the time segment \( Y_M \) containing \( M \) sample points is predicted based on \( N \) sample points in the past.

\[ \begin{align*} y_1, y_2, \ldots, y_M \end{align*} \]

Figure 1: A time segment with \( M \) sample points to be predicted from \( N \) known sample points.

2.2 Overview of the Proposed System

Figure 2 presents a block diagram of the prediction process. After preprocessing, the EEG data are divided into training, validation and test sets. The trained predictor is then validated using the validation data set. Different predictors are trained for the temporal based prediction and the frequency based prediction separately. The resulting predictions are then combined using weights that optimize the results in the training data. Based on the validation performance, a set of weights is selected and used to generate the final result in the testing process.

![Figure 2: Block diagram of the prediction system.](image)

The raw EEG data are normalized by rescaling the signal to the range [0,1] to meet the high convergence in the neural network based training.
process. This pre-processing step is illustrated by Equation (2):

\[ Y_i = \frac{y_i - y_{\min}}{y_{\max} - y_{\min}} \]  

(2)

2.4 Proposed Prediction Algorithm

Figure 3 shows the workflow of our proposed prediction algorithm. Our proposed prediction algorithm is divided into two main steps. In the first step, the temporal domain data and its corresponding frequency domain data are predicted separately. In the frequency based prediction, Fast Fourier Transformation is used to convert the time signal into the frequency domain. From the frequency response data, the amplitude and the phase are computed. Two neural networks are built to predict the amplitude and the phase separately. In Figure 3, these predictors are shown in two blocks named Amplitude Predictor and Phase Predictor. The predicted frequency response data are reconstructed using the predicted amplitude and the predicted phase as shown in Equation (3).

\[ f(t) = \text{amplitude } (t) \times e^{ix \text{phase } (t)} \] 

(3)

Inverse Fast Fourier Transformation is applied to get the frequency based predicted data in time domain.

In the second step, the temporal and frequency based predicted data are combined by using weights obtained from the analysis of the prediction error for each frequency band of each predicted signal during the validation process.

2.5 Predictive Model

The Neural Network parameters are obtained using a two fold cross validation process. For the temporal and frequency based predictions, gradient descent with momentum and adaptive learning rate back-propagation is used. Parameters are optimized separately to get the best performance in each domain. Learning rate in the range 0.01-0.03 and momentum of 0.3-0.9 gives better performance. Iteration range is 2000-2500 to train the predictor. A three layered back-propagation neural network is used for the system. The number of the input and output nodes are equal to the known segment length and predicted segment length respectively. The number of hidden nodes is optimized both in temporal and frequency domain. The number optimized hidden nodes are in the range of 36-120. Log-sigmoid functions are used as transformation function.

Our proposed prediction algorithm is a general framework and it can work regardless of the predictive model to be applied. The proposed method is checked with the gradient decent learning without momentum and the overall performance is lower than the case of gradient decent with momentum. The neural network parameters such as the value of momentum and transformation function are varied and it is found that the performance is very similar with negligible difference.

2.6 Weighted Combining

From the predicted results of the temporal and frequency domains, the weights are optimized from the validation set. There are two possible ways to combine the temporal and frequency based predicted data: 1) in frequency domain and 2) in time domain. Combining in the frequency domain has the advantage of being able to put more emphasis in a particular frequency band if the corresponding prediction signal is shown to be more accurate. Next we will show how we calculate the weights.

Figure 4 illustrates the process for computing the weights used to combine the temporal-based predicted frequency response and the frequency-based predicted frequency response. Each frequency response is divided into \( n \) frequency bands. The frequency bands \( F_{Rt1}, F_{Rt2}, \ldots, F_{Rtn} \) represent the frequency response of the signal predicted in the temporal domain. The frequency bands \( F_{Rf1}, F_{Rf2}, \ldots, F_{Rfn} \) represent the frequency response of the signal predicted in the frequency domain. With the validation set, the ground truth frequency
response FRg₁, FRg₂, … , FRgn of the actual signal is known. During the validation, this ground truth information is compared with the prediction from the temporal domain and the prediction from the frequency domain at each frequency band.

Figure 4: Weight calculation and the combining process of predicted frequency response data.

Errors are then computed in order to determine how good the prediction is in each domain. The error $E_t$ denotes the error between the i-th frequency band of the temporal based predicted frequency response and that of the ground truth frequency response. Similarly, the error $E_f$ denotes the error between the i-th frequency band of the frequency based predicted frequency response and that of the ground truth frequency response. The smaller the error is, the better the corresponding prediction is.

The weights for combining the temporal based predicted frequency response and the frequency based predicted frequency response for the i-th frequency band are denoted by $W_t$ and $W_f$ respectively. These weights are calculated by Equations (4) and (5):

$$W_t = \frac{E_f}{E_t + E_f}$$  \hspace{1cm} (4)

$$W_f = 1 - W_t$$  \hspace{1cm} (5)

It can be seen from Equations (4) and (5) that if the error for a particular prediction method is small at a frequency band, then the corresponding weight will be set to be higher. For example, if temporal based prediction yields a small error $E_t$, then this means that the error from the frequency based prediction $E_f$ is relatively larger. From Equations (4) and (5), it can be observed that the weight $W_t$ will become larger than the $W_f$, thus putting more emphasis on the temporal based prediction.

After calculating the weights, the corresponding frequency bands are then multiplied and added to get the combine frequency band FRc₁, FRc₂, … , FRcn. Inverse Fourier Transformation is used to transform the combined response back to the time domain signal.

2.7 Performance Measures

Eight different performance measures are used to check the system performance. These performance measures are defined by Equations (6)-(13). For the first five measures MSE, NMSE, MAE, NMAE, MAPE, the larger the values are, the worse the performance is. For the last three measures SNR, PSNR, CCORR, the larger the values are, the better the performance is.

- **Mean Square Error (MSE)**
  $$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$  \hspace{1cm} (6)

- **Normalized Mean Square Error (NMSE)**
  $$NMSE = \frac{1}{\sigma_y} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$  \hspace{1cm} (7)

- **Mean Absolute Error (MAE)**
  $$MAE = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i|$$  \hspace{1cm} (8)

- **Normalized Mean Absolute Error (NMAE)**
  $$NMAE = \frac{1}{\sigma_y} \sum_{i=1}^{n} |y_i - \hat{y}_i|$$  \hspace{1cm} (9)

- **Mean Absolute Percentage Error (MAPE)**
  $$MAPE = \frac{100}{n} \sum_{i=1}^{n} \frac{|y_i - \hat{y}_i|}{|y_i|}$$  \hspace{1cm} (10)

- **Signal-to-Noise Ratio (SNR)**
  $$SNR = 10 \log_{10} \left( \frac{\sum y_i^2 / MSE}{\sigma_y^2} \right)$$  \hspace{1cm} (11)

- **Peak Signal-to-Noise Ratio (PSNR)**
  $$PSNR = 10 \log_{10} \left( \frac{\sum_{i=1}^{n} y_i^2}{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2} \right)$$  \hspace{1cm} (12)

- **Cross Correlation (CCORR)**
  $$CCORR = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})}{\sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2 \sum_{i=1}^{n} (\hat{y}_i - \bar{\hat{y}})^2}}$$  \hspace{1cm} (13)
2.8 Effect of Weight on Performance

We examine how the performance is affected by the number of frequency bands $n$ considered. Figure 5 shows the effect of number of frequency bands on the performance with different error measures. From the analysis of the results, we found that for all performance measures if the number of frequency bands increases then the performance also increases.

Figure 5: Performance with different number of frequency bands $n$.

3 EXPERIMENT AND RESULTS

3.1 Data Acquisition

The EEG signals were collected with a Biosemi EEG system with 10/20 international standard (http://www.biosemi.com) from a total of 13 young adult subjects. The sampling frequency ($f_s$) was 512Hz. The behavioural task was an instrumental reward-based learning task adapted for humans (Peterson et al., 2009), based on a primate study designed to examine the firing rates of dopamine cells during decision making (Morris et al., 2006). The task is a modification of the classic two-armed bandit (Robbins, 1952). Subjects were presented with a series of trials in which they chose abstract visual images with a possibility of accruing a small reward on each trial. The task consists of two phases of 256 trials of reference and decision. Subjects were first given a brief practice session, with eight reference and four decision trials. The practice stimuli were four simple geometric shapes that were different from any of the stimuli used in the actual experiment. There were no feedback signals or rewards in this practice session in order to avoid teaching any associations prior to the actual experiment. Table 1 shows the number of sample points as well as the total time in second in which the EEG signal for each of the 13 subjects is recorded.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Number of Sample Points ($f_s$=512Hz)</th>
<th>Time (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1285120</td>
<td>2510</td>
</tr>
<tr>
<td>2</td>
<td>1126400</td>
<td>2200</td>
</tr>
<tr>
<td>3</td>
<td>949760</td>
<td>1855</td>
</tr>
<tr>
<td>4</td>
<td>1172480</td>
<td>2290</td>
</tr>
<tr>
<td>5</td>
<td>1246720</td>
<td>2435</td>
</tr>
<tr>
<td>6</td>
<td>1141760</td>
<td>2230</td>
</tr>
<tr>
<td>7</td>
<td>1077760</td>
<td>2105</td>
</tr>
<tr>
<td>8</td>
<td>1100800</td>
<td>2150</td>
</tr>
<tr>
<td>9</td>
<td>1226240</td>
<td>2395</td>
</tr>
<tr>
<td>10</td>
<td>1044480</td>
<td>2040</td>
</tr>
<tr>
<td>11</td>
<td>1231360</td>
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<td>1044480</td>
<td>2040</td>
</tr>
<tr>
<td>13</td>
<td>1008640</td>
<td>1970</td>
</tr>
</tbody>
</table>

3.2 Data Preparation

The data for each subject are segmented into different sections as shown in Figure 6. Each section is further divided into two equal subsections, one used for training and the other used for validation purpose.

The unused portions of the data shown in Figure 6 are used to for testing. Figure 7 illustrates the test set generation for a subject.

After splitting the data for a subject, the training sets are processed for input into the predictive model (Neural Network in our case). Based on the $N$ known sample points, $M$ sample points are to be predicted.

Figure 6: Training and validation set splitting for a subject.
Eight different pairs of parameters \((N, M)\) are used to check the performance of the proposed method: \((128, 32), (128, 64), (256, 32), (256, 64), (256, 128), (512, 64), (512, 128), (512, 256)\).

### 3.3 Results

The performance averaged over all subjects with different values of \((N, M)\) using our proposed method is shown in Figure 8. Temporal based prediction performance is better than the frequency based prediction for MSE, MAE, SNR and CCORR. On the other hand, frequency based prediction gives better performance for NMSE, NMAE, MAPE, and PSNR. It can be seen from Figure 8 that the case with \(N=128\) and \(M=32\) gives the best average result. It can also be observed that the performance degrades when the number of samples to be predicted becomes larger, i.e., when \(M\) is larger. An example prediction with a large value of \(M=256\) is shown in Figure 11 (Appendix). Another example prediction with a small value of \(M=32\) is shown in Figure 12 (Appendix).

Figure 9 compares the performance with different values of \(M\) \((M=32, 64, 128)\) at a fixed value of \(N=256\). From the analysis of the results, we found that for long segments, frequency based prediction gives better performance than temporal based prediction. For example, with the measures NMSE, NMAE and CCORR, frequency based prediction gives better performance with \(N=256\) and \(M=128\). With the measures MSE and MAE, temporal based prediction gives better performance in this case. The other three measures SNR, PSNR and MAPE give similar performance in both temporal and frequency based prediction. Similar results are found from the analysis of the cases \((N=512\) and \(M=64, 128, 256)\) shown in Figure 13 (Appendix).

Figure 10 shows the performance averaged among all subjects and among all the 8 parameter pairs of \((N, M)\).

It can be observed from Figure 10 that the performance of proposed combined prediction approach is better than the performance with the temporal based prediction or the frequency based prediction alone with all the 8 measures. Frequency based prediction gives better performance than temporal based prediction for the performance measures NMSE, MAE, NMAE, SNR and PSNR.
Figure 10: Performance averaged over all subjects and all parameter pairs ($N, M$).

For the other three measures MSE, MAE and CCORR, temporal based prediction gives better performance than frequency based prediction.

### 3.4 Statistical Test

The t-tests (one tailed and paired) were performed to test the statistical significance of the final results of the eight different performance measures. We tested the significance of differences between $S_{td}$-$S_{fd}$ and $S_{rd}$-$S_{cd}$ pairs, where $S_{td}$, $S_{fd}$ and $S_{cd}$ are performance of all subjects in temporal, frequency and combined domain based prediction results, respectively. The differences in test scores had approximately normal distributions. A significance level of $\alpha=0.1$ was used. Table 2 shows the $t$-values and $p$-values for different performance measures. We have accepted most of the measures, because in most of the cases $p<0.1$. Subscript of error measures R and L represent right and left tailed test respectively.

### Table 2: Statistical t-Test Result.

<table>
<thead>
<tr>
<th>Error Measure</th>
<th>$t$-Value ($df=12$)</th>
<th>$p$-Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{td}$-$S_{fd}$</td>
<td>$S_{rd}$-$S_{cd}$</td>
<td>$S_{rd}$-$S_{cd}$</td>
</tr>
<tr>
<td>MSE$_{E}$</td>
<td>3.037</td>
<td>1.502</td>
</tr>
<tr>
<td>NMSE$_{E}$</td>
<td>5.003</td>
<td>1.662</td>
</tr>
<tr>
<td>MAE$_{E}$</td>
<td>6.003</td>
<td>1.647</td>
</tr>
<tr>
<td>NMAE$_{E}$</td>
<td>4.191</td>
<td>0.629</td>
</tr>
<tr>
<td>MAPE$_{E}$</td>
<td>4.611</td>
<td>0.935</td>
</tr>
<tr>
<td>SNR$_{L}$</td>
<td>4.429</td>
<td>1.612</td>
</tr>
<tr>
<td>PSNR$_{L}$</td>
<td>4.427</td>
<td>1.612</td>
</tr>
<tr>
<td>CCORR$_{I}$</td>
<td>3.103</td>
<td>3.422</td>
</tr>
</tbody>
</table>

### 4 CONCLUSIONS AND FUTURE WORK

In this paper, we propose a method for predicting time series data. Our approach works by combining temporal based prediction and frequency based prediction. We apply our proposed method to the prediction of EEG signals recorded from 13 subjects. From the experiments, it is found that frequency based prediction gives better performance than the temporal prediction and that the combined final result gives the best performance. In our experiments, eight different performance measures were used to evaluate the performance since different performance measure may be preferred in different applications.

In future studies, we will apply the system to predict future brain activity, future user intention for decision-making and arm movements in an instrumental reward-based learning task. We will also use different methods of signal decomposition to achieve better prediction performance.

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### REFERENCES


http://www.biosemi.com


**APPENDIX**

![Figure 11: Prediction result of a longer segment (M=256).](image)

![Figure 12: Prediction result of a shorter segment (M=32).](image)

![Figure 13: Performance comparison for different values of M at a fixed value of N=512.](image)