C-FUZZY DECISION TREES IN DEFAULT PREDICTION OF SMALL ENTERPRISES

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Abstract: This work uses fuzzy c-tree in order to predict default in small and medium enterprises in Brazil, using indexes that reflect the financial situation of enterprise, such as profitable capability, operating efficiency, repayment capability and situation of enterprise’s cash flow, etc. Fuzzy c-trees are based on information granules—multivariable entities characterized by high homogeneity (low variability). The results are compared with those produced by the “standard” version of the decision tree, the C4.5 tree. The experimental study illustrates a better performance of the C-tree.

1 INTRODUCTION

The issue of credit availability to small firms has garnered world-wide concern recently. Small and Medium Enterprises (SMEs) are almost 99% of the total number of firms in Brazil, and they offer 78% of the jobs in the country. But, around 80% of SMEs is shut down before one year of activity. Many public and financial institutions launch each year plans in order to sustain this essential player of nation economies (Altman, Sabato, 2006). Borrowing remains undoubtedly the most important source of external SME financing.

SMEs in Brazil share some characteristics with the private individuals:

− Large number of applications
− Small profit margins
− Irregular available information (especially for the micro companies).

Small firms may be particularly vulnerable because they are often so informationally opaque, and the informational wedge between insiders and outsiders tends to be more acute for small companies, which makes the provision of external finance particularly challenging (Berger, Udell, 2002). Some financial ratios are used in the context of default prediction in small and micro firms operating in a state of Brazil and we choose some of them, as described in Section 3.

Although the enterprise’s wish of returning loan, which is represented by the rate of returning interests, we often don’t have any information about the amount of interests that has been repaid by enterprises that are requiring a loan for the first time. In this case, the prediction of default relies on information in the balance sheet of these enterprises. Decision trees (Quinlan, 1986) are the commonly used architectures of machine learning and classification systems, particularly in default prediction or scoring. They come with a comprehensive list of various training and pruning schemes, a diversity of discretization (quantization) algorithms, and a series of detailed learning refinements. In spite of such variety of the underlying development activities, one can easily find several fundamental properties that are common in the entire spectrum of the decision trees. First, the trees operate on discrete attributes that assume a finite (usually quite small) number of values. Second, in the design procedure, one attribute is chosen at a time. More specifically, one selects the most “discriminative” attribute and expands (grows) the tree by adding the node whose attribute’s values are located at the branches originating from this node. The discriminatory power of the attribute
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2 CLUSTER DECISION TREE

The architecture of the cluster–based decision tree develops around fuzzy clusters that are treated as generic building blocks of the tree. The training data set \( X = \{x(k), y(k)\}, k=1,2, \ldots, N \), where \( x(k) \in \mathbb{R}^d \) is clustered into clusters so that the similar data points are put together. These clusters are completely characterized by their prototypes (centroids). They are positioned at top nodes of the tree structure. The way of building the clusters implies a specific way in which we allocate elements of \( X \) to each of them. Each cluster comes with a subset of \( X \), namely \( X_1, X_2, \ldots, X_c \). The process of growing the tree is guided by a certain heterogeneity criterion that quantifies a diversity of the data (with respect to the output variable \( y \)) falling under the given cluster (node). We can choose the nodes with the highest heterogeneity values and treat them as candidates for further refinement.

The process is repeated by selecting the most heterogeneous node out of all final nodes. The growth of the tree is carried out by expanding the nodes and building their consecutive levels that capture more details of the structure. It is noticeable that the node expansion leads to the increase in either the depth or width (breadth) of the tree. The pattern of the growth is very much implied by the characteristics of the data as well as influenced by the number of the clusters.

2.1 Tree Development

Fuzzy clustering is a core functional part of the overall tree. It builds the clusters and the standard fuzzy C-means (FCM) (Bezdek, 1981) is used.

For the purpose of clustering the ordered pairs \( \{x(k),y(k)\} \) are concatenated. This implies that the clustering takes place in the \((d+1)\) dimensional space and involves the data distributed in the input and output space. Likewise, the resulting prototype \((f)\) is positioned in \( \mathbb{R}^{d+1} \). The coordinates of the prototype are split into two parts as follows:

\[
\begin{align*}
\mathbf{v}_i &= \{v_{i1}, v_{i2}, \ldots, v_{in}\} = \{f_{i1}, f_{i2}, \ldots, f_{in}\} \\
\mathbf{w}_i &= f_{i+1}.
\end{align*}
\]

The first part, \( \mathbf{v}_i \), describes a prototype located in the input space and it is used in the classification (prediction) mode.

The growth process of the tree is pursued by quantifying the diversity of data located at the individual nodes of the tree and splitting the nodes that exhibit the highest diversity. This criterion takes into account the variability of the data, finds the node with the highest value of the criterion, and splits it into \( c \) nodes that occur at the consecutive lower level of the tree.

The \( i \)-th node \( N_i \) can be represented as an ordered triple

\[
N_i = \langle X_i, Y_i, U_i \rangle \tag{1}
\]

\( X_i \) denotes all elements of the data set that belong to this node in virtue of the highest membership grade

\[
X_i = \{x(k) \mid u_i(k) > u_j(k) \text{ for all } j \neq i\} \tag{2}
\]

The index \( j \) pertains to the nodes originating from the same parent.

The second set collects the output coordinates of the elements that have already been assigned to \( X_i \). Likewise, \( U_i = [u_i(x(1)), u_i(x(2)), \ldots, u_i(x(N))] \) is a vector of the grades of membership of the elements in \( X_i \).

We define the representative of this node positioned in the output space as the weighted sum (note that in the construct hereafter we include only those elements that contribute to the cluster so the summation is taken over \( X_i \) and \( Y_i \)), as follows:

\[
m_i = \frac{\sum_k u_i(x(k))y(k)}{\sum_k u_i(x(k))} \tag{3}
\]
The variability of the data in the output space existing at this node \( V_i \) is taken as a spread around the representative \((m_i)\) where again we consider a partial involvement of the elements in \( X_i \) by weighting the distance by the associated membership grade,

\[
V_i = \sum_{(x(k), y(k)) \in X_i} u_i(x(k))(y(k) - m_i)^2. \tag{4}
\]

In the next step, we select the node of the tree (leaf) that has the highest value of \( V_i \), and expand the node by forming its children by applying the clustering of the associated data set into \( c \) clusters. The process is then repeated: we examine the leaves of the tree and expand the one with the highest value of the diversity criterion.

The growth of the tree is controlled by conditions under which the clusters can be further expanded (split). We envision two intuitively conditions that tackle the nature of the data behind each node.

The first one is self-evident: a given node can be expanded if it contains enough data points. With clusters, we require this number to be greater than the number of the clusters; otherwise, the clusters cannot be formed.

The second stopping condition pertains to the structure of data that we attempt to discover through clustering. It becomes obvious that once we approach smaller subsets of data, the dominant structure (which is strongly visible at the level of the entire and far more numerous data set) may not manifest that profoundly in the subset. It is likely that the smaller the data, the less pronounced its structure. This becomes reflected in the entries of the partition matrix that tend to be equal to each other and equal to \( 1/c \).

If no structure becomes present, this equal distribution of membership grades occurs across each column of the partition matrix.

The diversity criterion (sum of variabilities at the leaves) can be also viewed as another termination criterion.

### 2.2 Classification (Prediction) Mode

Once the C-tree has been constructed, it can be used to classify a new input \((x)\) or predict a value of the associated output variable (denoted here by \( y \)).

In the calculations, we rely on the membership grades computed for each cluster as the standard fuzzy C-means (FCM). The calculations pertain to the leaves of the C-tree, so for several levels of depth we have to traverse the tree first to reach the specific leaves. This is done by computing \( u_i(x) \) and moving down. At some level, we determine the path that maximizes \( u_i(x) \). The process repeats for each level of the tree. The predicted value occurring at the final leaf node is equal to \( m_i \) defined in Equation (3).

### 3 DEFAULT PREDICTION

The sample data set comes from a state-owned commercial bank. The dataset of 243 samples represent SMEs. Among these enterprises, the number of the enterprises which can repay the loan is 123, the rest 120 are those which can not repay the loan.

In order to evaluate the performance of the tree a fivefold cross-validation was used. More specifically, in each pass, an 80–20 split of data is generated into the training and testing set, respectively, and the experiments are repeated for five different splits for training and testing data.

The binary default variable \( Y_i = 1 \) if firm \( i \) defaults, and \( Y_i = 0 \) otherwise.

Our model is an accounting based model. In this kind of model, accounting balance sheets are used and the input indexes include the enterprise’s capability of returning loan and wish of returning loan. The wish of returning loan is measured by the rate of returning interests, namely

\[
X_0 = \frac{\text{Amount of interests that has been repaid}}{\text{Amount of interests that should be repaid}}.
\]

Each index represented the average of three periods before the prediction period.
characterized by the highest value of the variability index, and expand it by forming two children nodes by applying the FCM algorithm to data associated with this original node. The decision tree grown in this manner is visualized in Fig. 1. Nodes in gray represent zero variability. All leaves of the tree have zero variability. Therefore, the error in training dataset was zero.

![Decision Tree Diagram](image)

Figure 1: Fuzzy c-tree structure. Gray circles have zero variability.

It is of interest to compare the results produced by the C-decision tree with those obtained when applying “standard” decision trees, namely C4.5.

The results are summarized in Table 1. We report the mean values of the error. For the C-decision trees, the number of nodes is equal to the number of clusters multiplied by the number of iterations. The C-tree is more compact (in terms of the number of nodes). This is not surprising as its nodes are more complex than those in the C4.5 decision tree. The results on the training and test sets are better for the C-trees.

<table>
<thead>
<tr>
<th>Decision Tree</th>
<th>Error Training</th>
<th>Error Test</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-tree</td>
<td>0%</td>
<td>20%</td>
<td>10</td>
</tr>
<tr>
<td>C4.5</td>
<td>10%</td>
<td>35%</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 1: C-decisions tree and C4.5 results.

4 CONCLUSIONS

The C-decision trees are classification constructs that are built on a basis of information granules—fuzzy clusters. In contrast to C4.5-like trees, all features are used once at a time, and such a development approach promotes more compact trees and a versatile geometry of the partition of the feature space. The experimental study illustrates a better performance of the C-tree. Further research should be conducted to test the potential improvements associated with such approach. New strategies for splitting nodes can be developed as well as for stopping criterions. We intend conduct research in order to extract rules with improved interpretability. Others comparisons could be experimented. In spite of the simplicity adopted, the experimental results confirm the effectiveness of C-trees in default prediction of SMEs.

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