SUBOPTIMAL DUAL CONTROL ALGORITHMS FOR DISCRETE-TIME STOCHASTIC SYSTEMS UNDER INPUT CONSTRAINT

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Abstract: The paper considers a suboptimal solution to the dual control problem for discrete-time stochastic systems under the amplitude-constrained control signal. The objective of the control is to minimize the two-step quadratic cost function for the problem of tracking the given reference sequence. The presented approach is based on the MIDC (Modified Innovation Dual Controller) derived from an IDC (Innovation Dual Controller) and the TSDSC (Two-stage Dual Suboptimal Control. As a result, a new algorithm, i.e. the two-stage innovation dual control (TSIDC) algorithm is proposed. The standard Kalman filter equations are applied for estimation of the unknown system parameters. Example of second order system is simulated in order to compare the performance of proposed control algorithms. Conclusions yielded from simulation study are given.

1 INTRODUCTION

The problem of the optimal control of stochastic systems with uncertain parameters is inherently related with the dual control problem where the learning and control processes should be considered simultaneously in order to minimize the cost function. In general, learning and controlling have contradictory goals, particularly for the finite horizon control problems. The concept of duality has inspired the development of many control techniques which involve the dual effect of the control signal. They can be separated in two classes: explicit dual and implicit dual (Bayard and Eslami, 1985). Unfortunately, the dual approach does not result in computationally feasible optimal algorithms. A variety of suboptimal solutions has been proposed, for example: the innovation dual controller (IDC) (R. Milito and Cadorin, 1982) and its modification (MIDC) (Królikowski and Horla, 2007), the two-stage dual suboptimal controller (TSDSC) (Maitelli and Yoneyama, 1994) or the pole-placement (PP) dual control (N.M. Filatov and Keuchel, 1993).

Other controllers like minimax controllers (Sebald, 1979), Bayes controllers (Sworder, 1966), MRAC (Model Reference Adaptive Controller) (Åström and Wittenmark, 1989), LQG controller where unknown system parameters belong to a finite set (D. Li and Fu,) or Iteration in Policy Space (IPS) (Bayard, 1991) are also possible.

The IPS algorithm and its reduced complexity version were proposed by Bayard (Bayard, 1991) for a general nonlinear system. In this algorithm the stochastic dynamic programming equations are solved forward in time ,using a nested stochastic approximation technique. The method is based on a specific computational architecture denoted as a H block. The method needs a filter propagating the state and parameter estimates with associated covariance matrices.

In (Królikowski, 2000), some modifications including input constraint have been introduced into the original version of the IPS algorithm and its performance has been compared with MIDC algorithm.

In this paper, a new algorithm, i.e. the two-stage innovation dual control (TSIDC) algorithm is proposed which is the combination of the IDC approach and the TSDSC approach. Additionally, the amplitude constraint of control input is taken into consideration for algorithm derivation.

Performance of the considered algorithms is il-

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lustrated by simulation study of second-order system with control signal constrained in amplitude.

2 CONTROL PROBLEM FORMULATION

Consider a discrete-time linear single-input single-output system described by ARX model

$$A(q^{-1})y_k = B(q^{-1})u_k + w_k, \tag{1}$$

where $A(q^{-1}) = 1 + a_{1,k}q^{-1} + \cdots + a_{na,k}q^{-na}$, $B(q^{-1}) = b_{1,k}q^{-1} + \cdots + b_{nb,k}q^{-nb}$, y_k is the output available for measurement, u_k is the control signal, $\{w_k\}$ is a sequence of independent identically distributed gaussian variables with zero mean and variance σ_w^2. Process noise w_k is statistically independent of the initial condition y_0. The system (1) is parametrized by a vector θ_k containing $na+nb$ unknown parameters $\{a_{i,k}\}$ and $\{b_{i,k}\}$ which in general can be assumed to vary according to the equation

$$\underline{\theta}_{k+1} = \Phi\underline{\theta}_k + \underline{e}_k \tag{2}$$

where Φ is a known matrix and $\{\underline{e}_k\}$ is a sequence of independent identically distributed gaussian vector variables with zero mean and variance matrix R_e. Particularly, for the constant parameters we have

$$\underline{\theta}_{k+1} = \underline{\theta}_k = \underline{\theta} = (b_1, \cdots, b_{nb}, a_1, \cdots a_{na})^T, \tag{3}$$

and then $\Phi = I$, $\underline{e}_k = 0$ in (2).

The control signal is subjected to an amplitude constraint

$$|\, u_k \,| \leq \alpha \tag{4}$$

and the information state I_k at time k is defined by

$$I_k = [y_k, ..., y_1, u_{k-1}, ..., u_0, I_0] \tag{5}$$

where I_0 denotes the initial conditions.

An admissible control policy Π is defined by a sequence of controls $\Pi = [u_0, ..., u_{N-1}]$ where each control u_k is a function of I_k and satisfies the constraint (4). The control objective is to find a control policy Π which minimizes the following expected cost function

$$J = E[\sum_{k=0}^{N-1}(y_{k+1} - r_{k+1})^2] \tag{6}$$

where $\{r_k\}$ is a given reference sequence. An admissible control policy minimizing (6) can be labelled by CCLO (Constrained Closed-Loop Optimal) in keeping with the standard nomenclature, i.e. $\Pi^{\text{CCLO}} = [u_0^{\text{CCLO}}, ..., u_{N-1}^{\text{CCLO}}]$. This control policy has no closed form, and control policies presented in the following section can be viewed as a suboptimal approach to the Π^{CCLO}.

3 SUBOPTIMAL DUAL CONTROL ALGORITHMS

In this section, we shall briefly describe algorithms being an approximate solution to the problem formulated in Section 2. To this end, the method for estimation of system parameters $\underline{\theta}_k$ is needed.

3.1 Estimation Method

The system (1) can be expressed as

$$y_{k+1} = \underline{s}_k^T \underline{\theta}_{k+1} + w_{k+1} \tag{7}$$

where

$$\begin{aligned} \underline{s}_k &= (u_k, u_{k-1}, \ldots, u_{k-nb+1}, -y_k, \ldots, -y_{k-na+1})^T = \\ &= (u_k, \underline{s}_k^{*T})^T. \end{aligned} \tag{8}$$

The estimates $\hat{\underline{\theta}}_k$ needed to implement dual control algorithms can be obtained in many ways. A common way is to use the standard Kalman filter in a form of suitable recursive procedure for parameter estimation, i.e.

$$\hat{\underline{\theta}}_{k+1} = \Phi\hat{\underline{\theta}}_k + \underline{k}_k\varepsilon_k \tag{9}$$

$$\underline{k}_k = \Phi P_k \underline{s}_{k-1}[\underline{s}_{k-1}^T P_k \underline{s}_{k-1} + \sigma_w^2]^{-1} \tag{10}$$

$$P_{k+1} = [\Phi - \underline{k}_k \underline{s}_{k-1}^T] P_k \Phi^T + R_e, \tag{11}$$

$$\varepsilon_k = y_k - \underline{s}_{k-1}^T \hat{\underline{\theta}}_k, \tag{12}$$

where ε_{k+1} is the innovation which will be used later on to construct the suboptimal dual control algorithm.

The following partitioning is introduced for parameter covariance matrix P_k

$$P_k = \begin{bmatrix} p_{b_1,k} & \underline{p}_{b_1\underline{\theta}^*,k}^T \\ \underline{p}_{b_1\underline{\theta}^*,k} & P_{\underline{\theta}^*,k} \end{bmatrix} \tag{13}$$

corresponding to the partition of $\underline{\theta}_k$

$$\underline{\theta}_k = (b_{1,k}, \underline{\theta}_k^{*T})^T \tag{14}$$

with

$$\underline{\theta}_k^* = (b_{2,k}, \ldots, b_{nb,k}, a_{1,k}, \ldots, a_{na,k})^T. \tag{15}$$

3.2 Two-stage Dual Suboptimal Control (TSDSC) Algorithm

The TSDSC method proposed in (Maitelli and Yoneyama, 1994) has been derived for system (1) with stochastic parameters (2). Below this method is extended for the input-constrained case. The cost function considered for TSDSC is given by

$$J = \frac{1}{2}E[(y_{k+1} - r)^2 + (y_{k+2} - r)^2 | I_k] \tag{16}$$

and according to (Maitelli and Yoneyama, 1994) can be obtained as a quadratic form in u_k and u_{k+1}, i.e.

$$J = \frac{1}{2}[au_k + bu_{k+1} + cu_k u_{k+1} + du_k^2 + eu_{k+1}^2] \quad (17)$$

where a,b,c,d,e are expressions depending on current data $\underline{s}_k^*$, reference signal r and parameter estimates $\hat{\underline{\theta}}_k$ (Maitelli and Yoneyama, 1994). Solving a necessary optimality condition the unconstrained control signal is

$$u_k^{\text{TSDSC,un}} = \frac{bc - 2ae}{4de - c^2}. \quad (18)$$

This control law has been taken for simulation analysis in (Maitelli and Yoneyama, 1994). Imposing the cutoff the constrained control signal is

$$u_k^{\text{TSDSC,co}} = sat(u_k^{\text{TSDSC,un}}; \alpha). \quad (19)$$

The cost function (27) can be represented as a quadratic form

$$J = \frac{1}{2}[\underline{u}_k^T A \underline{u}_k + \underline{b}^T \underline{u}_k] \quad (20)$$

where $\underline{u}_k = (u_k, u_{k+1})^T$, and

$$A = \begin{bmatrix} d & \frac{1}{2}c \\ \frac{1}{2}c & e \end{bmatrix}, \underline{b} = \begin{bmatrix} a \\ b \end{bmatrix}. \quad (21)$$

The condition $4de - c^2 > 0$ together with $d > 0$ implies positive definitness and guarantees convexity. Minimization of (30) under constraint (4) is a standard QP problem resulting in $\underline{u}_k^{\text{TSDSC,qp}}$. The constrained control $u_k^{\text{TSDSC,qp}}$ is then applied to the system in receding horizon framework.

3.3 Two-stage Innovation Dual Suboptimal Control (TSIDSC) Algorithm

A modified version of the TSDSC algorithm is given below where innovation term is included to the cost function

$$J = \frac{1}{2}E[(y_{k+1} - r)^2 + (y_{k+2} - r)^2 - \lambda_{k+1}\varepsilon_{k+1}^2 | I_k] \quad (22)$$

where $\lambda_{k+1} \geq 0$ is the learning weight, and ε_{k+1} is the innovation, see (16). Incorporating the term $-\lambda_{k+1}\varepsilon_{k+1}^2$ in the cost function makes the parameter estimation process to accelerate and consequently to improve the future control performance. Taking (2) and (7) into account it can be seen that

$$\varepsilon_{k+1} = \underline{s}_k^T[\Phi(\underline{\theta}_k - \hat{\underline{\theta}}_k) + (\Phi - I)\hat{\underline{\theta}}_k] + \underline{s}_k^T \underline{e}_k + w_{k+1}, \quad (23)$$

and consequently

$$\begin{aligned} E[\varepsilon_{k+1}^2 | I_k] &= \underline{s}_k^T \Phi P_k \Phi^T \underline{s}_k + \underline{s}_k^T (\Phi - I)\hat{\underline{\theta}}_k \hat{\underline{\theta}}_k^T (\Phi - I)^T + \\ &+ \underline{s}_k^T R_e \underline{s}_k + \sigma_w^2 = \\ &= \underline{s}_k^T[\Phi P_k \Phi^T + (\Phi - I)\hat{\underline{\theta}}_k \hat{\underline{\theta}}_k^T (\Phi - I)^T + R_e]\underline{s}_k + \sigma_w^2 = \\ &= \underline{s}_k^T \Sigma_k \underline{s}_k + \sigma_w^2. \end{aligned} \quad (24)$$

Introducing the partitioning for matrix Σ_k

$$\Sigma_k = \begin{bmatrix} \sigma_{11,k} & \underline{\sigma}_{1,k}^T \\ \underline{\sigma}_{1,k} & \Sigma_k^* \end{bmatrix}. \quad (25)$$

Keeping (8) in mind we have

$$E[\varepsilon_{k+1}^2 | I_k] = f u_k^2 + g u_k + h, \quad (26)$$

where $f = \sigma_{11,k}$, $g = \underline{\sigma}_{1,k}^T \underline{s}_k^*$, $h = \underline{s}_k^{*T} \Sigma_k^* \underline{s}_k^* + \sigma_w^2$ are expressions known at the current time instant k.

Finally, the cost function J including terms depending only on u_k and u_{k+1} takes the form

$$\begin{aligned} J = \frac{1}{2}[&au_k + bu_{k+1} + cu_k u_{k+1} + du_k^2 + eu_{k+1}^2 - \\ &-\lambda_{k+1}(f u_k^2 + g u_k)] \end{aligned} \quad (27)$$

Solving a necessary optimality condition the unconstrained control signal is

$$u_k^{\text{TSIDSC,un}} = \frac{bc - 2ae - 2eg}{4de - c^2 - 4ef\lambda_{k+1}}. \quad (28)$$

Imposing the cutoff the constrained control signal is

$$u_k^{\text{TSIDSC,co}} = sat(u_k^{\text{TSIDSC,un}}; \alpha). \quad (29)$$

The cost function (27) can again be represented as a quadratic form

$$J = \frac{1}{2}[\underline{u}_k^T A \underline{u}_k + \underline{b}^T \underline{u}_k] \quad (30)$$

where $\underline{u}_k = (u_k, u_{k+1})^T$, and correspondingly to (21)

$$A = \begin{bmatrix} d - \lambda_{k+1} f & \frac{1}{2}c \\ \frac{1}{2}c & e \end{bmatrix}, \underline{b} = \begin{bmatrix} a - \lambda_{k+1} g \\ b \end{bmatrix}. \quad (31)$$

The weight λ_{k+1} has influence on positive definitness of the quadratic form. Minimization of (30) under constraint (4) is again the QP problem resulting in $\underline{u}_k^{\text{TSIDSC,qp}}$. The constrained control $u_k^{\text{TSIDSC,qp}}$ is then applied to the system in receding horizon framework.

4 SIMULATION TESTS

Performance of the described control methods is illustrated through the example of a second-order system with the following true values: $a_1 = -1.8$, $a_2 = 0.9$, $b_1 = 1.0$, $b_2 = 0.5$, where the Kalman filter algorithm

(9)-(12) was applied for estimation. The initial parameter estimates were taken half their true values with $P_0 = 10I$. The reference signal was a square wave ± 3, and then the minimal value of constraint α ensuring the tracking is $\alpha_{min} = 3\frac{|A(1)|}{|B(1)|} = 0.2$. Fig. 1 shows the reference, output and input signals during tracking process under the constraint $\alpha = 1$ for both TSDSC and TSIDSC control policies. The controls $u_k^{\mathrm{TSDSC,qp}}$ and $u_k^{\mathrm{TSIDSC,qp}}$ were obtained solving the minimization of quadratic forms (20), (31) using MATLAB function *quadprog*. The performance of these control algorithms is surprisingly essentially inferior with respect to $u_k^{\mathrm{TSDSC,co}}$ and $u_k^{\mathrm{TSIDSC,co}}$. On the other hand, as expected, the control $u_k^{\mathrm{TSIDSC,co}}$ performs better than $u_k^{\mathrm{TSDSC,co}}$.

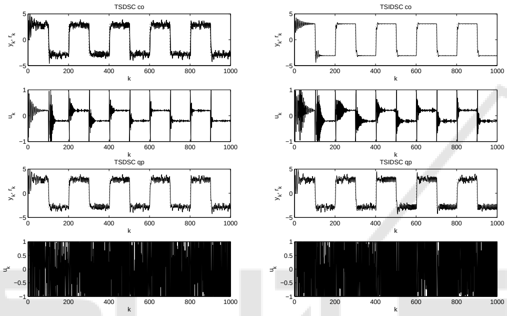

Figure 1: Reference, output and control signals for TSDSC, TSIDSC; $\alpha = 1$; constant parameters.

For the control policy Π^{TSIDSC} the constant learning weight was $\lambda_k = \lambda = 0.98$.

The performance index

$$\bar{J} = \sum_{k=0}^{N-1} (y_{k+1} - r_{k+1})^2$$

was considered for simulations. The plots of $\bar{J}$ versus the constraint α are shown in Fig.2 for $\sigma_w^2 = 0.05$, and $N = 1000$.

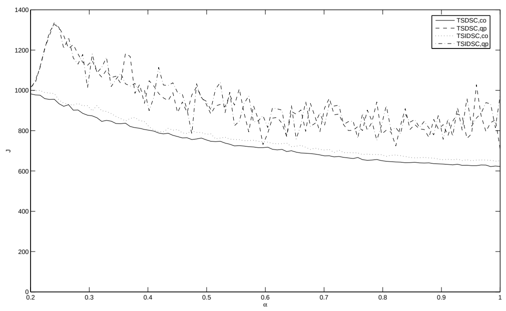

Figure 2: Plots of performance indices for TSDSC, TSIDSC.

In the case of varying parameters (2), $\Phi = I$ and $R_e = 0.05I$ have been taken. Fig.3 shows the performance of the tracking process under the constraint $\alpha = 1$ for both TSDSC and TSIDSC control policies. An examplary run of parameter estimates is shown in Figs.4,5 for control policies $\mathrm{TSIDSC,co}$ and $\mathrm{TSIDSC,qp}$, respectively.

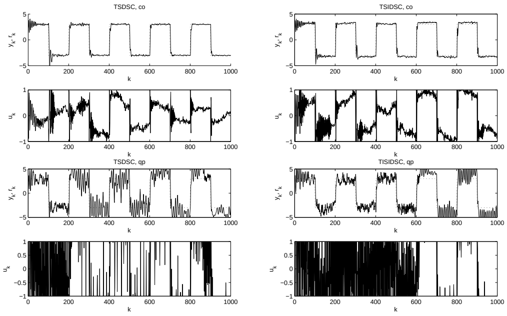

Figure 3: Reference, output and control signals for TSDSC, TSIDSC; $\alpha = 1$; varying parameters.

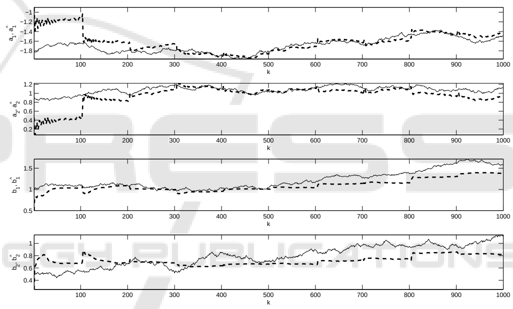

Figure 4: Parameter estimates for $\mathrm{TSIDSC,co}$.

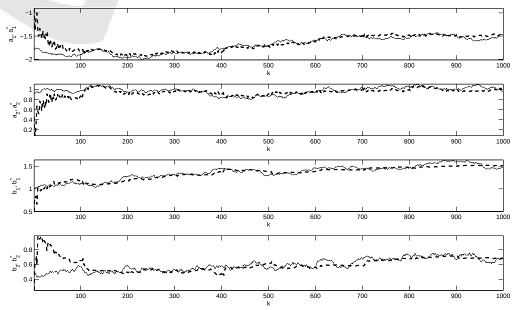

Figure 5: Parameter estimates for $\mathrm{TSIDSC,qp}$.

5 CONCLUSIONS

This paper presents a problem of discrete-time dual control under the amplitude-constrained control signal. A simulation example of second-order system is given and the performance of the presented two control policies is compared by means of the simulated performance index.

A new control policy TSIDSC was proposed as suboptimal dual control approach. The method exhibits good tracking properties for both constant and time-varying unknown system parameters.

It was found that both control policies $u_k^{\mathrm{TSDSC,qp}}$ and $u_k^{\mathrm{TSIDSC,qp}}$ derived via QP optimization do not yield a tracking improvement compared to the cut-off controls $u_k^{\mathrm{TSDSC,co}}$ and $u_k^{\mathrm{TSIDSC,co}}$.

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