REAL-CODED GENETIC ALGORITHM IDENTIFICATION OF A FLEXIBLE PLATE SYSTEM

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Abstract: Parametric modelling deals with determination of model parameters of a system. Parametric modelling of systems may benefit from advantages of real coded genetic algorithms (RCGAs), as they do not suffer from loss of precision during the processes of encoding and decoding compared with Binary Coded Genetic Algorithm. In this paper, RCGA is used to identify the best model order and associated parameters characterising a thin plate system. The performance of the approach is assessed on basis mean-squared error, time and frequency domain response of the developed model in characterising the system. A comparative assessment of the approach with binary coded GA is also provided. Simulation results signify the advantages of RCGA over two further algorithms in modelling the plate system are also provided.

1 INTRODUCTION

Parametric modelling is defined as the process of estimating parameters of a model characterising a plant. The technique basically searches for numerical values of the parameters so that to give the best agreement between the predicted (model) output and the measured (plant) output. Parametric modelling can include both the parameter estimates and the model structure. Statistical validation procedures, based on correlation analysis, are utilised to validate parametric models.

Several advantages motivating research intention in a flexible structure are due to light weight, lower energy consumption, smaller actuator requirement, low rigidity requirement and less bulky design. These advantages lead to extensive usage of flexible plates in various applications such as space vehicles, automotive industries, and the construction industry. Modelling is the first step in a model-based control development of a system. Accordingly, the accuracy of the model is crucial for the desired performance of the control system.

Artificial intelligence approaches such as genetic algorithm (GA), particle swarm optimisation (PSO), fuzzy logic and neural networks have been utilised in system identification applications. Among these GAs have shown great potential in parametric modelling of dynamic systems.

The utilisation of binary-coded GA (BCGA) and real-coded GA (RCGA) for parameter estimator of models of dynamic systems has been reported in various applications. Zamanan et al. (2006) have reported the use of RGA as an optimization technique for tracking harmonics on power systems. Mitsukura et al. (2002) have reported using BCGA and RCGA to (i) determine a function type and (ii) the coefficient of the function and time delay, respectively. They have tested the technique successfully in determining the hammer stain model and music data model. BCGA also has been used to estimate the parameters of a plate structure (Intan, 2002). However, precision in BCGA is affected due to the processes of encoding and decoding. Moreover, BCGA is susceptible to the Hamming Cliff effect, which can be problematic when searching a continuous search space. Instead of working on the conventional bit by bit operation in BCGA, an RCGA approach is chosen in a wide range of applications where both the crossover and mutation operators are handled with real-valued numbers. A real coded GA leads to reduced computational complexity and faster convergence compared to a binary coded GA.

In this work, RCGA is proposed for parametric modeling of a flexible plate structure in comparison to a binary-coded GA. The rest of the paper is structured as follows: Section 2 describes the flexible plate system and formulates the problem. Section 3 presents the parametric models with RCGA and parametric system identification respectively. Section 4 presents implementation of
the algorithms in modeling the system using various excitation signals such as finite duration step, random and pseudo random binary signal (PRBS). Results and discussions of the model validity through input/output mapping, mean square of output error and frequency domain response are also presented. Parametric modelling is also confirmed with convergence of fitness values and time run. Finally, the paper is concluded in Section 5.

2 THE FLEXIBLE PLATE SYSTEM

Dynamic simulation of a plate structure using the finite differences (FD) method is considered in this paper. The finite difference method is used to discretise the governing dynamic equation considered with no damping and the lateral deflection of plates is obtained using central finite difference method. It then transformed into state space equation as the following equation.

\[ W_{i,j,k+1} = (A+2B)W_{i,j,k} + BW_{i,j,k} + CF \]  

Where \( A_{i,j} \) represents the diagonal elements of \( A \) \((2/\Delta t)\), \( B = (\Delta t/\rho) \), \( C = -(DC) \), and \( W_{i,j,k+1} \) is the deflection of grid points \( i = 1, 2, \ldots, n+1 \) and \( j = 1, 2, \ldots, m+1 \) at time step \( k+1 \). \( W_{i,j,k} \) and \( W_{i,j,k} \) are the corresponding deflections at time steps \( k \) and \( k-1 \) respectively. \( A \) is constant \( (n+1)(m+1) \times (n+1)(m+1) \) matrix whose entries depend on physical dimensions and characteristics of the plate, \( B \) is a diagonal matrix of \(-I\) corresponding to \( W_{i,j,k} \) and \( C \) is a scalar related to the given input and \( F \) is an \( (n+1)(m+1) \times 1 \) matrix known as the forcing matrix. The algorithm is implemented in Matlab/SIMULINK with applied external force or disturbance into all clamped edges plate. Twenty two equal divisions of plate elements with dimension 1.0mm \( \times \) 1.0mm \( \times \) 0.00032m is measured at the detection and observation points (Figure 1). Parameters of the plate considered comprise mass density per area, \( \rho = 2700 \) kg/m\(^2\), Young’s Modulus, \( E = 7.11 \times 10^{10} \) N/m\(^2\), second moment of inertia, \( I = 5.1924 \times 10^{-11} \) m\(^2\) and Poisson ratio, \( \nu = 0.3 \) with sampling time 0.001.

![Figure 1: The flexible plate system.](image)

3 REAL CODED GENETIC ALGORITHM

In most of practical engineering problems, the real-coded GA is more suitable than the binary-coded GA, as transformations from real number to binary digits may suffer from loss of precision. Genetic operations are very important to the success of specific GA applications. In this work, real-coded representation is used to determine the model order of the plant and subsequently identify parametric model of the system. The initial population is created randomly within [-1,1] range. The main three genetic operators involved are described below.

3.1 Selection

Selection is the process of determining the number of times or trials a particular individual in the population is chosen for reproduction (Chipperfield, 1994). The process includes two steps, namely selection probability and sampling algorithm. Selection probability is concerned with transformation of raw fitness values into real as expected of an individual to reproduce. Sampling algorithm reproduces individuals based on the selection probabilities computed before. This process is repeated as often as individuals must be chosen. There are many methods reported such as roulette wheel selection, stochastic universal sampling and tournament selection, etc. The stochastic universal sampling (SUS) method is used in this work that randomly copies chromosomes and simulates N equally distributed pointers. SUS is a simpler algorithm, and as individuals are selected entirely on their position in the population, SUS has zero bias. After selection has been carried out, the construction of the intermediate population is complete and the crossover and mutation operators are then applied.

3.2 Crossover (Recombination)

Crossover produces new individuals that have some parts of both parent’s genetic material (Chipperfield, 1994). However, Mühlenbein et. al (1991) have distinguished between recombination and crossover. The mixing of the variables was called recombination and the mixing of the values of a variable was named crossover. Line recombination employed in this work performs an exchange of variable values between the individuals. By using a real-valued encoding of the chromosome structure,
line recombination is a method of producing new phenotypes around and between the values of the parents’ phenotypes (Mühlenbein and Schlierkamp, 1993). For the line recombination, let \( x = (x_1, ..., x_n) \) and \( y = (y_1, ..., y_n) \) be the parent strings. Then, the offspring \( z = (z_1, ..., z_n) \) is computed by

\[
z_i = x_i + \alpha (y_i - x_i)
\]

where \( \alpha \) is chosen uniform randomly in \([-0.25, 1.25]\). Each variable in the offspring is the result of combining the variables in the parents according to (2). Line recombination can generate any point on the line defined by the parents within the limit of the perturbation, \( \alpha \), for a recombination in two variables. This operator can overcome limitations in variables decision and help improve in exploration during recombination.

### 3.3 Mutation

The mutation operator arbitrarily alters one or more components, genes, of a selected chromosome so as to increase the structural variability of the population. The role of mutation in GAs is that of restoring lost or unexplored genetic material into the population to prevent the premature convergence of GA to suboptimal solutions; it insures that the probability of reaching any point in the search space is never zero. Each position of every chromosome in the population undergoes a random change according to a probability defined by a mutation rate, the mutation probability, \( p_m \) (Herrera et.al, 1998). The probability of mutating a variable is set to be inversely proportional to the number of variables (dimensions). The more dimensions one individual has the smaller the mutation probability of it will be. A mutation rate of \( 1/m \), (where \( m \) is the number of variables) produced good results for a broad class of test function. However, the mutation rate was independent of the size of the population (Mühlenbein and Schlierkamp, 1993). The mutation operator for the real coded GA uses a non-linear term for the distribution of the range of mutation applied to gene values. Real value mutation is used in this work.

### 3.4 The Fitness Function

In this study, minimum mean square error is used as a fitness function of the algorithm, while number of generations is used as stopping criterion. The fitness function, \( X \), is set to minimize (3), in such a way that it approaches zero;

\[
X = \min \left\{ \frac{1}{n} \sum_{i=1}^{n} \left( y(i) - \hat{y}(i) \right)^2 \right\}
\]

where \( y(i) \) is the actual system output subjected to a disturbance signal, \( \hat{y}(i) \) is the response of the estimated system under the same disturbance, and \( i=1,2,\ldots,n \); \( n \) is total number of input/output sample pairs. The algorithm of all executions predefined a maximum number of generations as stopping criteria.

### 3.5 Values of Real-coded Genetic Parameters

The real-coded GA parameters used are presented in Table 1.

<table>
<thead>
<tr>
<th>RCGA Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Size</td>
<td>100</td>
</tr>
<tr>
<td>Selection rate</td>
<td>0.9</td>
</tr>
<tr>
<td>( P_{m_{\text{max}}}, P_{m_{\text{min}}} )</td>
<td>0.67</td>
</tr>
<tr>
<td>( P_{m_{\text{max}}}, P_{m_{\text{min}}} )</td>
<td>( 1/n ) ((n=\text{no of variables}))</td>
</tr>
<tr>
<td>Selection Method</td>
<td>SUS</td>
</tr>
<tr>
<td>Crossover Method</td>
<td>Line Recombination</td>
</tr>
<tr>
<td>Mutation method</td>
<td>Real-value mutation</td>
</tr>
</tbody>
</table>

### 4 PARAMETRIC SYSTEM IDENTIFICATION

The transfer function of the model used corresponds to the ARMA model structure by neglecting the noise, \( \eta \) term;

\[
\hat{y}(k) = -a_1 y(k-1) - \cdots - a_4 y(k-4) + b_0 u(k-1) + \cdots + b_3 u(k-4)
\]

In matrix form, the above equation can be written as

\[
\hat{y}(k) = \begin{bmatrix} y(k-1), y(k-2) \end{bmatrix}^T + \begin{bmatrix} b_0, b_1, b_2, b_3 \end{bmatrix}^T u(k-1), u(k-2), u(k-3), u(k-4)
\]

The first four variables are assigned to \( b_0,\ldots,b_3 \) and the next four to \( a_1,\ldots,a_4 \) as indicated in (5). Once the model is determined, the model needs to be verified to determine whether it is well enough to represent the system. Correlation tests including autocorrelation of the error, cross correlation of input-error, input*input-error are carried out to test
and validate the model. Each simulation was observed over 7000 samples of data for each set. The first five resonance frequencies of vibration of the plate found from spectral density of the predicted output of the RCGA model were 9.971 rad/s, 34.51 rad/s, 56.76 rad/s, 78.23 rad/s and 99.71 rad/s.

5 RESULTS

In order to determine appropriate model order for system model using RCGA, different model orders were tested. The results of these tests with model orders of 4 to 12 are summarized in Table 2. The results include time run, standard deviation, mean value and mean square error. The accuracy of the model, for different model orders, is presented in terms of standard deviation, mean value and MSE normalized with \(10^{-15}\), run time represented in minutes, and values averaged for each 5 runs. As noted in Table 2, a model order of 4 achieved minimum mean square error of 1.195 with the smallest standard deviation computational time, and this was thus chosen for obtaining a model of the flexible plate.

Table 2: Accuracy of model order.

<table>
<thead>
<tr>
<th>Model Order</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Deviation</td>
<td>5.825</td>
<td>6.492</td>
<td>10.10</td>
<td>7.332</td>
<td>10.86</td>
</tr>
<tr>
<td>Mean Value</td>
<td>2.208</td>
<td>2.383</td>
<td>2.996</td>
<td>3.097</td>
<td>3.939</td>
</tr>
<tr>
<td>Normal MSE</td>
<td>1.195</td>
<td>1.203</td>
<td>1.196</td>
<td>1.281</td>
<td>1.562</td>
</tr>
<tr>
<td>Time Run (min)</td>
<td>34.84</td>
<td>34.93</td>
<td>42.55</td>
<td>42.13</td>
<td>43.02</td>
</tr>
</tbody>
</table>

In subsequent attempts, model order of four (4) has been used to obtain unknown parameters of RCGA model system in comparison to binary coded genetic algorithm (BCGA). In BCGA, the design parameters are similar to those in RCGA with single point crossover and mutation rate of 0.0001. For RCGA, the time-domain and frequency-domain results with random disturbance are shown in Figure 2 and Figure 3 respectively. Both figures show agreement between the actual and predicted output in modelling the plate. The normalized error between the two outputs as depicted in Figure 4 is reasonably small. The corresponding correlation test results are shown in Figure 5 using random signals for RCGA, and these are in general within the 95% confidence level. Thus, this confirms the accuracy of the model in representing the dynamic behaviour of the plant system.

Small or less significant parameter variations with BCGA indicate convergence to local minima and/or pre-mature convergence. The MSE values achieved after 500/1000 generations (Figure 6 – Figure 8) with BCGA and RCGA are shown in Table 3. RCGA achieved faster convergence compared to BCGA. The RCGA achieved better convergence than BCGA over 500 generations or less.

Table 3: Mean squared output error with the Gas.

<table>
<thead>
<tr>
<th>Algorithm (Generation)/Disturbance</th>
<th>Mean Squared Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCGA (500)</td>
<td>9.51350 1.83940</td>
</tr>
<tr>
<td>RCGA (1000)</td>
<td>9.51070 1.84130</td>
</tr>
<tr>
<td>BCGA (500)</td>
<td>12.02200 4.41720</td>
</tr>
</tbody>
</table>
Figure 5: Correlation validation tests (a) – (e).

Figure 6: Convergence with random signal.

Figure 7: Convergence with PRBS Signal.

Figure 8: Convergence with step signal.

(recommended about 350) with all the test signals. It was noted that a larger number of generations did not improved the convergence rate, but took more time to compute. Figures 9 and 10 show the convergence of parameter estimates with RCGA as compared to BCGA.

The estimated system model parameters \([a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3]\) with the tested disturbance signals at the end of 500 generations with RCGA and BCGA are shown below.

i) Random disturbance

RCGA: \([0.07336, 0.1579, 0.1716, 0.07099, 1, 0.5824, -1, 0.392]\).

BCGA: \([-0.375, 0.6445, -0.107, 0.1354, 0.9176, 0.5837, -0.7937, 0.2734]\).

ii) PRBS

RCGA: \([0.1355, -0.2193, 0.3892, -0.2897, 1, 0.6084, -1, 0.3739]\).

BCGA: \([-0.5263, 0.3177, 0.0453, 0.3203, 1, 0.3285, -0.9275, 0.5801]\).

iii) Finite duration step

RCGA: \([0.1850, -0.0002, -0.5244, 0.3418, 1, 0.4964, -0.0352, -0.4639]\).

BCGA: \([-0.0576, 0.7715, -0.9993, 0.3186, 0.4695, 0.3206, 0.4653, -0.4607]\).

Figure 11 shows the MSE (in 10^-4) and associated computer run time (in hours) for convergence with RCGA and BCGA. It is noted that in general the RCGA required less computing time as well as achieved lower MSE values as compared to BCGA.
6 CONCLUSIONS

Parametric modelling of a flexible plate system has been carried out. Real-coded GA has been used for estimation of order and parameters of the model characterising the dynamic behaviour of the plate system. The approach has been evaluated in comparison to equivalent binary-coded GAs with three different test signals. It is noted that the models obtained with RCGA have performed better in characterising the system in comparison to those obtained with BCGA.

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REFERENCES


