SOLVING THE NON-SPLIT WEIGHTED RING ARC-LOADING PROBLEM IN A RESILIENT PACKET RING USING PARTICLE SWARM OPTIMIZATION

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Abstract: Massive growth of the Internet traffic in last decades has motivated the design of high-speed optical networks. Resilient Packet Ring (RPR), also known as IEEE 802.17, is a standard designed for the optimized transport of data traffic over optical fiber ring networks. Its design is to provide the resilience found in SONET/SDH networks but instead of setting up circuit oriented connections, providing a packet based transmission. This is to increase the efficiency of Ethernet and IP services. In this paper, a weighted ring arc-loading problem (WRALP) is considered which arises in engineering and planning of the RPR systems (combinatorial optimization NP-complete problem). Specifically, for a given set of non-split and uni-directional point-to-point demands (weights), the objective is to find the routing for each demand (i.e., assignment of the demand to either clockwise or counter-clockwise ring) so that the maximum arc load is minimized. This paper suggests four variants of Particle Swarm Optimization (PSO), combined with a Local Search (LS) method to efficient non-split traffic loading on the RPR. Numerical simulation results show the effectiveness and efficiency of the proposed methods.

1 INTRODUCTION

This paper concerns load balancing problems on RPR, where the RPR is offered by IEEE 802.17 (RPR Alliance, 2004). The RPR is in essence, a distributed Ethernet switch, in which the RPR nodes are connected with two counter-rotating rings (clockwise and counter-clockwise ring). The ring spans are either SONET of Gbit Ethernet. The (unidirectional) point-to-point traffic demands (10/100/1000 Ethernet and/or TDM) can be carried on either ring.

Given a network and a set D of communications requests, a fundamental problem is to design a transmission route (direct path) for each request such that high load on the arcs/edges is avoided, where an arc is an edge endowed with a direction. The load of an arc is defined to be the total weight of those requests that are routed through the arc in its direction (WRALP) and the load of an edge is the number of routes traversing the edge in either direction (WRELP). In general each request is associated with a non-negative integer weight. Practically, the weight of a request can be interpreted as a traffic demand or the size of the data to be transmitted.

The load balancing problems can be classified into two formulations: with demand splitting (WRALP) or without demand splitting (non-split WRALP). Split loading allows the splitting of a demand into two portions to be carried out in two directions, while a non-split loading is one in which each demand must be entirely carried out in either the clockwise or counter-clockwise direction. In either split or non-split cases, WRELP/WRALP ask for a routing scheme such that maximum load on arcs/edges is minimized. In this paper we study the WRALP without demand splitting.

For research on the no-split WRELPh, Cosares and Saniee (1994) and Dell’Amico et al. (1998) studied the problem on SONET rings. Cosares and Saniee (1994) proved that the formulation without
demand splitting is NP-complete. This means that we cannot guarantee to find the best solution in a reasonable amount of time. For the split problem, various approaches are summarized by Schrijver et al. (1998) and their algorithms compared in Myung and Kim (2004) and Wang (2005).

The non-split WRALP considered in the present paper is identical to the one described by Kubat and Smith (2005) - non-split WRALP, Cho et al. (2005) - non-split WRALP and WRALP and Yuan and Zhou (2004) - WRALP. Their objective is to produce feasible solutions in a reduce amount of time (using algorithms that produce approximate solutions). Our objective is to compare the performance of our algorithms in achieving the optimal solution. A heuristic method can greatly improve the quality of a solution as the domain knowledge is introduced, but this process will cost much time.

In this article we report the results of the application of four different variants of PSO, all of them newer implementations to solve this problem and we also present a novel binary local search PSO (LS-PSO) to solve this problem.

The paper is structured as follows. In Section 2 we present the problem; in section 3 we describe the algorithms implemented while in Section 4 we show the computational results obtained and, finally, in Section 6 we report about the conclusions.

2 PROBLEM DEFINITION

To effectively use the RPR’s potential, namely spatial reuse, statistical multiplexing and bi-directionality, it is necessary to route the demands efficiently. Given a set of point-to-point unidirectional customer traffic demands of specified bandwidth, the demands should be assigned to the best performance.

Let \( R_n \) be a \( n \)-node bidirectional ring with nodes \( \{ n_1, n_2, \ldots, n_n \} \) labelled clockwise. Each edge \( \{ e_k, e_{k+1} \} \) of \( R_n \), \( 1 \leq k \leq n \) is taken as two arcs with opposite directions, in which the data streams can transmit in either direction.

\[
\begin{align*}
a^+_k &= (e_k, e_{k+1}), \\
a^-_k &= (e_{k+1}, e_k)
\end{align*}
\]

A communication request on \( R_n \) is an ordered pair \((s, t)\) of distinct nodes, where \( s \) is the source and \( t \) is the destination. We assume that data can be transmitted clockwise or counter-clockwise on the ring without splitting. We use \( P^+(s, t) \) to denote the directed \((s, t)\) - path clockwise around \( R_n \), and \( P^-(s, t) \) the directed \((s, t)\) - path counter-clockwise around \( R_n \).

Often a request \((s, t)\) is associated with an integer weight \( w \geq 0 \); we denote this weighted request by \((s, t ; w)\). Let \( D = \{(s_1, t_1 ; w_1), (s_2, t_2 ; w_2), \ldots, (s_m, t_m ; w_m)\} \) be a set of integrally weighted requests on \( R_n \). For each request/pair \((s_i, t_i)\) we need to design a directed path \( P_i \) of \( R_n \) from \( s_i \) to \( t_i \). A collection \( P = \{ P_i : i = 1, 2, \ldots, m \} \) of such directed paths is called a routing for \( D \).

In this work, the solutions are represented using binary vectors. If a position has the value \( 1 \) the demand flows by the clockwise direction, \( 0 \) otherwise (see Table 1).

<table>
<thead>
<tr>
<th>Pair(s, t) Demand</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: (1, 2)</td>
<td>15C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2: (1, 3)</td>
<td></td>
<td>3CC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3: (1, 4)</td>
<td></td>
<td></td>
<td>6CC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4: (2, 3)</td>
<td></td>
<td></td>
<td></td>
<td>15CC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5: (2, 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15CC</td>
<td></td>
</tr>
<tr>
<td>6: (3, 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14C</td>
</tr>
</tbody>
</table>

Table 1: Chromosome representation.

We assume that weights cannot be split, that is, for some integer \( x_i = 1, 1 \leq i \leq m \), the total amount of data is transmitted along \( P^+(s, t) \); \( x_i = 0 \), the total amount of data is transmitted along \( P^-(s, t) \). The vector

\[
x = (x_1, x_2, \ldots, x_m)
\]

determines a routing scheme for \( D \).

The WRALP is formulated as follows:

\[
\begin{align*}
W_1, \ldots, W_m \rightarrow \text{demands of the pairs } (s_1, t_1), \ldots, (s_m, t_m) \\
x_1, \ldots, x_m & = 0 \rightarrow P^+(s_i, t_i) \\
1 & \rightarrow P^-(s_i, t_i) \quad (1)
\end{align*}
\]

Load on arcs:

\[
L(x, a^+_k) = \sum_{i, a^+_i \in P^+(s_i, t_i)} w_i \\
L(x, a^-_k) = \sum_{i, a^-_i \in P^-(s_i, t_i)} w_i
\]

\[
\forall k = 1, \ldots, n; \quad \forall i = 1, \ldots, m
\]

Fitness Function:

\[
\max \{ \max_{k=1} L(x, a^+_k), \max_{k=1} L(x, a^-_k) \} \quad (4)
\]

Constraints (1) in conjunction with constraints (3) state that each demand is routed in either clockwise (C) or counter-clockwise (CC) direction.
For an arc, the load is the sum of $w_k$ for clockwise or counter-clockwise between nodes $e_k$ and $e_{k+1}$. The objective is to minimize the maximum load on the arcs of a ring (4).

3 PARTICLE SWARM OPTIMIZATION

PSO is an intelligent optimization algorithm, originally developed by Kennedy and Eberhart in 1995, inspired by the behaviour of bird flock's looking for food (Kennedy and Eberhart, 1995, 1997). Like Genetic Algorithms (GA), PSO is a population-based optimization algorithm.

The initial population (P) of particles can be created randomly or in a deterministic form. The deterministic form is based in a Greedy Algorithm proposed by Bernardino et al. (2008). Initially a deterministic strategy is followed and in a second phase is used the PSO algorithm to optimize the solution.

Procedure Greedy:

\begin{verbatim}
FOR each pair
    Give a direction (C - 1, CC - 0)
    pos = random (numberPairs)
    FOR k=j=pos until j=numberPairs + pos
        IF (j > numberPairs)
            k=j- numberPairs
        Change direction pairk
        IF fitnessNewSolutionk<fitnessOldSolutionk
            Replace the previous value of pairk
        k++
    j++
\end{verbatim}

Whether continuous or discrete, the original and most essential idea of PSO is: difference in position leads to velocity and velocity leads to search. Supposing that the searching space is $D$-dimensional and $m$ particles form a swarm, each particle is looked as a point in the $D$-dimensional space, and the $i$th particle represents a $D$-dimensional vector $x_i=(x_{i1}, x_{i2},..., x_{iD})$. According to the fitness value, the particle is updated to move towards the better area by the corresponding operators till the best point is found. In the iterative process, each particle's previous best position is remembered and denoted $p_i=(p_{i1}, p_{i2},..., p_{iD})$, and the globally best position in the whole swarm is recorded as $pg=(pg_1, pg_2,..., pg_D)$. The $i$th particle's "flying" velocity is also a $D$-dimensional vector, represented as $v_i=(v_{i1}, v_{i2},..., v_{iD})$ ($i=1, 2,..., p$). At each step, the velocity of all particles is adjusted as a sum of its local best value, global best value and its present velocity, multiplied by the three constants $W$, $C_1$ and $C_2$ respectively, shown in (5); the position of each particle is also modified by adding its velocity to the current position, see (6).

$$
v^{k+1} = W \times v^k + C_1 \times r_1 \times (p^k - x^k) + C_2 \times r_2 \times (p^{g} - x^k)
$$

$$
x^{k+1} = x^k + v^{k+1}
$$

In (5-6) $k$ represents the iteration number; $r_1$, $r_2$ are two random numbers selected from a uniform distribution in [0.0, 1.0]; $W$ is the inertia weight. $C_1$ and $C_2$ are two constant numbers, which are often called the acceleration coefficients.

The four PSO variants used to solve the WRALP are extensions of the basic PSO and were used to solve discrete binary problems. To improve the performance of the PSO algorithms developed we apply a separate local search (LS) process to refine individuals.

The performance of the child vector and its parent is compared and the better one is selected. If the parent is better, it is retained in the population. The algorithm continues until a certain number of cycles defined by the user, have passed.

3.1 Discrete Binary Particle Swarm Optimization

As the basic PSO operates in continuous and real number space, it can’t be used to optimize the pure discrete binary problem. To handle this problem, Kennedy and Eberhart (1997) proposed a discrete binary PSO (KBPSO) algorithm, where the particles take the values of binary vectors of length $p$ and the velocity defined the probability of bit $x_{i1}$ to take the value 1. KBPSO reserved the updating formula of the velocity (see (5)) while velocity was constrained to the interval [0.0, 1.0] by a limiting transformation function, that is, the particle changes its bit value by (7-8) in KBPSO:}

$$
S(v^k_{ig}) = 1/(1 + e^{-v^k_{ig}})
$$

$$
\hat{v}^k_{ig} = \min (\max (\hat{v}^k_{ig}, 0), 1)
$$
\[ x_{ij} = \begin{cases} 1 & \text{if } \text{rand}() \leq S(v_{ij}) \\ 0 & \text{otherwise} \end{cases} \]  

where the value of \( \text{rand}() \) drawn from the interval \([0.0, 1.0]\) and the function \( S(v) \) is a sigmoid limiting transformation.

### 3.2 Constriction Coefficient Particle Swarm Optimization

The constriction coefficient (CBPSO) was introduced by Clerc and Kennedy (2002) as an outcome of a theoretical analysis of swarm dynamics. Velocities are constricted, with the following change in the velocity update:

\[ v_{ij}^{k+1} = \kappa \times (v_{ij}^k + \varphi_1 \times r_1 \times (p_{gj}^k - x_{ij}^k) + \varphi_2 \times r_2 \times (p_{ij}^k - x_{ij}^k)) \]

where \( \kappa \) is the constriction factor determined from the following two equations:

\[ \varphi = \varphi_1 + \varphi_2; \varphi > 4 \]
\[ \kappa = \frac{2}{\sqrt{2 - \varphi - \sqrt{\varphi^2 - 4\varphi}}} \]

It should be noted, however, that Clerc’s velocity (9) is simply a special case of the original velocity (5) where the constants \( W, C_1 \) and \( C_2 \) are chosen according to (10) and (11).

### 3.3 Modified Discrete Binary Particle Swarm Optimization

According to an information sharing mechanism of PSO, a modified discrete PSO (MBPSO) was proposed by Shen and Jiang (2004) as follows. The velocity \( v_{ij} \) of every individual is a random number in the range of \([0.0, 1.0]\). The resulting change in position is then defined by the following rule:

\[ x_{ij} = \begin{cases} x_{ij}^\text{new} & \text{if } 0 < v_{ij} < \alpha \text{ then } x_{ij}^\text{new} = x_{ij}^\text{old} \\ p_{ij} & \text{if } 0 < v_{ij} < \frac{1}{2}(1 + \alpha) \text{ then } x_{ij}^\text{new} = p_{ij} \\ g_{ij} & \text{if } \frac{1}{2}(1 + \alpha) < v_{ij} < 1 \text{ then } x_{ij}^\text{new} = g_{ij} \end{cases} \]

where \( \alpha \) is a random value in the range of \([0.0, 1.0]\) named static probability.

To circumvent convergence to local optima and improve the ability of the modified PSO algorithm to overcome local optima, five percent of particles are randomly selected, and each site of the selected particles has a probability of 0.5 to vary the value in a stochastic manner.

Using a static probability that decreases and some percent of randomly fling particles to overcome local optima, the MBPSO remains having satisfactory converging characteristics.

### 3.4 The Probability Binary Particle Swarm Optimization

Wang et al. (2008) propose a novel probability binary PSO (PBPSO). In PBPSO, a novel updating strategy is adopted to update the swarm and search the global solution. The variant equations (5) and (6) are all reserved for iterative evolution in PBPSO, and a different formula is used to determine a binary bit \( p_{x_{ij}} \), which can be denoted as follows:

\[ L(x_{ij}) = \frac{(x_{ij} - R_{\text{min}})}{(R_{\text{max}} - R_{\text{min}})} \]

\[ p_{x_{ij}} = \begin{cases} 1 & \text{if } \text{rand}() \leq L(x_{ij}) \\ 0 & \text{otherwise} \end{cases} \]

where \( L(x) \) is a linear function, its output value belongs to \((0, 1)\); \( \text{rand}() \) is a stochastic number selected from a uniform distribution in \([0.0, 1.0]\); and \([R_{\text{max}}, R_{\text{min}}]\) is a predefined range for gaining the probability value with \( L(x) \) function.

Compare to KBPSO, now the vector \( x_i \) is a real number vector rather than the binary vector. To obtain a probability value distributed in \([0.0, 1.0]\), is used the linear function \( L(x) \) to calculate it, which determines \( p_{x_{ij}} \) to be 1 or 0. The binary vector \( p_{x_{ij}} = (p_{x_{i1}}, p_{x_{i2}}, \ldots, p_{x_{iD}}) \) can be worked out, and then we can apply this binary vector into the combinatorial optimization problem.

### 4 STUDIED EXAMPLES

We evaluate the utility of the algorithms using identical examples produced by Cho et al. (2005). The studied examples arise by considering six different ring sizes – 5, 10, 15, 20, 25 or 30 nodes. A ring in a telecommunication network will typically contain between 5 and 20 nodes. Thus, we consider the 5, 10 and 15 node rings to be ordinary-sized rings and the 20, 25 and 30 node rings to be extremely large rings.

For convenience, they are labeled \( C_{ij} \), where \( 1 \leq i \leq 6 \) represents the ring size and \( 1 \leq j \leq 3 \) represents the demand case.
5 RESULTS

Since its conception, much work has been done to understand and develop the ideal parameters for PSO implementation. The goal was to develop an algorithm with an optimal balance between global exploration and exploitation of local maxima. One of the first issues encountered during PSO implementation was the ability to control the search space explored by the swarm. Early work done by the KBPSO developers (mostly trial and error) suggested that the best choice for $C_1$ and $C_2$ is 2.0 for each (Eberhart and Shi, 2001). This essentially became standard in the literature until recent results called the values into question. Several values of inertial weights have been suggested, attempting to strike a balance between global exploration and local exploitation. It was suggested varying the inertial weight linearly from 0.9 to 0.4 over the course of the run (Eberhart and Shi, 2001).

Parametric studies, using CBPSO have suggested that the optimal choice for $\varphi_1$ and $\varphi_2$ is 2.8 and 1.3, respectively (Carlisle and Dozier, 2001).

Shen and Jiang (2004) consider $\alpha=0.5$ when using MBPSO. Static probability $\alpha$ normally starts with a value of 0.5 and decreases to 0.33 when the iteration terminates.

In the work of Wang et al. (2008) small values of $R_{max}$ and $R_{min}$ are harmful to PBPSO as the algorithm cannot perform meticulous search well, and the optimization results are both poorest in the executions made. The simulation results showed that the $R_{min}=-50$ and $R_{max}=50$ may be encouraged as PBPSO both achieving the best optimization results. In our tests that is not the case (see Figure 1).

Population size is another parameter that needs a careful selection. Large populations, while providing the most thorough exploration of the solution space, increase the cost of more fitness evaluations and computation time. For the PSO, it has been found that relatively small population sizes can sufficiently explore a solution space while avoiding excessive fitness evaluations. Parametric studies have found that a population size of about 30 is optimal for many problems (Carlisle and Dozier, 2001).

Obviously, the parameters of the PSO variants will seriously affect the real optimization performance (see Figure 1). To know the PSO variants well, we study and test all the combination parameters of the different variants. Previous works have proven that the traditional values of parameters in PSO can keep algorithm work well, but since this problem has a different specificity we perform a new parameter studying using the test instance C32 (see table 2).

<table>
<thead>
<tr>
<th>Problem</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>KBPSO</td>
<td>0.5&lt;\omega&lt;0.9, 0.8&lt;C_1=C_2&lt;2.0</td>
</tr>
<tr>
<td>MBPSO</td>
<td>0.5&lt;\alpha&lt;0.7, \text{Descend = (true,false)}</td>
</tr>
<tr>
<td>CBPSO</td>
<td>0.6&lt;\varphi_1&lt;3.2, 1.2&lt;\varphi_2&lt;3.3</td>
</tr>
<tr>
<td>PBPSO</td>
<td>-15\leq R_{min} \leq -20, -15\leq R_{max} \leq 20</td>
</tr>
</tbody>
</table>

With or without varying linearly the inertial weight / $\alpha$ at the course of the run the results produced are very similar. With 30 particles the algorithms can reach in a reasonable amount of time a high number of optimal solutions.

Table 3 presents the best obtained results. The first column represents the problem number (Prob.), the second and the third columns show the number of nodes (Nodes) and the number of pairs (Pairs), the fourth column shows the minimum fitness values obtained and finally the fifth column shows the number of iterations used to test each instance. The number of iterations was selected based upon preliminary observation. The algorithms have been implemented using C++ and were executed using a processor Intel Core Duo (2.66 GHZ, Windows XP). The algorithms were tested using randomly initial solutions and deterministic initial solutions.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Nodes</th>
<th>Pairs</th>
<th>Optimal Fitness</th>
<th>Number Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>C11</td>
<td>5</td>
<td>10</td>
<td>161</td>
<td>200</td>
</tr>
<tr>
<td>C12</td>
<td>5</td>
<td>8</td>
<td>116</td>
<td>100</td>
</tr>
<tr>
<td>C13</td>
<td>5</td>
<td>6</td>
<td>116</td>
<td>10</td>
</tr>
<tr>
<td>C21</td>
<td>10</td>
<td>45</td>
<td>525</td>
<td>250</td>
</tr>
<tr>
<td>C22</td>
<td>10</td>
<td>23</td>
<td>243</td>
<td>200</td>
</tr>
<tr>
<td>C23</td>
<td>10</td>
<td>12</td>
<td>141</td>
<td>200</td>
</tr>
<tr>
<td>C31</td>
<td>15</td>
<td>105</td>
<td>1574</td>
<td>300</td>
</tr>
<tr>
<td>C32</td>
<td>15</td>
<td>50</td>
<td>941</td>
<td>250</td>
</tr>
<tr>
<td>C33</td>
<td>15</td>
<td>25</td>
<td>563</td>
<td>200</td>
</tr>
<tr>
<td>C41</td>
<td>20</td>
<td>190</td>
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<td>C42</td>
<td>20</td>
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</tr>
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<td>C53</td>
<td>25</td>
<td>61</td>
<td>912</td>
<td>300</td>
</tr>
<tr>
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<td>30</td>
<td>435</td>
<td>5762</td>
<td>2500</td>
</tr>
<tr>
<td>C62</td>
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<td>201</td>
<td>2696</td>
<td>1000</td>
</tr>
<tr>
<td>C63</td>
<td>30</td>
<td>92</td>
<td>1453</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 4 presents the best-obtained results. The first column represents the problem number (Prob.) and the remaining columns show the results obtained (T – run time in seconds and I – number of iterations). The run time corresponds to the average
time that the algorithms need to obtain the best solution. For each instance/variant, we perform 100 executions and the average computation time of the algorithms is calculated using the 20 best results. For the executions we use different seeds and we just consider the best combination parameters. All the algorithms reach the optimal solution.

Table 4: Results – run times and number of iterations.

<table>
<thead>
<tr>
<th>Prob.</th>
<th>KBPSO</th>
<th>LS-KBPSO</th>
<th>CBPSO</th>
<th>LS-CBPSO</th>
<th>PBPSO</th>
<th>LS-PBPSO</th>
<th>MBPSO</th>
<th>LS-MBPSO</th>
<th>PBPSO</th>
<th>LS-PBPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T</td>
<td>I</td>
<td>T</td>
<td>I</td>
<td>T</td>
<td>I</td>
<td>T</td>
<td>I</td>
<td>T</td>
<td>I</td>
</tr>
<tr>
<td>C11</td>
<td>&lt;0.001</td>
<td>&lt;5</td>
<td>&lt;0.001</td>
<td>&lt;5</td>
<td>&lt;0.001</td>
<td>&lt;5</td>
<td>&lt;0.001</td>
<td>&lt;5</td>
<td>&lt;0.001</td>
<td>&lt;5</td>
</tr>
<tr>
<td>C12</td>
<td>&lt;0.001</td>
<td>&lt;5</td>
<td>&lt;0.001</td>
<td>&lt;3</td>
<td>&lt;0.001</td>
<td>&lt;5</td>
<td>&lt;0.001</td>
<td>&lt;3</td>
<td>&lt;0.001</td>
<td>&lt;5</td>
</tr>
<tr>
<td>C13</td>
<td>&lt;0.001</td>
<td>1</td>
<td>&lt;0.001</td>
<td>1</td>
<td>&lt;0.001</td>
<td>1</td>
<td>&lt;0.001</td>
<td>1</td>
<td>&lt;0.001</td>
<td>1</td>
</tr>
<tr>
<td>C21</td>
<td>&lt;0.001</td>
<td>&lt;15</td>
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Figure 1: Problem C32 – Comparison between parameters.

Figure 2: Average number of iterations / execution times using the best parameters combination (only best solutions).
In comparison, the LS-PSO obtains results in smaller number of iterations. LS-PBPSO is the faster algorithm (see Figure 2). For large problems it produces solutions with a smaller number of iterations and in a smaller time. The main advantage of including the LS algorithm is that it obtains almost always a good solution with the correct combination of parameters. In 100 executions with the best combination parameters and the same number of iterations it obtains a higher number of optimal solutions as can be seen in Figure 3.

6 CONCLUSIONS

This paper proposes a novel LS algorithm combined with four binary PSO variants to solve the WRALP. The performance of all algorithms is compared. The four PSO binary variants (with or without LS algorithm) used to solve the WRALP prove to be very effective in the resolution of the WRALP. LS-PSO exhibits better optimization performance in terms of speed and global search. LS-PBPSO variant provides solutions in smaller number of iterations and in a smaller execution time.

The continuation of this work will be the search and implementation of new methods for speeding up the optimization process.

REFERENCES


