IMPLEMENTATION OF A FUZZY LOGIC SYSTEM ON A FPGA FOR A SERVO CONTROLLER

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Abstract: In this paper we propose a digital fuzzy logic system implemented on a field programmable gate array (FPGA) in order to control a servo controller. The fuzzy logic controller (FLC) is designed as a combinational circuit and does not depend on a clock signal. So the advantage is that the fuzzy system is enough fast to control a servo controller. For the implementation of the membership functions (MF) we propose to use dynamic MF, i.e. the parameters that define the each MF are adapted on line. Also, for the design of fuzzy system a new methodology was developed so the design and implementation of the fuzzy system is easy to do. The fuzzy system was programmed in MATLAB and was proved that the fuzzy system is capable to control a servo motor. Finally the performance of the fuzzy system was proved directly on the FPGA.

1 INTRODUCTION

There had been many implementations of hardware FLC since the first one (Togai, 1986). Each one with some differences related to the processing way.

A FLC can be implemented in three forms: Runtime Computing (RTC), Lookup Computing (LUC) or mixed. It can also be combined with several computer architecture techniques like combinatorial, sequential, pipelined and mixed processing, implemented in a computer, a microcontroller or any programmable device.

The use of FPGA in a reconfigurable device has been suitable when talking about versatility to make any digital design by means of costs and design time. It is a dare to play with these architectures and techniques to make an appropriated FLC design, by which it is necessary to change the way of designing algorithms to describe an improved FLC.

This paper shows a practical approach for the design of a combinational FLC for a DC servo using simple construction modules on a FPGA in order to speed up the FLC prototyping.

2 FUZZY LOGIC CONTROLLER

This FLC uses a Mamdani fuzzy model. The Fuzzy system is described by the next equation:

\[ y = \frac{\sum_{i=1}^{g} b_i \mu_i(u_1, u_2, ..., u_n)}{\sum_{i=1}^{g} \mu_i(u_1, u_2, ..., u_n)} \]  

(1)

Where \( y \) is the system output, \( \mu_i(u_1, u_2, ..., u_n) = \text{Mamdani}(\mu_{A_1}^{f}, \mu_{A_2}^{g}, ..., \mu_{A_n}^{h}) \), and \( A_1 = \text{input 1}, A_2 = \text{input 2}, ..., A_n = \text{input n}; b_i = \text{center of the output of each membership function}. \) Each input is assigned to a set of membership functions, for instance \( \mu_{A_1}^{f} \) represents all possible membership functions for the first input variable. The FLC only uses triangles, S and Z membership functions for fuzzy sets.

This FLC has only two inputs and one output, each input is described by three membership functions which were implemented in hardware according to equations 2-4, which are shown in Figure 1.

The Triangle function is described by:
The S function is described by:

\[
y(x) = \begin{cases} \frac{\mu_{\text{max}}(x - c + a)}{a}, & c - a < x < c \\ \frac{\mu_{\text{max}}(x - c - a)}{a}, & c < x < c + a \\ \mu_{\text{max}}, & x = c \\ 0, & \text{else} \end{cases}
\] (2)

And the Z function is described by:

\[
y(x) = \begin{cases} \frac{\mu_{\text{max}}(x - c + a)}{a}, & c - a < x < c \\ \mu_{\text{max}}, & x \geq c \\ 0, & \text{else} \end{cases}
\] (3)

3 DESIGN

FPGA technology facilities and advantages let the designer to build any fuzzy system in short time. We purpose a fast and simple approach for FLC design which consists with eight steps in order to implement it on FPGA:

1. Establish whatever the designer want to control and which variables will be related. Define the number of inputs and outputs of the FLC.
2. Define the universe of discourse for each input and output variable based on the process characteristics.
3. Define the number and shapes of fuzzy sets for every variable based on the last step.
4. Establish the FLC configuration by means of the fuzzy inference rules according to the wished operation and based on the expert knowledge about the process.
5. Build the fuzzifier using simple membership functions connected in parallel.
7. Build the defuzzification stage by means of multiplication and division modules using parallelism.
8. Implement the design on FPGA.

Considering the FPGA limitations, it is possible to build any fuzzy system.

STEP 1. Assume a fuzzy control system for a DC servomotor which consists of two inputs and one output, as shown in Section 2. First input variable \( u_1 \) is the rotor “position error” which is the difference between current position and desired position. Second input variable \( u_2 \) is the change of that position error (“change in position”) in order to know if the error is increasing or decreasing. The output variable \( y_1 \) is the needed applied voltage to move the rotor servo to a certain position (Figure 3).

STEP 2. Before choosing the number of membership functions or fuzzy sets for this system, it is needed to establish the size of the discourse universe for every variable. Expert should purpose those values. So, position error must be expressed in radians \((\text{rad})\) and it must not be greater than half turn \(\left[ -\frac{\pi}{2}, \frac{\pi}{2}\right]\text{rad}\). And change in position should accept non- abrupt changes in error position \(\left[ -\frac{\pi}{2}, \frac{\pi}{2}\right]\text{rad}\); this variable rises from position error derivative. For every input it is required to get the minimum possible error as possible or zero. This ranges let
the FLC to control the servo rotor position by means of varying the voltage supply from -5 to 5 volts.

STEP 3. Once the universe of discourse is defined, the designer may purpose the number and the shape (membership function) of the fuzzy sets and establish their linguistic meaning. By simplicity, for every variable, we purpose three fuzzy sets that describe the triangular, S and Z membership functions, and their linguistic labels for every variable. For position error: NE (negative error), ZE (zero error) and PE (positive error); for change in error variable: NC (negative change), ZC (zero change) and PC (positive change); and for voltage variable: NV (negative voltage), ZV (zero voltage) and PV (positive voltage). The fuzzy sets distribution is a task for the designer also.

STEP 4. The FLC configuration is made by their rule set. The number of rules $R$ (Equation 5) is defined by the number of fuzzy sets per input variable ($N_1$ number of fuzzy sets for input position error, $N_2$ number of fuzzy sets for input change in position):

$$R = \prod_{i=1}^{3} N_i = N_1 \times N_2 = (3) \times (3) = 9$$

These nine rules are nine possible cases that may occur in the FLC inputs, shown in Figure 4.

Figure 4: Graphical representation of nine rule cases for the servo application.

In order to get the right FLC configuration it is needed to analyze the process entirely for covering all the possible cases in which it exist a non- desired operation. So, FLC can assure there is a concrete and exact value in its output that contributes for a right control action.

Then, the rule set can be built this way:

1. IF $eP$ is NE AND $cP$ is NC THEN $V$ is NV
2. IF $eP$ is NE AND $cP$ is ZC THEN $V$ is NV
3. IF $eP$ is NE AND $cP$ is PC THEN $V$ is NV
4. IF $eP$ is ZE AND $cP$ is NC THEN $V$ is NV
5. IF $eP$ is ZE AND $cP$ is ZC THEN $V$ is ZV
6. IF $eP$ is ZE AND $cP$ is PC THEN $V$ is PV
7. IF $eP$ is PE AND $cP$ is NC THEN $V$ is PV
8. IF $eP$ is PE AND $cP$ is ZC THEN $V$ is PV
9. IF $eP$ is PE AND $cP$ is PC THEN $V$ is PV

STEP 5. Due to the use of RTC approach, it is possible to build a combinatorial FLC that may change its fuzzy set parameters on line, regardless on their shape. Symmetric trapezoidal, triangular, S and Z membership functions are practical and suitable fuzzy sets for implementing on FPGA.

STEP 6. All these membership functions, shown in Figure 1, use the arithmetic operation of division and multiplication for calculating their corresponding slopes. In step 3, it was chosen triangular, S and Z functions for the fuzzy sets of every variable of the FLC fuzzifier, so it is needed to establish their parameters according to their corresponding range. This range must be scaled according to the FLC bits number. This way, fuzzifiers in Figure 5 show parallel operation. Notice every fuzzy set parameter can be changed simply by modifying their register values.

Figure 5: Fuzzifiers for servo application. Every input variable has three fuzzy membership values.
STEP 7. Inference Machine is made using Mamdani Implication Method, where all every output relates all the implied premises using simple MIN/ MAX operations according to the rule set, established in step 4. Figure 6 shows this rule set by means of combinatorial circuit connections.

STEP 8. Defuzzification is needed for converting all fuzzy values onto crisp values in order to use them for a specific control action. RTC combinatorial fuzzification stage let the expert change the singleton positions over the output discourse universe on line. This defuzzifier uses centroid technique, where every singleton position is divided (modified non-restoring division) by the summation of the membership values obtained from the inference machine and this result is multiplied by every fuzzy value obtained from the inference machine. Finally every multiplication result is summed in order to obtain the crisp output. Figure 7 shows this RTL design for this servo application.

STEP 9. Finally, once this design is synthesized for a FPGA it is ready to upload. Designer should take in account the pins number in order to fit the specific application. Due this application has only two inputs and one output there is no any problem to fit the most of recent FPGAs.

4 RESULTS

A general result for FPGA implementation was obtained and this is resumed in Table 1 for Xilinx Spartan FPGA starter kit. This results mean show the delay time each module takes and the entire FLC performance. Also, it is shown the total used LUTs in FPGA resources. Division algorithm is the slowest module in the FLC. So, the more the division algorithm performance improvement is made the more the FLC performance will increase.

This fuzzy system was built in MATLAB Fuzzy Toolbox first (FIS). Then it was chosen a set of representative values called test bench and it is applied to the FIS in order to prove the FLC performance once it is implemented in FPGA kit.

DC servo FLC needs 84 ns to make an inference. Then, its processing data rate is 11.9 MFLIPS. Table 2 shows a set of FLC implemented on FPGA with RTC, LUT and mixed approaches, with similar FLC configuration, but for different applications. Each implemented FLC has similar and higher performance compared with the purposed in this paper, but the advantage of this method is the easy scaling for system improvement or other application designs.

Table 1: FPGA timing and resource results obtained for DC servo control.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Delay (ns)</th>
<th>LUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 bits non-restoring division</td>
<td>48.50</td>
<td>644</td>
</tr>
<tr>
<td>Modified 8 bits non-restoring division</td>
<td>28.83</td>
<td>208</td>
</tr>
<tr>
<td>8 bits multiplication</td>
<td>13.17</td>
<td>36</td>
</tr>
<tr>
<td>Isosceles triangle MF</td>
<td>36.70</td>
<td>251</td>
</tr>
<tr>
<td>8-step MF</td>
<td>36.70</td>
<td>249</td>
</tr>
<tr>
<td>Z-step MF</td>
<td>36.70</td>
<td>251</td>
</tr>
<tr>
<td>Fuzzifier</td>
<td>37.42</td>
<td>755</td>
</tr>
<tr>
<td>Defuzzifier</td>
<td>41.49</td>
<td>677</td>
</tr>
<tr>
<td>Mamdani inference machine</td>
<td>19.32</td>
<td>242</td>
</tr>
<tr>
<td>MIN-MAX operations</td>
<td>9.36</td>
<td>16</td>
</tr>
<tr>
<td>FLC</td>
<td>84.01</td>
<td>2689</td>
</tr>
</tbody>
</table>
Table 2: Other hardware FLC with similar fuzzy systems. Velocity performance comparison.

<table>
<thead>
<tr>
<th>MFLIPS</th>
<th>Device</th>
<th>Autor</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.00</td>
<td>Undefined</td>
<td>(Manzoul, 1995)</td>
</tr>
<tr>
<td>50.00</td>
<td>Undefined</td>
<td>(Lund, 2000)</td>
</tr>
<tr>
<td>15.38</td>
<td>Xilinx XC3S1500</td>
<td>(Deliparaschos, 2005)</td>
</tr>
<tr>
<td>11.90</td>
<td>Xilinx XC3S200</td>
<td>(Téllez, 2008)</td>
</tr>
<tr>
<td>10.00</td>
<td>Xilinx XC3S1000</td>
<td>(Kim, 1997)</td>
</tr>
<tr>
<td>9.00</td>
<td>Xilinx XC4008</td>
<td>(Hung, 1994)</td>
</tr>
<tr>
<td>8.00</td>
<td>Altera EPF8820</td>
<td>(Aranguren, 1997)</td>
</tr>
<tr>
<td>5.56</td>
<td>Xilinx XC3S200</td>
<td>(González, 2007)</td>
</tr>
<tr>
<td>3.13</td>
<td>Xilinx XC3S1000</td>
<td>(Sánchez, 2007)</td>
</tr>
<tr>
<td>2.00</td>
<td>Xilinx XC6000E</td>
<td>(Gaona, 2003)</td>
</tr>
<tr>
<td>0.08</td>
<td>VLSI</td>
<td>(Togai, 1986)</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

A fuzzy logic controller was designed and implemented on FPGA. Specifically the FLC was implemented on a Xilinx design board. The architecture of the system was implemented by arithmetic modules by using combinational logic. In order to implement the fast FLC prototyping we proposed a methodology design of 8 basic steps. The digital design was development with VHDL. Each module was proved with ModelSim XE III Starter version. The FLC has the characteristic that the parameters of the membership functions are variable on line. The complete fuzzy system for a servo motor was probed with MATLAB and ModelSim.

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