ON THE USE OF SYNTACTIC POSSIBILISTIC FUSION FOR COMPUTING POSSIBILISTIC QUALITATIVE DECISION

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Abstract: This paper describes the use of syntactical data fusion to computing possibilistic qualitative decisions. More precisely qualitative possibilistic decisions can be viewed as a data fusion problem of two particular possibility distributions (or possibilistic knowledge bases): the first one representing the beliefs of an agent and the second one representing the qualitative utility. The proposed algorithm computes a pessimistic optimal decisions based on data fusion techniques. We show that the computation of optimal decisions is equivalent to computing an inconsistency degree of possibilistic bases representing the fusion of agent’s beliefs and agent’s preferences.

1 INTRODUCTION

This paper presents a computation of pessimistic decisions based on syntactic possibilistic fusion operations. Qualitative possibilistic decisions can be viewed as a data fusion problem of two particular possibility distributions: the first one representing the beliefs of an agent and the second one representing the qualitative utility (or agent’s preferences). A possibilistic decision model (Dubois and Prade, 1995) allows a gradual expression of both agent’s preferences and knowledge. The preferences and the available knowledge about the world are expressed in ordinal way. In (Dubois and Prade, 1995), the authors have proposed two qualitative criteria for ordinal decision approaches under uncertainty: the pessimistic and the optimistic decisions criteria. The first one being more cautious than the second one for computing optimal decisions.

A method for computing optimal decisions, based on ATMS, has been proposed in (Dubois et al., 1998). Using the pessimistic criteria, the procedure is translated to a problem tractable by an ATMS (Kleer, 1986a)(Kleer, 1986b). In (Berre, 2000), Le Berre has implemented the optimistic algorithm and the pessimistic one. This implementation can not deal with an important number of variables (Berre, 2000).

The rest of this paper is organized as follow. Section 2 gives a brief backgrounds on possibilistic logic, qualitative decision and data fusion in possibilistic logic. Section 3 contains an efficient and unified way of computing pessimistic qualitative decisions based on syntactic counterpart of data fusion problem. Section 4 concludes the paper.

2 BACKGROUNDS

2.1 Possibilistic Logic

This section gives a brief refresher on possibilistic logic and qualitative decision theory. See (Dubois et al., 1994b) for more details on possibilistic logic. A possibility distribution (Dubois et al., 1994b) \( \pi \) is a mapping from a set of interpretations \( \Omega \) into the unit interval \( [0,1] \). \( \pi(\omega) \) represents the degree of compatibility of the interpretation \( \omega \) with available pieces of information.

Given a possibility distribution \( \pi \), two dual measures are defined on the set of propositional formulas:

- The possibility measure of a formula \( \phi \), defined by:
  \[ \Pi(\phi) = \max\{\pi(\omega) : \omega \models \phi \text{ and } \omega \in \Omega\} \]
The pessimistic utility function is expressed in terms of combining possibility distributions associated with a possibilistic knowledge base (Dubois et al., 1994a): 

\[ \pi_{\phi_i}(\omega) = \begin{cases} 1 - \alpha_i & \text{if } \omega \not\models \phi_i \\ 1 & \text{otherwise} \end{cases} \]  

(3)

More generally, the possibility distribution associated with a weighted formula \( (\phi_i, \alpha_i) \) is (Dubois et al., 1994b): 

\[ \pi_{\phi_i}(\omega) = \bigoplus \{ \pi_{(\phi_i, \alpha_i)}(\omega) : (\phi_i, \alpha_i) \in \Sigma \} \]  

(4)

where \( \bigoplus \) is in general either equal to the minimum operator (in standard possibilistic logic), or to the product operator (*). 

2.2 Qualitative Decision

Let \( D = \{ l_i \} \) be a set of decision variables, where \( l_i \) are distinguished variables of the language L. Let \( d \subseteq D \), then the decision \( d^\wedge \) is the logical conjunction of literals in the chosen subset. Each set of decision \( d \) induces a possibility distribution \( \pi_{K_d} \) in the following way (Dubois et al., 1994a):

\[ \pi_{K_d}(\omega) = \begin{cases} 1 & \text{if } \forall (\phi_i, \alpha_i) \in K, \omega \models \phi_i \text{ and } \omega \models d^\wedge \\ \min_{(\phi_i, \alpha_i) \in K \cup \{ \{d, 1\} \} } (1 - \alpha_i) & \text{if } \omega \not\models d^\wedge \\ 0 & \text{otherwise} \end{cases} \]  

Where \( \alpha_i \) represents the degrees of necessity of the formulas in the corresponding layers of \( K \cup \{ \{d, 1\} \} \). 

The utility function \( \mu \) is built over \( \Omega \) in a similar way:

\[ \mu(\omega) = \begin{cases} 1 & \text{if } \forall (\psi_j, \beta_j) \in P, \omega \models \psi_j \\ \min_{\psi_j, \beta_j \in P} (1 - \beta_j) & \text{otherwise} \end{cases} \]  

where \( \beta_j \) represents a degree of priority of a formulas in \( P \). 

Making a decision amounts to choosing a subset \( d \) of the decision set \( D \). The objective is to rank-order decisions on the basis of \( K \) and \( P \). The pessimistic utility function is expressed in terms of the possibility distribution \( \pi_{K_d} \) and the utility function \( \mu \) (Dubois et al., 1999):

\[ u_s(d) = \min_{\omega \in \Omega} \max(1 - \pi_{K_d}(\omega), \mu(\omega)) \]  

(5)

In the pessimistic case, the decision \( d \) must satisfy (Dubois et al., 1997):

\[ K^\wedge \cap d^\wedge + P^\wedge_{(1-\alpha)} \]  

(6)

The decision \( d \) associated with the most certain part of \( K \) entails the satisfaction of the goals, even those with low priority. The pessimistic utility \( u_s \) of decision \( d \), defined at the syntactic level, takes the form (Dubois et al., 1997):

\[ u_s(d) = \begin{cases} \max_{\omega} K^\wedge \cap d^\wedge + P^\wedge_{(1-\alpha)} \wedge K^\wedge \cap d^\wedge \neq \bot & 0 \\ 0 & \text{if } \{ K^\wedge \cap d^\wedge + P^\wedge_{(1-\alpha)} \wedge K^\wedge \cap d^\wedge = \bot \} \end{cases} = \emptyset \]  

2.3 Fusion in Possibilistic Logic

Let \( \Sigma_1, \Sigma_2 \) be two possibilistic bases and \( \pi_1, \pi_2 \) be their associated possibilistic distributions. Let \( \oplus \) be a two-place function whose domain is \( [0,1] \times [0,1] \), to be used for aggregating \( \pi_1 \) and \( \pi_2 \). The only requirements for \( \oplus \) are the following properties (Benferhat et al., 1997):

• \( 1 \oplus 1 = 1 \),
• \( \text{if } \forall \omega, \omega' \text{ if } \pi_1(\omega) \geq \pi_1(\omega') \text{ and } \pi_2(\omega) \geq \pi_2(\omega'), \text{ then } \pi_1(\omega) \oplus \pi_2(\omega) \geq \pi_1(\omega') \oplus \pi_2(\omega') \).

The syntactic counterpart of the fusion of \( \pi_1 \) and \( \pi_2 \) is the following possibilistic base, denoted by \( \Sigma = \Sigma_1 \oplus \Sigma_2 \), which is made of the union of:

• the initial bases, however with new necessity degrees defined by :
  \( \{ (\phi_i, 1 - (1 - \alpha_i) \oplus 1) : (\phi_i, \alpha_i) \in \Sigma_1 \} \cup \{ (\psi_j, 1 - (1 - \beta_j) \oplus 1) : (\psi_j, \beta_j) \in \Sigma_2 \} \)
• and the knowledge common to \( \Sigma_1 \) and \( \Sigma_2 \) defined by :
  \( \{ (\phi_i \vee \psi_j, 1 - (1 - \alpha_i) \oplus (1 - \beta_j)) : (\phi_i, \alpha_i) \in \Sigma_1 \text{ and } (\psi_j, \beta_j) \in \Sigma_2 \} \)

The conjunctive operators exploit the symbolic complementarities between sources. \( \oplus \) is said to be a conjunctive operator if \( \forall a \in [0,1], \oplus(a, a) = \oplus(1, a) = a \).

The operator minimum (\( \min \)) is an idempotent conjunctive one. At the syntactic level, the base associated to \( \pi_{\min} \), such that \( \pi_{\min}(\omega) = \min(\pi_1(\omega), \pi_2(\omega)) \), is \( \Sigma_{\min} = \Sigma_1 \cup \Sigma_2 \) (Benferhat et al., 1997);
3 COMPUTATION OF QUALITATIVE PESSIMISTIC OPTIMAL DECISIONS BASED ON DATA FUSION TECHNICAL

A good pessimistic decision $d$ maximizing $u_s(d)$ is such that:

$$u_s(d) = \min_{\omega \in \Omega} \max(1 - \pi_{K_d}(\omega), \mu(\omega))$$

which is equivalent to:

$$u_s(d) = 1 - \max_{\omega \in \Omega} \min(\pi_{K_d}(\omega), 1 - \mu(\omega))$$

Besides, the syntactic counterpart of $\min(\pi_{K_d}(\omega), 1 - \mu(\omega))$ is the possibilistic base $\Sigma_{\min} = \Sigma_1 \cup \Sigma_2$. Thus, combining these results, the corresponding base $\Sigma_{\min}$ associated to $\min(\pi_{K_d}(\omega), 1 - \mu(\omega))$ is the possibilistic base $K \cup n_p \cup \{(d, 1)\}$, such that $n_p$ is the possibilistic base corresponding to the utility function $1 - \mu(\omega)$.

3.1 Transformations Steps

In this subsection, we define the possibilistic base $n_p$ corresponding to the utility function $1 - \mu(\omega)$, from the preferences base $P$.

Let $P = \{(\phi, \alpha) : i = 1, \ldots, n\}$ be a preferences base. We assume that: $\alpha_0 = 0 < \alpha_1 < \ldots < \alpha_n$. The following definition gives the possibilistic knowledge base associated with the negation of $P$.

Definition 1. The negated base of $P = \{(\phi, \alpha) : i = 1, \ldots, n\}$ is a possibilistic base, denoted by $n_p$, and defined by:

$$n_p = \{(d_i, 1 - \alpha_i) : i = 1, \ldots, n\} \cup \{(\bot, 1 - \alpha_n)\}$$

where $d_i = \phi_1 \lor \neg \phi_2 \lor \ldots \lor \neg \phi_n$.

The following proposition shows that $n_p$ is indeed encodes the negation of $P$.

Proposition 1. Let $P = \{(\phi, \alpha) : i = 1, \ldots, n\}$ be a preference base, and $n_p$ its negated base obtained using definition 7. Let $\mu_p$ and $\mu_n$ be the utility distributions associated with $P$ and $n_p$ respectively. Then:

$$\forall \omega \in \Omega, \mu_P(\omega) = 1 - \mu_n(\omega)$$

The obtained base $n_p$ must be put in clausal form. So, we get $C_{np}$.

If $\alpha_n$ is different to 1, then the utility function $1 - \mu(\omega)$ is not normalized. In this case, it will be necessary to add contradiction to the possibilistic base $C_{np}$ with priority degree $1 - \alpha_n$. Let $C'_{np}$ be this base. Then $C_{np}$ and $C'_{np}$ are equivalents.

Lemma 1. Let $\Sigma = \{(\phi, \alpha), i = 1, n\}$ be a preferences base and let $\alpha_1, \ldots, \alpha_n$ be the distinct valuations appearing in $\Sigma$, ranked increasingly: $0 \leq \alpha_1 \leq \cdots \leq \alpha_n \leq 1$ and let $\mu(\omega)$ be the utility function associated to the preferences base $\Sigma$. Let $\mathcal{N}_\Sigma = \{(\psi, \beta), i = 1, n\}$ be the preferences base associated to the utility function $\mu_{inc}(\omega) = 1 - \mu(\omega)$, where $\mathcal{N}_\Sigma = n_{\Sigma} \cup \{(\bot, 1 - \alpha_n)\}$ is equivalent to $n_{\Sigma}$. We have:

$$\forall \omega \in \Omega, \mu_{inc}(\omega) = \mu_{\mathcal{N}_\Sigma}(\omega).$$

3.2 Computation of Pessimistic Decisions

We recall that:

$$u_s(d) = 1 - \max_{\omega \in \Omega} \min(\pi_{\Sigma_{\min}}(\omega), 1 - \mu(\omega))$$

where $\pi_{\Sigma_{\min}}(\omega) = \min(\pi_{K_d}(\omega), 1 - \mu(\omega))$ and $\Sigma_{\min} = K \cup n_p \cup \{(d, 1)\}$.

On the other hand, the inconsistency degree of a possibilistic base $K$, $Inc(K)$ is defined as follow (Dubois et al., 1994b):

$$Inc(K) = 1 - \max\{\pi_{K_d}(\omega)\}$$

Proposition 2. Then clearly, the pessimistic utility function associated to decision $d$ is:

$$u_s(d) = Inc(\Sigma_{\min})$$

Where $Inc(\Sigma_{\min})$ represents the inconsistency degree of the base $K \cup n_p \cup \{(d, 1)\}$.

Then, the computation of optimal pessimistic decisions is obtained using the following algorithm.

Algorithm: Computation of Optimal Pessimistic Decisions

Input: $K$: knowledge base,
$n_p$: renews preferences base,
$N$: number of decision variables,
$D$: set of decisions,

Output: Decision: optimal decisions,

Begin

$i := 1;$
$max := 0;$
$Inc := 1;$

For $i = 1$ to $N$

Begin

$Inconsi(K \cup n_p \cup \{(d_i, 1)\}, Inc, bool);$ /s $d_i \in D^o$

if (bool=true) then

if $Inc > max$ then

$max := Inc;$

Decision := $\{d_i\};$

else

End
The computation of inconsistency degree is performed by a call to the function Inconsi \( B \cup \{¬φ, 1\} \), Inc, bool). This function has three parameters: a stratified knowledge base, an integer representing current inconsistency degree and a boolean variable. More precisely, the function Inconsi is defined as follows:

**Function Inconsi** \( B \cup \{¬φ, 1\} \), Inc, bool) **Input** :

- B: stratified base,
- φ: weighted formula,
- n: number of strate in base B,
- Output : Inc: inconsistency degree, bool: boolean,

**Begin**

\[
\begin{align*}
    l & := 0; /*initially pointed on the last strate of the base*/ \\
    u & := n; /*initially pointed on first strate of the base*/ \\
    bool & := true; \\
    \text{while} \ (l < u) \ \text{do} \\
    \begin{align*}
        r & := [(l + u)/2]; /*pointer uses for dichotomy*/ \\
        \text{if}(B^\perp_{l} \land ¬φ \ \text{consistent}) \\
        \quad \text{then} \\
        \quad \quad u := r - 1; \\
        \quad \text{if} \check{\text{check inconsistency in most big base}}*/ \\
        \text{else} \\
        \quad l := r; \\
        \quad \text{if} \check{\text{check the inconsistency delimited by u,l}}*/ \\
    \end{align*}
\]

**End**

\[
\begin{align*}
    \text{if}(\alpha_r < \text{inc}) \ \text{then} \ bool & := false; \\
    \quad \text{else Inc := α_r}; \ \text{inc} = N(φ)*/ \\
\end{align*}
\]

**End**

**4 CONCLUSIONS**

The main contribution of this paper is a proposition of a new approach to compute a qualitative pessimistic decision problem. This problem is viewed as the one of computing inconsistency degrees of particular bases in the framework of possibilistic logic. The application exploits the syntactic counterparts of data fusion techniques. Our approach avoids the use of the ATMS to compute the pessimistic optimal qualitative decision developed in (Dubois et al., 1999) which is known to be a hard problem.

**REFERENCES**


